Rebuttal of Review 1

Dear reviewer,

Below you will find our comments and responses to your review.

General Comment

This paper is useless, and introduces no novelties concerning the estimate of the RP of high-dimensional occurrences. The work does not contain any relevant advance: it simply suggests (in a non-technical and superficial way) to use well known algorithms to find the critical points of multidimensional functions.

We do not agree with your valuation of our contribution as *useless*. This paper is the first part of a two parts paper, and we consider this distinction crucial to understand the role that our work intends to play. In our experience, multidimensional statistics, and specifically extremes, is a complicated subject where, if it is true that a lot of literature exists, more often than not methodological details are missing. Our aim with this paper was twofold. First, we wanted to make sure that our methodology was fully reproducible, and for that, we wanted to make sure that all the relevant points were covered. Due to the fragmentation of much of the research on the topic, we considered to present an extended methodology that anyone could follow. The second aim, once we realized the extension of such a methodology, was to try to be as inclusive as possible and carry a literature review of the different options that exist. In this way we provide the reader with a more clear picture of the choices made in the paper, without giving the impression that our way to proceed forward was the only one.

Even considering both parts, it may seem that this paper includes more additional details than needed, and that is because there is a third paper, were we deal with other different variables, were the selection of events and the treatment of the distribution are quite different. Thus, including all the required information into this paper to make references to it from all our posterior works. In this regard, we were discouraged to submit the third paper as a third part, but we can make it available to the reviewers should you consider to evaluate if our argumentation is more understandable with all the judgement elements at hand.

In response to concerns regarding the novelty and relevance of our work, we find it essential to make clear how our research not only integrates existing literature but directly addresses one of the key limitations identified in the field: the estimation of multivariate return periods in high dimensions and the definition of the associated critical layer (or hypersurface). As noted by G. Salvadori, De Michele, and Durante (2011), as well as in recent studies (Gräler et al. (2013); Brunner, Seibert, and Favre (2016), Xu, Wang, and Bin (2023); Brunner (2023)), there is a consensus within the scientific that the mathematical and computational challenges of extending and aplying existing techniques to higher-dimensional spaces (n>3) remain underexplored. Our paper advances in this area through:

1. *Integration of existing methodological approaches for joint return periods (JRPs)*: We present a comprehensive approach that organizes and synthesizes existing methodologies, offering practical guidelines and structured steps to enhance reproducibility and applicability. Given the complexity and fragmentation of methods in the literature, this work aims to provide a well-documented and cohesive framework that integrates statistical and hydrological techniques, ensuring broader accessibility for researchers and practitioners.

2. *Definition of the multidimensional critical layer*: We propose a novel approach to define the critical hypersurface associated with compound events in high-dimensional spaces. This approach integrates advanced statistical techniques with Gaussian Process Regression (GPR), thus optimizing the computation of the critical layer and significantly reducing the associated computational cost.

3. *Reduction of computational cost*: Unlike traditional approaches that rely on intensive simulations, our methodology employs a combination of Monte Carlo simulation and GPR-based regression models. This enables the efficient computation of copula values in large datasets, making high-dimensional analysis feasible without compromising accuracy.

4. *Generalizable and applicable methodology*: Although our methodology is initially presented for analyzing the spatial dependence of rainfall regimes, its flexible and adaptable design allows for its application to different types of compound events, including multivariate, preconditioned, temporally compounding, and spatially compounding events, as classified by Zscheischler et al. (2020). Thanks to its adaptability, our approach is applicable to a broad range of environmental and climate-related challenges.

5. *Opening avenues for future research*: As detailed in our paper, once the critical layer in high-dimensional spaces is established, it opens the door for more detailed studies on the selection of design events and uncertainty analysis. We thus consider that our contribution is neither superficial nor a mere application of well-known methods. Rather, it constitutes a methodological advancement that tackles a well-documented challenge in the scientific literature while providing practical tools for implementation.

We would also like to emphasize that this work constitutes the first part of a broader investigation, with the second and third parts presenting specific results from its application to real-world case studies. This integrated and efficient approach is not only novel but also establishes a solid foundation for future research and practical applications in the analysis of high-dimensional compound events.

Not to say about the "Conceptual Framework": it is already well known in hydrological/geophysical Literature since about 20 years. All the formulas presented (sometimes inexact from a mathematical point of view) do not show anything new, nor they represent any advance with respect to the present knowledge.

