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2 **Ratio Limits of Water Storage and Outflow in Rainfall-runoff Process**

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20

21 **Abstract**

22 Flash floods typically occur suddenly within hours of heavy rainfall. Accurate forecasting of flash floods in
23 advance using the two-dimensional (2D) shallow water equations (SWEs) remains a challenge, due to the
24 governing equations of SWEs being difficult-to-solve partial differential equations (PDEs). Aiming at
25 shortening the computational time and gaining more time for issuing early warnings of flash floods, a new
26 relationship between water storage and outflow in the rainfall-runoff process is attempted to be constructed
27 by assuming the catchment as a water storage system. Through numerical simulations of the diffusion wave
28 (DW) approximation of SWEs, the water storage and discharge are found to be limited to envelope lines, and
29 the discharge/water depth process lines during water rising and falling showed a grid-shaped distribution.
30 Furthermore, if a catchment is regarded as a semi-open water storage system, there is a nonlinear relationship
31 between the inside average water depth and the outlet water depth, namely the water storage ratio curve,
32 which resembles the shape of a “plume”. In the case of an open channel without considering spatial
33 variability, the water storage ratio curve is limited to three values (i.e., the upper, the steady, and the lower
34 limit), which are found to be independent of meteorological (rainfall intensity), vegetation (Manning’s
35 coefficient), and terrain (slope gradient) conditions. Meteorological, vegetation, and terrain conditions only
36 affect the size of the “plume” without changing its shape. Rainfall, especially weak rain (i.e. when rainfall
37 intensity is less than 5.0 mm h^{-1}) significantly affects the fluctuations of the water storage ratio, which can be
38 divided into three modes, that is Mode I (inverse S-shape type) during the rainfall beginning stage, Mode II
39 (wave type) during the rainfall duration stage, and Mode III (checkmark type) during rainfall end stage.
40 Results indicate that the determination of the nonlinear relationship of the water storage ratio curve under
41 different geographical scenarios will provide new ideas for simulation and early warning of flash floods.

42

43 1. Introduction

44 Flood disaster is a significant global health and economic threat. Disastrous floods have caused millions
45 of fatalities in the twentieth century and billions of **dollars in direct economic losses** each year (Merkuryeva,
46 et al., 2015; Merz, et al., 2021; Ruidas, et al., 2022). According to statistics (Lee, et al., 2020), from 2001 to
47 2018, over 2,900 floods caused over 93,000 deaths and over 490 billion USD in economic damages
48 worldwide. **Based on 250-meter resolution daily satellite images of 913 major flood events during the same**
49 **period, the total area inundated by floods is estimated to be 2.23 million km² and the directly affected**
50 **population is estimated to be 255 to 290 million** (Tellman, et al., 2021). With the influence of climate change
51 and extreme El Niño events (Ward, et al., 2014; Cai, et al., 2014), flood events caused by extreme
52 precipitation are occurring frequently in many regions around the world (Kirezci, et al., 2020; Najibi and
53 Devineni, 2018; Almazroui, 2020). From 2020 to 2023, catastrophic floods caused by several extreme
54 rainfall events were reported in Germany (Tradowsky, et al., 2023), China (Hsu, et al., 2021), Italy (Valente,
55 et al., 2023), Japan (Kobayashi, et al., 2023), Pakistan (Nanditha, et al., 2023) and other developed or
56 developing countries and regions, even in some desert areas, **e.g. in the Taklimakan Desert and the Atacama**
57 **Desert, as reported by Li and Yao (2023) and by Cabré et al. (2023) respectively**. Research show that under a
58 high emissions scenario, in latitudes above 40° north, compound flooding could become more than 2.5 times
59 as frequent by 2100 compared to the present (Bevacqua, et al., 2020). It means that in the future, the fraction
60 of the global population at risk of floods will be growing.

61 Flood simulation provides an effective means of flood forecasting to reduce property and life losses in
62 flood-threatened areas around the world. **Particularly, weather prediction-based distributed**
63 **hydrological/hydraulic models are considered to be an effective strategy for flood simulation (Ming, et al.,**
64 **2020). Hence, a large number of scholars are committed to improving the simulation efficiency or simulation**

65 accuracy of distributed hydrological/hydraulic models. Accordingly, they have developed many forms of
66 hydrological models and hydrodynamic models in the past decades. Among them, the hydrological models
67 include Stanford Watershed Model IV-SWM (Crawford and Linsley, 1966), SHE/MIKESHE model (Abbott,
68 et al., 1986), Tank model (Sugawara, 1995), Soil and Water Assessment Tool-SWAT (Arnold and Williams,
69 1987), and TOPMODEL (Beven and Kirkby, 1979), etc. The hydrodynamic models include the
70 one-dimensional (1D) Saint-Venant equation (Köhne, et al., 2011), the two-dimensional (2D) SWEs
71 (Camassa, et al., 1994), and the three-dimensional (3D) integrated equations of runoff and seepage (Mori, et
72 al., 2015). In addition, a variety of hydrological-hydrodynamic coupling models have also been proposed by
73 Kim, et al. (2012); Liu, et al. (2019); Hoch, et al. (2019), and other scholars. Particularly, SWEs are the main
74 governing equations for simulating floods. However, flood simulation based on SWEs is a time-consuming
75 process due to its governing equations being a hyperbolic system of first-order nonlinear partial differential
76 equations (PDEs) (Li and Fan, 2017). Therefore, many scholars attempted to improve the efficiency and
77 accuracy of flood simulation through computer technology e.g. applying GPU parallel computing (Crossley,
78 et al., 2010) or advanced numerical scheme (Sanders, et al., 2010). For hydrological studies, the performance
79 of hydrological modeling is usually challenged by model calibration and uncertainty analysis during
80 modeling exercises (Wu, et al., 2021).

