

Reviewer 2:

General comments

While the topic covered could be of interest, using a Gaussian covariance (or double exponential, as it is called in the paper) without any nugget effect renders the comparison exercise a purely academic exercise with little or no practical bearing. Besides the fact that the underlying random function model behind the K-L expansion is the multiGaussian one, another decision that is far from reality.

In summary, for this paper to have any practical interest, the comparison exercise should be performed using a clearly non-multiGaussian random function with the kind of spatial variability one is expected to find in the field, not the unrealistic smooth spatial variability induced by a Gaussian covariance function.

Response:

We sincerely appreciate the reviewer's insightful comments and the time invested in evaluating our work. We fully recognize the importance of non-Gaussian random fields in real-world applications. In our original manuscript, the two cases were primarily designed to highlight the impact of variations in model parameter dimensions on the comparison of inverse algorithm performance, ignoring their real-world applicability. To address this, we will supplement the manuscript by discussing real-world scenarios that exhibit similar nonlinear characteristics to those in the examples presented, and explaining how the TNNA algorithm can handle inversion problems involving non-Gaussian random fields.

In Case 1, the domain is divided into a finite number of homogeneous zones, each with different parameter values. This parameterization approach is applicable to scenarios where the goal is to uncover the dynamics law of subsurface systems at a macroscopic level. For example, basin-scale groundwater models typically divide aquifers into homogeneous zones based on lithology and geomorphological features. In some field scale cases, divide the domain into homogeneous parameter zones based on weathering extent or fracture density is also a commonly adopted model simplification approach when data is insufficient. Regarding the Gaussian random field assumption in Case 2, when the aquifer consists of a single lithofacies with relatively uniform sedimentary structures, the permeability distribution can be considered to transition smoothly in space. In such scenarios, characterizing the permeability with a Gaussian random field proves to be an effective approach.

However, as the reviewer rightly pointed out, many real-world scenarios require accounting for non-Gaussian random field heterogeneity. For example, in braided bimodal geostatistical models of deltaic aquifers, the heterogeneous parameter fields exhibit spatial variability that does not transition smoothly, making it difficult to approximate using methods like K-L expansion. In such scenarios, both the TNNA algorithm and traditional metaheuristic algorithms may not be directly applicable. A promising approach to address this challenge is to use generative machine learning methods to establish a mapping between the non-Gaussian random field and low-dimensional latent vectors. The inversion of these latent vectors can indirectly reconstruct the non-Gaussian random field. By combining generative machine learning methods with the TNNA algorithm, it becomes possible to solve the inversion problems of non-Gaussian random fields.

We will include this discussion in the revised manuscript. Thank you once again for your

valuable feedback and constructive suggestions.