We acknowledge that the "Conceptual Framework" is well known in the hydrological and geophysical literature, having provided a solid theoretical basis for the past two decades. However, our contribution does not lie in *redefining* this framework, but rather in its integration into a novel methodological approach designed to address specific challenges in high-dimensional spaces. By combining this theoretical foundation with advanced regression techniques (such as Gaussian Process Regression), efficient computational simulations, and an optimized approach for defining critical layers, we provide practical solutions that are not directly available in the existing literature.

Moreover, reproducibility is a cornerstone of scientific progress, and incorporating established formulas ensures that our work remains transparent, accessible and verifiable by the broader research community. However, what sets our contribution apart is not merely the use of these concepts but their targeted application and the significant advancements in computational efficiency. These innovations provide deeper insights into the analysis of multivariate return periods in hydrology, and represent an ongoing effort to overcome persistent limitations that have yet to be fully addressed in the field. While we build upon two decades of research, our work strives to go beyond a mere repetition of existing methods. Rather, it seeks to integrate, adapt, and optimize established approaches to to enhance the reliability of multivariate return period estimation.

In addition, the fact that the occurrences considered are Compound Events does not emerge in any part of this work: only superficial comments and descriptions are presented, but the true core problem is nowhere investigated. Furthermore, sort of indications for carrying out a multivariate analysis are sketched, but they are too generic, never discussed in details concerning the problems they are expected to solve, and most of all they are all already well known in Literature: this paper adds nothing to knowledge, all what is written has already been more precisely (and mathematically correctly) introduced in already published works, so what?

We appreciate the reviewer's comments and understand the concern about the "true core problem," which, however, seems somewhat ambiguous to us, as it is not entirely clear which specific aspect the reviewer is referring to. From our perspective, one of the many fundamental problems in this field—widely discussed in the literature— is the definition of the multidimensional critical layer and its effective application to hydrological problems involving compound events. Our research is grounded in the pivotal contributions of G. Salvadori, De Michele, and Durante (2011), Gräler et al. (2013), Brunner, Seibert, and Favre (2016), Xu, Wang, and Bin (2023), and Brunner (2023), but seeks to extend this methodological framework by addressing several key limitations:

1. *From theory to a reproducible and practical methodology:* While previous studies have emphasized the theoretical importance of multivariate return periods and critical surfaces, they often lack practical guidelines on how to define these layers in high-dimensional spaces. To the best of our knowledge, no study has developed and applied a general methodology, including vine copulas, to derive multivariate return periods beyond three-dimensions, highlighting the challenges in this field. These challenges include high computational costs associated with managing complex dependence structures and the lack of a clear methodological framework to efficiently address these issues. Our work bridges this gap by providing a detailed and reproducible procedure to identify the critical layer, integrating statistical models and computational optimizations that make its application feasible in higher-dimensional settings.

2. *Applications in hydrology and coastal systems:* While the general framework for compound events is well established, our contribution lies in its adaptation to hydrological problems with high-dimensional dependencies, such as rainfall regimes and their spatial distribution. Moreover, the proposed methodological framework may be extendable to the study of compound events in coastal and estuarine environments, where interactions between river discharge, storm surges, and extreme winds can have significant impacts on coastal dynamics and infrastructure.

3. *Challenges in high-dimensional modeling:* As noted by Brunner (2023), modeling dependence in multivariate environments is feasible in low dimensions (e.g., bivariate cases) but becomes increasingly complex and computationally demanding as the number of interdependent variables increases. Identifying suitable dependence structures in high dimensions is not always straightforward and requires more flexible approaches to simultaneously account for temporal, spatial, and variable dependencies. In fact, these aspects have been recognized in the literature as *outstanding challenges and future research directions* (Brunner 2023), further reinforcing the relevance of our work in addressing these issues through computational optimizations, including efficient Monte Carlo simulations and regression-based models, enabling scalable and accurate analyses within the critical layer framework.