81 Efficient and stable solution of the hydrodynamic model has long been an important issue in flood
82 forecasting. Since the SWEs are nonlinear hyperbolic PDEs, the increase in the calculation domain and the
83 increase in the degree of discreteness will greatly increase the difficulty of solving SWEs. In addition, when
84 using high-resolution terrain to improve model calculation accuracy, non-physical phenomena such as false
85 high flow velocity in steep terrain will also occur, resulting in calculation distortion and a sharp increase in
86 calculation time. Hence, we try to ignore the complex exchange/transfer process of mass and momentum

87 (hydrodynamic models), and also abandon the empirical relationships (hydrological models) between the
 88 input (precipitation), the transmission (flow rate), and the output (discharge) in the catchment area. A
 89 catchment is regarded as a semi-open water storage system, and the complex problem is simplified into three
 90 megascopic variables, i.e., inflow, water storage, and outflow. For one watershed, the complex internal flow
 91 processes could be ignored if the physical mechanism between inflow, water storage, and outflow can be
 92 found under different meteorological, geographical, and geological conditions. In other words, if we can give
 93 a physical-based relationship between the three megascopic variables, flood forecasting will become much
 94 simpler. For this goal, a “plume” shaped nonlinear relationship between the inside average water depth and
 95 the outlet water depth, namely the water storage ratio curve, was found by using the calculation results of the
 96 hydrodynamic model.

97 2. Methods

98 An arbitrary catchment (Fig. 1b) could be assumed to be a conceptual water tank (Fig. 1a). In this water
 99 tank, according to the law of conservation of mass, the complex confluence process of surface runoff could
 100 be neglected and it can be described only by the relationship between input, storage and output, which can be
 101 expressed as Eq. 1,

$$102 \quad \underbrace{A \times \frac{dH}{dt}}_{storage} = \underbrace{R \times A}_{rainfall} - \underbrace{I \times A}_{infiltration} + \underbrace{F \times A}_{exfiltration} - \underbrace{E \times A}_{evaporation} - \underbrace{\frac{Q}{A} \times A}_{discharge} \quad (1)$$

103 where A is catchment area (m^2); t is time (s); H is internal average water depth (m); R is rainfall
 104 intensity ($m s^{-1}$); I is infiltration ($m s^{-1}$); F is exfiltration ($m s^{-1}$); E is evaporation ($m s^{-1}$) and Q is discharge
 105 ($m^3 s^{-1}$).

106 In this section, attentions are focused on the surface flow of runoff, so the runoff-atmosphere moisture
 107 exchange (evaporation) and runoff-soil moisture exchange (infiltration and/or exfiltration) are

108 non-considered. Zhu et al. (2020) validated the effectiveness of a diffusion wave (DW) approximation of
 109 shallow water equations by numerical simulations for simulating ground surface runoff,

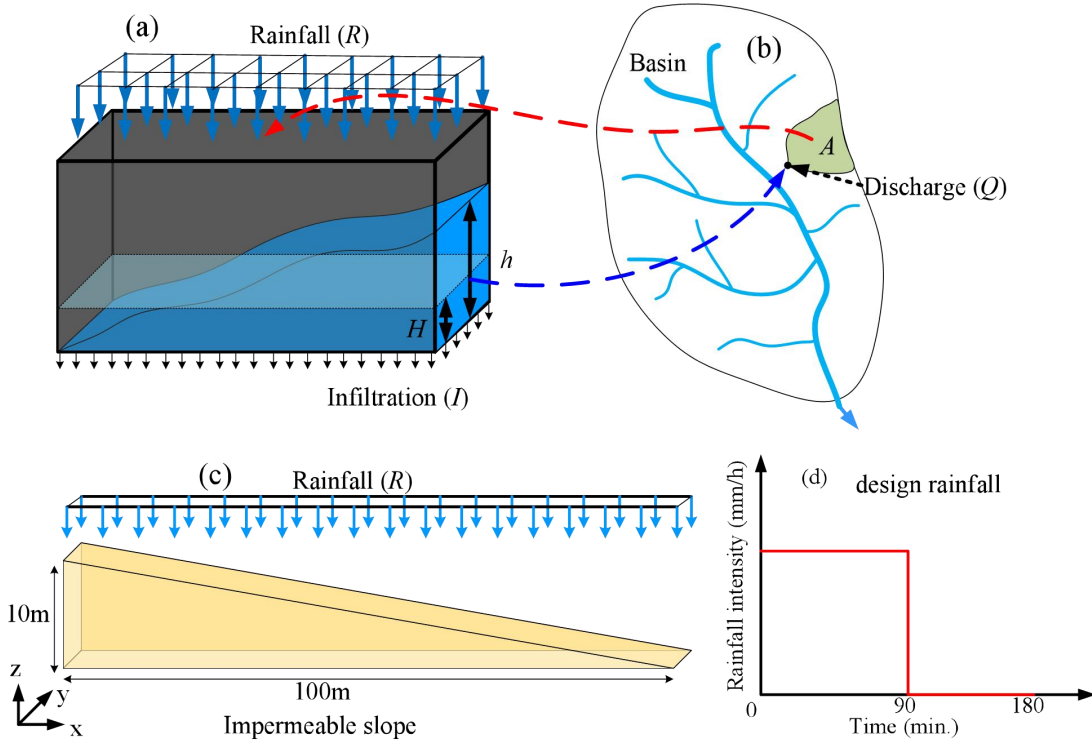
$$110 \quad \frac{\partial h}{\partial t} - \nabla \left(\frac{h^{5/3}}{n_m \sqrt{|S|}} \nabla (h + z) \right) = R \quad (2)$$

111 where h is water depth (m); z is elevation (m); n_m is Manning's coefficient ($s \text{ m}^{-1/3}$) and S is the slope
 112 gradient.

113 To improve the computational efficiency of the hydrodynamic model, after strict mathematical
 114 derivation according to the basic hydrodynamic equation and the law of conservation of mass, Zhu et al.
 115 (2022) proposed a hydrological-hydrodynamic integrated model, i.e., distributed runoff model (DRM) as,

$$116 \quad \begin{cases} \frac{dH}{dt} = R - q \\ H = \eta h = \eta \left(\frac{n_m}{\sqrt{S}} \right)^{0.6} q^{0.6} \left(\frac{A}{B} \right)^{0.6} \end{cases} \quad (3)$$

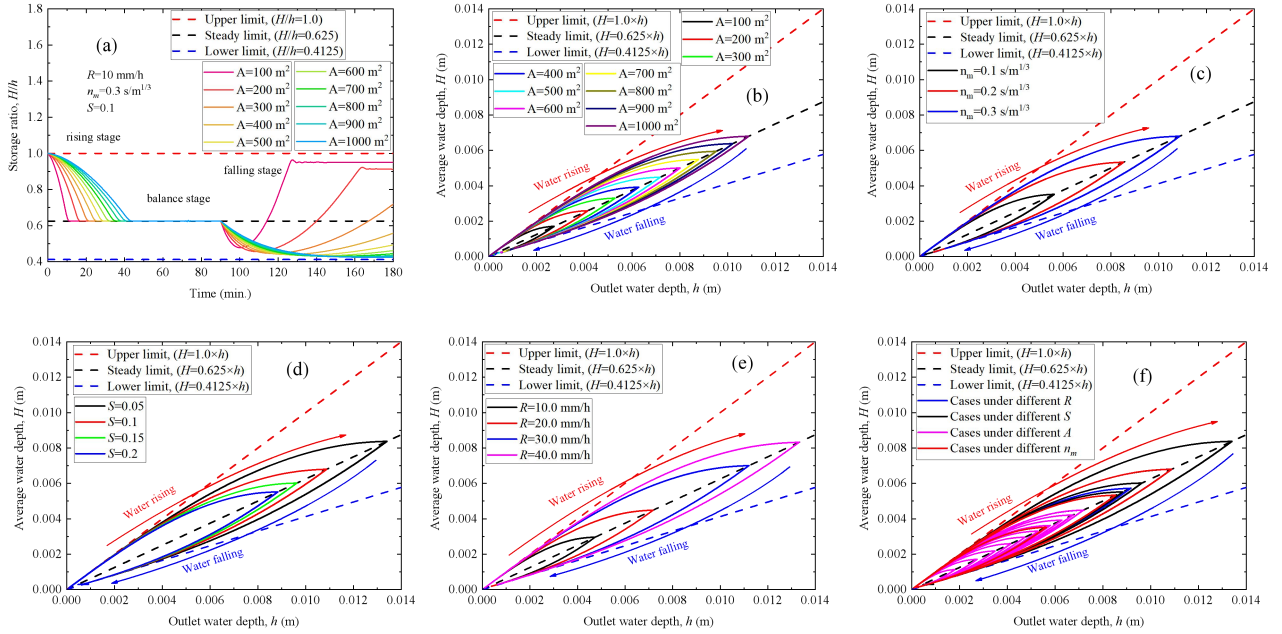
117 where $q=Q/A$ is conceptual outflow (m s^{-1}); η is the water storage ratio; B is the outlet width (m).



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 119 **Fig. 1. Conceptual schematic of the DRM and numerical model.** (a) conceptual water tank; (b)
 120 conceptual catchment; (c) impermeable conceptual slope model; (d) design rainfall.

121 3. Limits and “plume” shape of water storage ratio curve

122 The conceptual hydrological model takes the inside average water depth (H) in the catchment area as
123 the independent variable (Eq. 1). However, the hydrodynamic equations take the water depth at any outlet (h)
124 as an independent variable (Eq. 2). If a relationship between the inside average water depth (H) and outlet
125 water depth (h) can be established, then this relationship will have both hydrodynamic and hydrological
126 characteristics. Therefore, to find the H - h relationship, an impermeable conceptual slope model was built as
127 shown in Fig. 1c, and numerical simulations were performed using diffusion wave (DW) approximation (Eq.
128 2) of shallow water equations (SWEs). The water storage ratio is defined as the inside average water depth
129 (H) divided by the outlet water depth (h). Firstly, the numerical simulations are performed under a designed
130 rainfall condition, i.e., rainfall intensity is 10 mm h^{-1} and rainfall duration is 90 minutes with a total time of
131 180 minutes as shown in Fig. 1d. From the time-dependent water storage ratio (H/h) under different
132 catchment area (Fig. 2a), it can be seen that the continuous rainfall will cause the water storage ratio (H/h) to
133 gradually decrease from the initial value 1.0 (upper limit) to a stable value, which is approximately 0.625
134 (steady limit). When the rainfall ends, the value of the water storage ratio (H/h) decreases first and then
135 increases, showing a U-shaped curve with a lower limit, which is approximately 0.4125. Afterward, the
136 water storage ratio curves under ten kinds of catchment area (Fig. 2b), three kinds of Manning’s coefficient
137 (Fig. 2c), four kinds of slope gradient (Fig. 2d), and four kinds of rainfall intensity (Fig. 2e) conditions are
138 obtained from parametric analyses and collected in Fig. 2f.