4. *Compound floods with multiple drivers:* Xu, Wang, and Bin (2023) highlight that, in the context of climate change and urbanization, the interaction between extreme precipitation, storm surges, and rising sea levels will continue to intensify, making the study of compound events in coastal environments increasingly relevant. However, most studies have focused on two-dimensional scenarios (e.g., rainfall-storm surge or runoff-storm surge), while three-dimensional scenarios still present *methodological and computational challenges*. Our work contributes to close this gap by developing methodologies for identifying critical layers in *high dimensions*, allowing for a more realistic approach and improving the predictive capabilities of hydrological models .

This article represents the first part of a three-stage study. In this work, we establish the general framework and methodology for identifying the critical layer in high-dimensional compound events. The second part of this study, currently under review, focuses on applying the methodology to spatially and temporally compounding events, providing detailed numerical validations and case studies. Additionally, there is a third part of the study, which, although not submitted to this journal, applies the same methodological framework to multivariate compound events in estuarine environments, considering oceanic, hydrological and meteorological variables to analyze interactions between different extreme phenomena affecting these ecosystems.

From this perspective, our work complements and extends previous studies rather than replicating them. By making existing knowledge more accessible and applicable to hydrological, and more generally, environmental problems, we aim to contribute to the ongoing scientific effort to better understand and manage compound events in complex environments.

Finally, the mathematical notation is often wrong: e.g., the Authors confuse a function (say, F) with its value (e.g., F (x, y)), or even worse confuse F (x) with F (X), where the former is a real number, whereas the latter is a random variable.

We will revise the mathematical notation. It is true that none of the authors is a mathematician by training, and that some expressions were not rigorous. Luckily, these imprecisions pertain to the realm of wording and have not introduced any conceptual error into the methodological procedure itself.

Overall, the content of this paper can be summarized in a single paragraph, and recycled in the Introduction of the companion paper (Part II).

Again, we do not agree with you. Although we perfectly understand that highly-trained researchers in multivariate statistics may find some of the contents unnecessary and summarizable in a single paragraph, in our view, most hydrologists require more details and explanations to approach such a complex topic. For instance, the review for Part II mentions that we should not include approaches that are wrong just because practitioners use them. In our opinion, we researchers have a deontological obligation to convey our knowledge in the most comprehensible way possible. If there are professionals using these techniques wrongly, it may be because of their intellectual limitations, but the hypothesis that we favor is the one in which the problem is that nobody cared to build a better bridge for them to transit from lousy practice to rigorous application.

We stress the importance of providing an extensive and detailed presentation of the methodology to address the knowledge gap that still exists in the practical application of multivariate statistics in hydrology. While many of the concepts discussed may be familiar to experts in multivariate statistics, our goal is to present them in a clear and applicable manner for a broader audience, including readers and professionals in hydrology and coastal studies. These fields often involve complex dependencies and uncertainties, making it essential to complement theoretical knowledge with practical guidelines to ensure proper implementation.

As highlighted by studies such as G. Salvadori, De Michele, and Durante (2011) and Gräler et al. (2013), misapplications of statistical tools in practice often stem from a lack of clarity in the available literature. We see it as our responsibility to fill this gap by offering a comprehensive and well-organized framework that bridges the theoretical and practical aspects of defining multidimensional critical layers and estimating multivariate return periods. This cannot be effectively conveyed in a single paragraph without sacrificing important details that are crucial for reproducibility and understanding.

Furthermore, as this is the first part of a broader investigation, the level of detail provided here is essential to ensure that the second part (Part II) is well-supported and does not require unnecessary repetition of fundamental concepts. This structure allows for a more efficient and targeted discussion of the specific case studies and applications presented in Part II.

We hope that this explanation clarifies our perspective and illustrates the importance of the level of detail provided in this paper. We view it as a necessary step toward improving the practical application of advanced statistical methods in hydrology and addressing common sources of misinterpretation and misuse.

Specific comments

Line(s) 55–56 AUTHOR(s). Vine copulas are a flexible class of dependence models consisting of bivariate building blocks. REFEREE. The Authors do not mention the problems intrinsic to modeling via Vine

copulas.

Your comment raises an important point, as it allows us to delve deeper into some critical aspects of modeling with vine copulas. After reading both reviews, we identified that the conclusion of the paper could lead to misinterpretations by not explicitly addressing certain methodological limitations. Therefore, we have decided to revise the final sections of the paper. Instead of a *Discussion* and a *Conclusions* section, we have added a *Problems and Limitations* section and a *Concluding Remarks* section, where we discuss the technical challenges associated with the use of vine copulas and the strategies we have implemented to address them.