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Fig. 2. Water storage ratio curves. (a) time-dependent water storage ratio under different catchment areas with 10 mm h^{-1} ; (b) water storage ratio curves under ten kinds of catchment area; (c) water storage ratio curves under three kinds of Manning's coefficient; (d) water storage ratio curves under four kinds of slope gradient; (e) water storage ratio curves under four kinds of **rainfall intensity**; (f) **collection of the above twenty one water storage ratio curves**. Three limit lines envelop all water storage ratio curves, i.e., upper limit ($H/h=1.0$), steady limit ($H/h=0.625$), and lower limit ($H/h=0.4125$).

Finally, it is found that water storage ratio curves **resemble the shape of a “plume”**. When the water outlet depth is the same, the water storage ratio (H/h) of the water-rising limb is higher than that of the **water-falling limb**. Furthermore, in the case of an open channel without considering spatial variability, there are three limits (the upper, the steady, and the lower limit) of the water storage ratio curves, which are found to be independent of meteorological (rainfall intensity), vegetation (Manning's coefficient), and terrain (slope gradient) conditions. Meteorological, vegetation, and terrain conditions only affect the size of the “plume” without changing its shape which is anchored by three limits. This means that the three limits and the water storage ratio curves provide a key to establishing a relationship between the hydrodynamic models and the

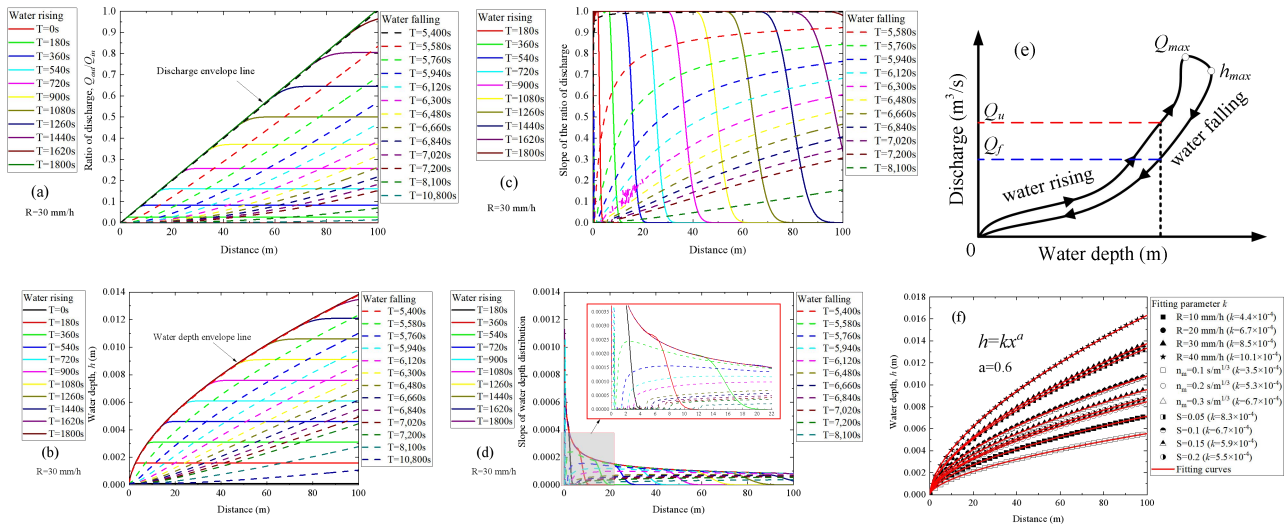
155 hydrological models.

156 **4. Grid-shaped cross-distribution of discharge/water depth process lines during water rising and**
157 **falling**

158 To obtain further insights into the causes for the formation of the water-rising limb and the water-falling
159 limb of the water storage ratio curve, the ratio of discharge (i.e., the ratio of the total outflows (Q_{out}) to the
160 total inflows (Q_{in})), and the water depth (h) along the slope are discussed in Fig. 3a and Fig. 3b, respectively.

161 Results indicate that there is an envelope line that controls the distribution of the discharge and water depth
162 along the slope, respectively. The discharge envelope line is a straight line with a slope of 1.0 (Fig. 3a), while
163 the water depth envelope line is a nonlinear curve controlled by a power function of general form $h=kx^a$ (Fig.
164 3b). It means that if the duration of rainfall with a constant intensity is long enough, the catchment system
165 will eventually reach an equilibrium state between inflow and outflow.

166 On the other hand, the process lines of discharge and water depth during water rising and falling present
167 a grid-shaped cross-distribution (Fig. 3a and Fig. 3b). Similarly, from the view of the gradient of the
168 discharge and water depth process lines during water rising and falling, the discharge gradient curves (Fig. 3c)
169 and the water depth gradient curves (Fig. 3d) also present a grid-shaped cross-distribution during water rising
170 and falling, which might be the cause of the looped rating curve (Fig. 3e), i.e., higher discharges for the
171 rising limb (Q_u) than for the recession limb (Q_f) at the same stage (Petersen-Øverleir, 2006). After fitting the
172 value of parameter k and a under different rainfall intensity (R), Manning's coefficient (n_m), and slope
173 gradient (S) conditions (Fig. 3f), it is found that the parameter a is a constant, while the change of parameter
174 k is positively correlated with the change of rainfall intensity (R) and Manning's coefficient (n_m), but
175 negatively correlated with the change of slope gradient (S).



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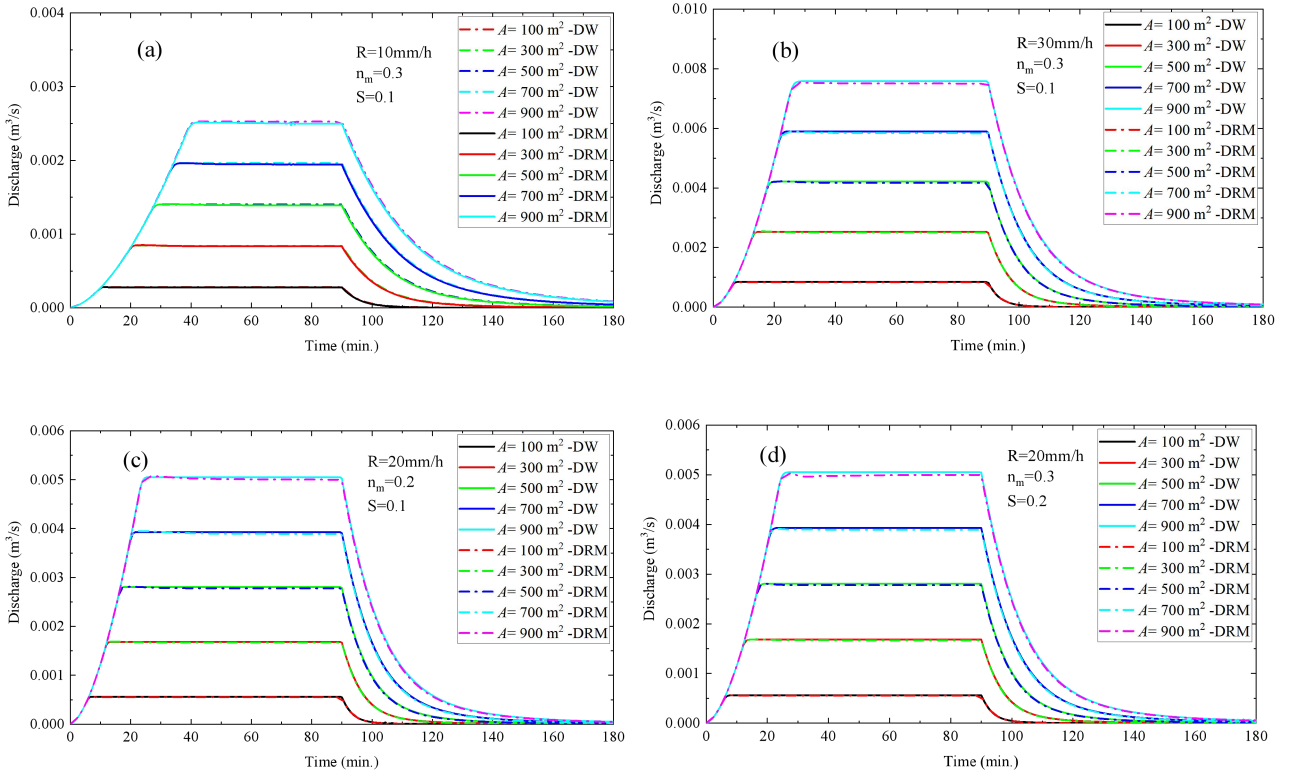
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Fig. 3. Discharge/water depth process lines during water rising and falling. (a) discharge process lines during water rising and falling; (b) gradient lines of discharge process line during water rising and falling; (c) schematic diagram of looped rating curve; (d) water depth process lines during water rising and falling; (e) gradient lines of water depth process lines during water rising and falling; (f) change of water depth envelope line under different rainfall intensity (R), Manning's coefficient (n_m), and slope gradient (S).

Based on the water storage ratio curve, a hydrological-hydrodynamic integrated model, namely the Distributed Runoff Model (DRM), is established with the governing equations in Eq. 3. To check the effectiveness and applicability of DRM, a comparative analysis of the numerical results obtained from the DRM and the DW model is implemented. We found that the DRM quickly reproduces the calculation results of the time-consuming DW model under different rainfall intensities (Fig. 4a and Fig. 4b), different Manning's coefficient, and different slope gradients (Fig. 4c and Fig. 4d). meaning that the water storage ratio curve will provide new ideas for simulation and early warning of floods. In addition, due to the governing equations of DRM being an ordinary differential equations (ODEs), the computational efficiency of DRM is much higher than the DW model, which is governed by nonlinear partial differential equations (PDEs). More attention should be paid to the determination of the nonlinear relationship of the water storage

193 ratio curve under different geographical scenarios, which will be beneficial to the proposal of more efficient
 194 flood forecasting methods or early warning systems.



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197 **Fig. 4. Comparative analyses of discharge calculated by DW and DRM under designed rainfall.** (a)
 198 controlled group; (b) compared with (a), only the rainfall intensity is changed; (c) compared with (a), rainfall
 199 intensity and Manning coefficient are changed; (d) compared with (a), rainfall intensity and slope gradient
 200 are changed.

201 **5. Validation of DRM by considering infiltration calculated by Horton infiltration method.**

202 In the above section, the simulations of DW and DRM are based on an impermeable conceptual slope
 203 model as shown in Fig. 1c. After considering infiltration in the DW and DRM, the Eq. 2 and Eq. 3 become:

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$$\frac{\partial h}{\partial t} - \nabla \left(\frac{h^5}{n_m \sqrt{|S|}} \nabla (h + z) \right) = R - I \quad (4)$$

$$\begin{cases} \frac{dH}{dt} = R - q - I \\ H = \eta h = \eta \left(\frac{n_m}{\sqrt{S}}\right)^{0.6} q^{0.6} \left(\frac{A}{B}\right)^{0.6} \end{cases} \quad (5)$$

Infiltration (I) is calculated by Horton's infiltration model (Horton, 1933), which suggests an exponential equation for modeling the soil infiltration capacity f_p (m s^{-1}):

$$f_p(t) = f_c + (f_0 - f_c)e^{-kt} \quad (6)$$

where f_0 is the initial infiltration capacities (m s^{-1}), f_c is the final infiltration capacities (m s^{-1}), k represents the rate of decrease in the capacity (s^{-1}). The infiltration parameter sets are listed in Table 1.