In the *Problems and Limitations* section, we will discuss the following key aspects:

1. *Computational complexity and scalability:* One of the main limitations of using vine copulas is the exponential growth in the number of parameters to be estimated as the dimensionality of the problem increases. This challenge is relevant in our context, as defining the critical layer involves handling multiple variables and conducting intensive simulations. To address this issue, we have implemented optimization techniques using Gaussian process regression (GPR), which reduces the need for exhaustive simulations. Furthermore, we plan to explore the use of parallel computing strategies and truncated vine copula models in future studies to further improve computational efficiency.

2. *Selection of the optimal structure and bivariate copula families:* The selection of the vine structure and the bivariate copula families is a critical process, as it directly impacts the accuracy of the results and the model's ability to capture extreme dependencies. We recognize that this process can be complex and computationally expensive, especially in high-dimensional applications. To address this limitation, we have implemented the Dißmann et al. (2013), which sequentially selects the optimal vine structure and copula families based on conditional dependence criteria and independence tests.

   The Dißmann algorithm optimizes the structural search by reducing the computational cost and avoiding the selection of irrelevant copulas at higher levels of the vine. This approach is particularly important in our work, where multiple hydrological variables with complex dependencies require well-defined structures. Additionally, to ensure that the selected structure properly represents the observed events, we perform a *validation of synthetic and observed events*. This process involves generating synthetic samples from the selected vine structure and comparing them with the observed data using graphical tools, such as quantile plots and joint empirical distributions. We also employ specific tests, such as the *upper tail dependence test* and *correlation test*, to verify that the model accurately captures extreme dependencies. This validation process, which is applied in the second part of our study, ensures that the selected structure is not only theoretically sound but also consistent with observed data.

   Nonetheless, we acknowledge that in more complex scenarios, future work could benefit from the development of hybrid methods that combine this algorithm with machine learning techniques to further enhance structural selection efficiency.

3. *Error propagation and simplified assumptions:* Another challenge is the potential propagation of errors in parameter estimation, especially in the presence of strong conditional dependencies. Although our approach uses cross-

validation and regularization of estimators to minimize this effect, we also recognize that, in certain applications, simplified assumptions can be problematic. Future work could explore the use of non-simplified copulas and Bayesian approaches to improve parameter estimation in highly dependent cases.

In the *Concluding Remarks* section, we will highlight that, despite these limitations, the use of vine copulas enables us to adequately capture complex dependencies among hydrological variables and accurately define the critical layer. The proposed improvements, including computational cost reduction, automated structural selection, and validation based on observed events, are important steps toward optimizing this approach and its applicability to real-world cases.

Line(s) 58–59 AUTHOR(s). For more theoretical details, please refer to (Sklar, 1959; Nelsen, 2006). REFEREE. None of the references is pertinent to Vine copulas: more recent and relevant paper must be cited, starting from the basic one: Aas, K., Czado, C., Frigessi, A., Bakken, H. (2009). Pair-copula constructions of multiple dependence. Insurance: Mathematics and Economics, 44(2), 182–198.

Following the reviewer's suggestion, we have incorporated more specific and updated references to further support the theoretical development of *vine copulas*. Our initial intention was for the citations provided (Sklar, 1959; Nelsen, 2006) to serve as general reference for the overall content. However, upon reviewing the text, we recognize that the phrasing of the sentence suggested that these references were directly related to *vine copulas*, which is not entirely accurate. Therefore, we will revise the sentence as follows:

"For more theoretical details on general copulas, please refer to (Sklar, 1959; Nelsen, 2006). For theoretical details on vine copulas, please refer to (Aas et al., 2009; Czado, 2019)."

Additionally, we will include the reference to Aas et al. (2009), as suggested by the reviewer, and add Czado (2019), one of the main sources of our research, which provides a comprehensive and practical guide to the construction and application of vine copulas. Although we had already included this reference in later sections, we recognize that it is also relevant in this context.

Line(s) 149–151 AUTHOR(s). For instance, Kendall's is more appropriate when the joint distribution is non-Gaussian (Serinaldi, 2008). Spearman's rank correlation is based on the rankings of variable values, whereas Kendall's rank correlation assesses the concordance and discordance between pairs of observations (Czado, 2019). REFEREE. None of the references is pertinent. Much better references are the two books by Nelsen (2006)—a more theoretical one—and Salvadori et al. (2007)—a more practical one.