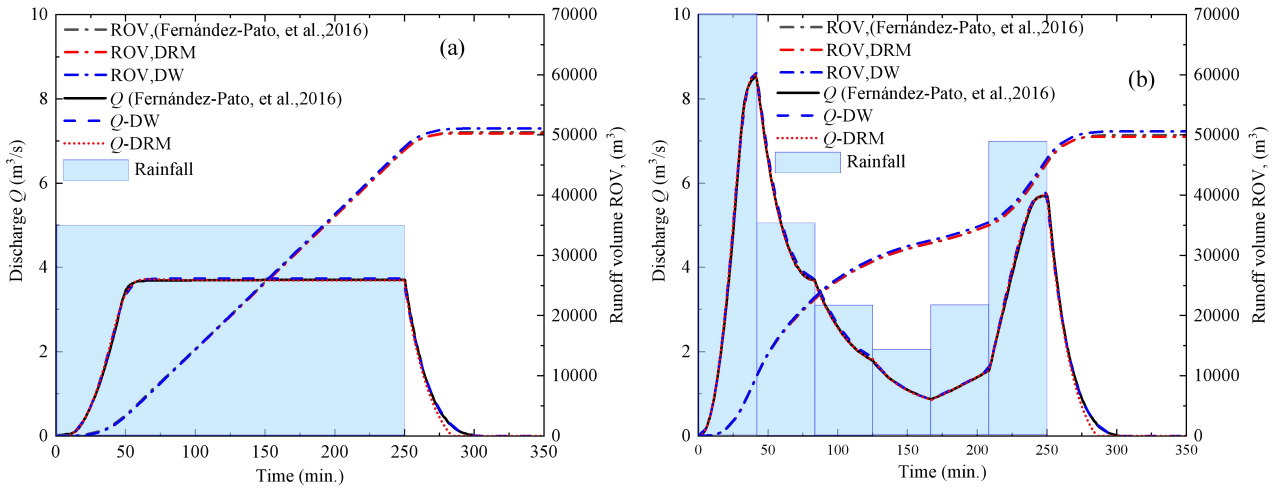
Table 1 Infiltration parameter sets.

Parameter	k (s^{-1})	f_c (m s^{-1})	f_0 (m s^{-1})
Value	2.43×10^{-3}	3.272×10^{-5}	1.977×10^{-4}

A rainfall event begins with a weak precipitation intensity. When the rainfall intensity is less than the infiltration capacity, all the rainwater will infiltrate into the soil. While, when the rainfall intensity exceeds the soil infiltration capacity, the surface water is generated, and Horton law (Eq. 6) applies:

$$I = \begin{cases} R(t) & \text{if } R(t) \leq f_p(t) \\ f_p(t) & \text{if } R(t) > f_p(t) \end{cases} \quad (7)$$

Results of outlet discharge (Q) and runoff volume (ROV) calculated by DW and DRM are compared with the reference results adopted from Fernández-Pato et al., (2016) as shown in Fig. 5. Fig. 5a shows the comparison of results under a uniform design rainfall. In this case, the rain volume is $75,000 \text{ m}^3$ with a duration of 250 minutes (min.). Fig. 5b shows the comparison of results under a non-uniform rainfall. Rain volume is $75,000 \text{ m}^3$ with a duration of 250 minutes (min.). From Fig. 5, it can be recognized that after considering infiltration, except that the calculation results of DRM are a little small at the end-stage of rainfall, the calculation results of DRM are still highly consistent with the calculation results of the DW model and reference results adopted from Fernández-Pato et al., (2016).



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Fig. 5. Outlet discharge (Q) and runoff volume (ROV) calculated by DW and DRM vs. reference

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results adopted from Fernández-Pato et al., (2016).

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6. Fluctuation of water storage ratio under natural rainfall conditions

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After implementing a real rainfall event in the impermeable conceptual slope model (Fig. 1c), the

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change of the water storage ratio is calculated as shown in Fig. 6. Rainfall data was recorded from 09 August

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2022 00:00 - 10 August 2022 00:00 in Aomori Prefecture, Japan and 29 August 2016 01:00 - 31 August 2016

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09:00 in Nissho Pass, Japan (<https://www.data.jma.go.jp>). The total simulation time is 30 hours and 56 hours,

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respectively. Results show that in addition to the fluctuations of water storage ratio in the beginning and end

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stages of rainfall, there are mainly ten fluctuation periods of water storage ratio during the rainfall duration

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stage, identified as 1[#], 2[#], 3[#], 4[#], and 5[#] in Fig. 6a and 6[#], 7[#], 8[#], 9[#], and 10[#] in Fig. 6b. The fluctuations are

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found to be mainly caused by weak rainfall (i.e. rainfall intensity is near 5.0 mm h^{-1}) as pointed by the red

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arrows in Fig. 6a and Fig. 6b. The magnitude of the fluctuations appears to be positively correlated with the

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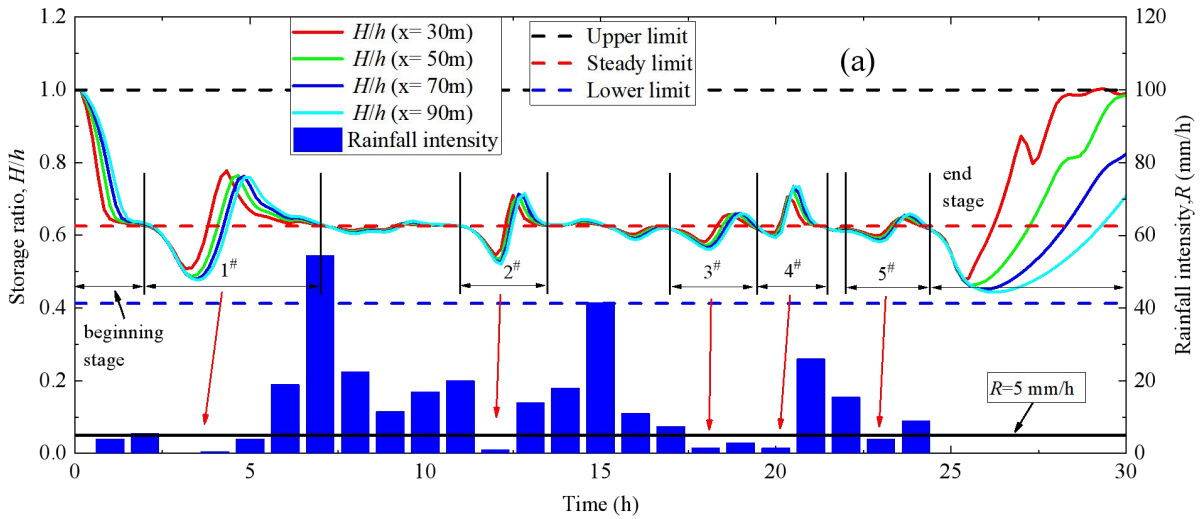
difference between rainfall intensity and 5.0 mm h^{-1} . When the rainfall intensity continues to be greater than

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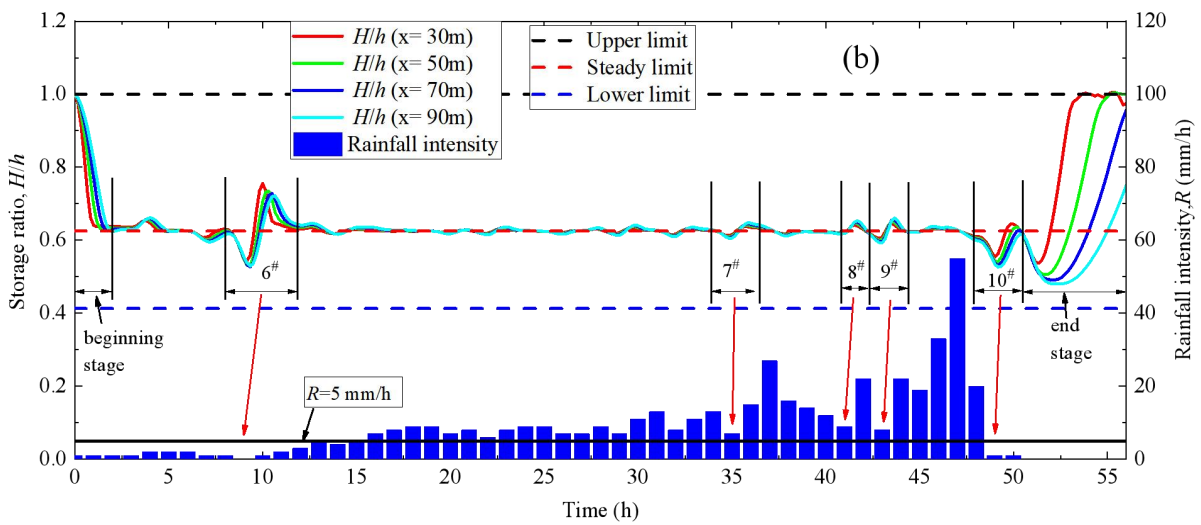
5.0 mm h^{-1} , the fluctuation of of water storage ratio is not obvious. The water storage ratio is stable near the

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steady limit, even if there is heavy rainfall during this period.



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243 **Fig. 6.** The fluctuation of water storage ratio and the effectiveness of DRM in natural rainfall events. (a)

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Aomori Prefecture; (b) Nissho Pass.

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Besides, the fluctuations of the water storage ratio can be divided into three modes, that is Mode I

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identified as the inverse S-shape type during the rainfall beginning stage (Fig. 7a), Mode II identified as

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wave type during the weak rainfall duration stage (Fig. 7b), and Mode III identified as checkmark type

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during rainfall end-stage (Fig. 7c). Among them, Mode I is that the water storage ratio drops from upper limit

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to steady limit in an inverse S-shape. Mode II is that the water storage ratio fluctuates around the steady limit.