As suggested, we have reviewed the relevance of the references included in this section. Serinaldi (2008) and Czado (2019) were chosen because they directly discuss the application of Kendall's and Spearman's in non-Gaussian contexts and in models involving multivariate dependence, making them relevant for our discussion. However, we recognize that the suggested references by *Nelsen (2006)* and *Salvadori et al. (2007)* provide a more comprehensive theoretical and practical perspective.

Consequently, we propose the following adjustments:

- We will retain Serinaldi (2008) and Czado (2019) due to their specific relevance to our application.
- We will add the suggested references as complementary sources to strengthen the discussion:
- Roger B. Nelsen (2006) will provide additional theoretical context on dependence measures within the framework of copulas.
- Gianfausto Salvadori et al. (2007) will enhance the practical aspect, linking the discussion to real-world hydrological applications.

The revised paragraph will read:

Modeling the dependence between variables is fundamental for understanding and analyzing their joint behavior. To achieve this, both parametric measures, such as the Pearson correlation coefficient, and non-parametric measures, such as rank-based correlations—Kendall's and Spearman's —are employed. Non-parametric measures are particularly favored in the estimation of dependence for compound events because the marginal distributions of these data often deviate from normality. For more theoretical background on dependence measures, see (Nelsen, 2006). For instance, Kendall's is more appropriate when the joint distribution is non-Gaussian (Serinaldi, 2008). Spearman's rank correlation is based on the rankings of variable values, whereas Kendall's rank correlation assesses the concordance and discordance between pairs of observations (Czado, 2019; Salvadori et al., 2007).

Line(s) 159–ff. AUTHOR(s). Graphical tools for analyzing dependence. . . REFEREE. For intellectual honesty, the Authors should warn the reader that the interpretation of graphical results always involves a degree of subjectivity, and should always be accompanied by objective formal tests.

This is an important observation, and we do agree that the interpretation of graphical tools for analyzing dependence, such as scatter plots, quantile-quantile plots, and dependence structure visualizations, can introduce a degree of subjectivity. To address this concern, we will include a clear recommendation in the manuscript emphasizing that graphical tools should be used as a complement to formal statistical tests.

The following revision aims to clarify the limitations of graphical tools while emphasizing their role as a complement to formal statistical analysis:

"Graphical tools provide valuable insights into the structure of dependencies between variables, offering a visual representation that can highlight nonlinear relationships, tail dependencies, and clusters (Genest et al., 2009; Salvadori et al., 2007). However, we caution readers that the interpretation of graphical results involves a degree of subjectivity and should be complemented by formal statistical tests, such as goodness-of-fit tests, upper tail dependence tests, or correlation tests (Joe, 2015; Nelsen, 2006), to ensure robust conclusions."

Line(s) 175–176 AUTHOR(s). They also found that the strong bias and associated uncertainty raise doubts about the reliability of most empirical results reported in the hydrological literature. REFEREE. See also Illustration 3.18 in Salvadori et al. (2007, p. 173), where numerical experiments were carried out both on empirical and simulated data.

As highlighted by Salvadori et al. (2007), the estimation of tail dependence coefficients λ_L and λ_U is particularly challenging in hydrology due to limited data availability in extreme regions, which can lead to unstable empirical estimates. This issue is shown in Figure 3.16 of Gianfausto Salvadori et al. (2007), where the estimates become unreliable as $t \to 0^+$ or $t \to 1^-$.

To address this known limitation, we have implemented a validation procedure that combines empirical data with synthetic data generated through simulations. Following the recommendations in Salvadori et al. (2007), this approach mitigates bias and improves the robustness of our estimates by providing additional data in regions where empirical observations are scarce.

The text will be modified as follows:

Estimating tail dependence coefficients in hydrology is prone to bias and instability due to the limited availability of extreme data (Salvadori et al., 2007). To address this issue, we validate our estimates by combining empirical data with synthetic data generated through simulations, following the recommendations in Salvadori et al. (2007), where numerical experiments showed that this combination improves the robustness of the analysis.

Line(s) 242 AUTHOR(s). The literature proposes various alternatives for combining multivariate analysis and non-stationarity. REFEREE. For intellectual honesty, the Authors should point out that, at present, non-stationarity is generally modeled by introducing a temporal dependence of the parameters of the marginals/copulas at play (e.g., by assuming linear and/or exponential changes of the parameters with time), but these remain mere mathematical exercises, not tested on empirical data.

This observation brings attention to a key consideration regarding non-stationarity in the multivariate framework. While many approaches have indeed relied on introducing temporal variability in the parameters of the marginals or copulas, this is neither the only possible method nor the only one that has been empirically applied

In the univariate case, non-stationarity is often addressed by incorporating covariates into the marginal distributions rather than relying solely on explicit time-dependent parameterizations. Méndez et al. (2007) applied a non-stationary Generalized Extreme Value (GEV) model to analyze monthly extreme sea levels, explicitly considering seasonal variability and long-term trends. Later, López and Francés (2013) explored non-stationary flood frequency analysis, highlighting the influence of both climatic and anthropogenic factors on extreme event distributions. Their study emphasized the importance of integrating external covariates, such as reservoir regulation effects and climate variability indices, to improve the characterization of hydrological extremes beyond traditional stationary assumptions. More recently, Urrea Méndez and Jesus (2023) incorporated non-stationary techniques into extreme rainfall estimation, demonstrating that covariates—such as climate indices—can be used to improve probabilistic estimations without exclusively assuming parametric temporal trends.These studies show that alternative approaches exist for modeling non-stationarity beyond the simple incorporation of time-dependent parameters in traditional models.

This approach moves beyond simplistic time-based formulations, allowing for a more physically interpretable modeling of variability in extreme events multivariate context, Boumis et al. (2025) propose the use of physics-informed dynamic copulas, where dependence parameters vary not only as a function of time but also in relation to climate indices such as the Oceanic Niño Index (ONI) and the

North Atlantic Oscillation (NAO). This approach represents a more realistic alternative, as it enables the modeling of multivariate dependencies based on observable climatic factors, avoiding the abstraction of time as the sole driver of change.

To better capture these considerations, we have updated the relevant sentence as follows:

"While much of the literature has focused on modeling non-stationarity by introducing temporal dependence in the parameters of marginal distributions or copulas (e.g., assuming linear or exponential changes over time), other approaches have also been explored. In the univariate setting, non-stationarity can be introduced through covariates in the marginal distributions (Méndez et al., 2007; López & Francés, 2013; Urrea-Méndez & del Jesus, 2023), allowing for greater flexibility in capturing long-term variability by incorporating climatic and anthropogenic influences. In the multivariate framework, alternative strategies such as physics-informed dynamic copulas have been proposed (Boumis et al., 2025), where dependence parameters evolve not only as a function of time but also in response to large-scale climate indices such as ONI or NAO, providing a physically consistent approach to modeling changing dependencies."

Line(s) 292–294 AUTHOR(s). This test evaluates the null hypothesis that the empirical copula comes from a specific copula;if the null hypothesis is rejected, the empirical copula does not follow the distribution of the specified copula. REFEREE. Statistically speaking, this sentence is not correct: Statistics can only offer guidance and suggestions, but never absolute truths. The words "copula does not" should be written as "copula may not".

Your correction is well taken, and we agree that the current wording could be interpreted as overly deterministic, which is not appropriate in a statistical context. As you correctly point out, goodness-of-fit test results provide probabilistic evidence rather than absolute conclusions. Therefore, we have revised the corresponding sentence to reflect this conceptual precision.

The text has been updated to:

"This test evaluates the null hypothesis that the empirical copula comes from a specific copula; if the null hypothesis is rejected, this suggests that the empirical copula may not follow the distribution of the specified copula."

This revision better reflects the probabilistic nature of statistical test results and eliminates any potential misinterpretation. Additionally, this correction aligns with the general understanding that statistical tests only provide a confidence level associated with the rejection or acceptance of a hypothesis.

Line(s) 422–ff. (3.6 Compound design events) REFEREE. The Authors are quite confused about the difference between a density and a probability distribution function: they are not the same, and they have different meanings. For instance, in the cited paper by Salvadori et al. (HESS 2011), the two different strategies proposed were based either on a probabilistic base (the Component-wise Excess one) or on a likelihood/density base (the Most Likely one). The description and the explanation given by the Authors is a superficial and confusing one, especially for practitioners. Here and elsewhere, use "most likely" instead of "most probable": in Probability Theory, a density induces a likelihood, not a probability (which, instead, is induced by a distribu-

tion function, i.e. the integral of a density).Line 427 refers to the Most Likely strategy outlined in the cited paper by Salvadori et al. (HESS 2011). Line 428 refers to the usage of "ensembles", as suggested in Salvadori et al. (HESS 2011).

We are not confused, but we acknowledge that our choice of wording was influenced by our *mother tongue and the way these terms are commonly used in Spanish.* The phrase *"most probable"* was used as a *direct linguistic translation* of how we refer to density-based likelihood in Spanish. However, to ensure rigor and consistency with probability theory, we will *adopt the term "most likely"* throughout the text, aligning with standard terminology in the field.

In G. Salvadori, De Michele, and Durante (2011), two approaches are proposed for identifying representative events in a multivariate setting:

- The *Component-wise Excess Approach*, which focuses on selecting events where all variables exceed specific thresholds, prioritizing the probability of joint exceedance.
- The *Most Likely Approach*, which selects events based on the highest joint probability density, ensuring that the chosen event is the most representative in terms of likelihood.

Our methodology follows the *Most Likely Approach*, as we identify the event in the critical layer that maximizes the *joint probability density function.* This is consistent with the theoretical framework outlined in G. Salvadori, De Michele, and Durante (2011), where this method is used to determine the most typical or expected realization of an extreme event.

To avoid confusion, we will explicitly clarify that *our approach does not correspond to the Component-wise Excess strategy*, which is based on probability exceedance rather than likelihood maximization. While both strategies are valid, they serve different purposes, and our focus aligns with the density-based selection outlined in G. Salvadori, De Michele, and Durante (2011).

To improve clarity and ensure consistency with standard probability terminology, we will modify this section as follows:

Original Title: Most probable compound event
Modified Title: Most likely compound event

The revised text now states:

"*The simplest way to select the* most likely *design event among all possible options is to choose the point with the highest joint probability density. This follows the likelihood-based approach outlined by Salvadori et al. (2011)."*

These modifications aim to *eliminate any ambiguity regarding terminology*, clarify our approach, and ensure consistency with the probabilistic framework established in the literature. We appreciate the reviewer's feedback, as it has allowed us to refine this section for better clarity and methodological alignment.

Eq. (9) is a special case of Eq. (13) in Salvadori et al. (HESS 2011), using the density f as a weight function w. Note that fXY ...W is the joint density of the distribution FXY ...W , not of the copula CXY ...W .

We will correct and adapt the wording to properly capture the concepts. Once again, the informal way in which we speak about all these concepts has percolated into the written paper.

For intellectual honesty, the Authors should make it clear, and warn the reader, that there is no guarantee that the maximum found by a numerical routine will be a global maximum, rather it is very likely that it will be a local maximum, and this will be more and more likely as the dimension of the problem increases.

A new section, *Problems and Limitations*, will be added, including a discussion on the objective behind this approach. In practice, even if the true global maximum is not found, a maximum close enough will serve, since the aim is to define an event as similar to the most likely one as possible.

We will also mention all the modern techniques used to try to improve the probability of finding the global maximum. Despite it being true that finding the global maximum in high dimensions is a tough problem, most of our machine learning and deep learning techniques depend on finding a good approximation to it, and the current development of the field indicates that the new algorithms perform well, although at a high computational cost.

Line(s) 489–ff. (Discussion, Conclusions) REFEREE. This is not a Discussion, it is at most a replica of generic sentences already written in previous parts of the manuscript. Actually, in my opinion, in this paper there little to discuss about. The Conclusions are a collection of statements that try to justify a paper with no content.

We do agree with the referee that the Discussion and Conclusions sections do not belong in the paper, since, without reading part II, it is difficult to provide solid conclusions to our study. We have removed both sections and instead included two new ones: *Problems and Limitations* and a *Concluding Remarks*.

The former will deal with all the topics that we have commented in this review that may render the applications of these techniques impractical or even undesirable. The latter will try to summarize the most important points to guide the reader towards the Part II of the paper.

References

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