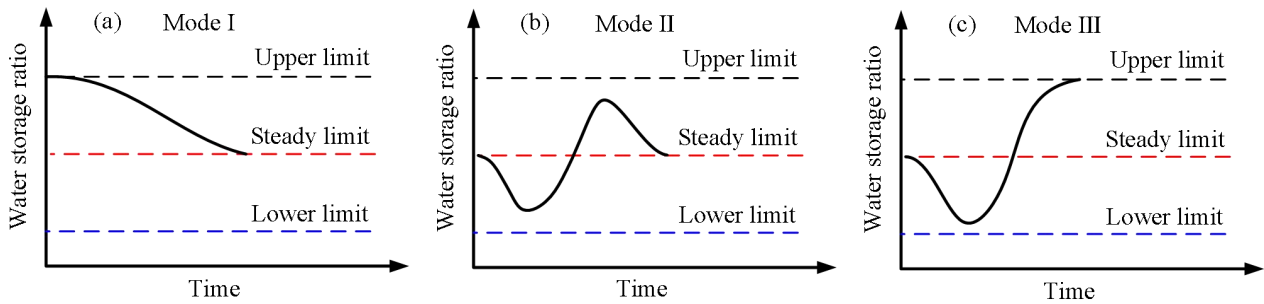
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Mode III is that the water storage ratio first drops from the steady limit to the lower limit and then rises to the

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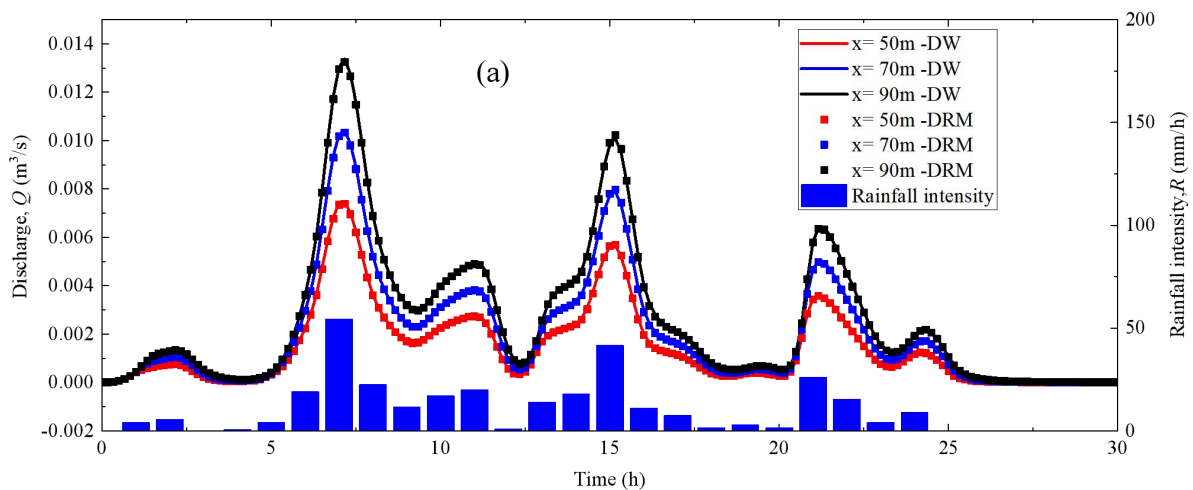
upper limit. This means that the certainty of the fluctuation modes will provide the possibility for

252 quantitative analysis of the fluctuation of the water storage ratio induced by the change in the rainfall
 253 intensity.

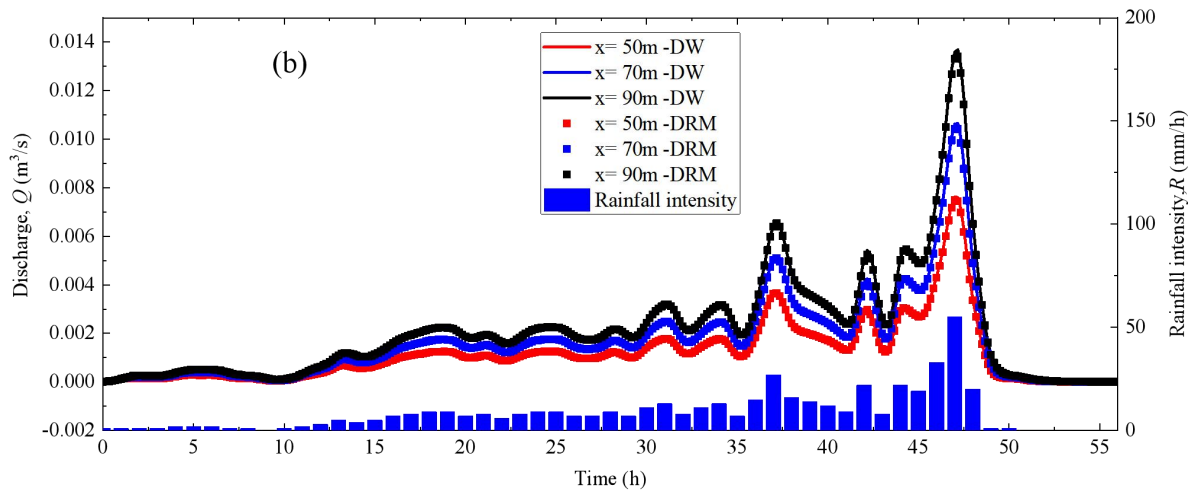


254
 255 **Fig. 7. Three kinds of water storage ratio fluctuation modes in natural rainfall events.** (a) Mode I during
 256 the rainfall beginning stage; (b) Mode II during the weak rainfall duration stage; (c) Mode III during the
 257 rainfall end stage.

258 Figures 8a and 8b show the simulation results of discharge calculated by the DRM and DW model using
 259 the rainfall data recorded in Aomori Prefecture and Nissho Pass, Japan, respectively. Results suggest that
 260 after the determination of the water storage ratio fluctuations, the calculation results of DRM are in good
 261 agreement with those of the DW model, meaning that DRM provides a new and more effective theoretical
 262 scheme for flood prediction.



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264

265 **Fig. 8. Time-dependent discharge calculated by DRM and DW model. (a) Aomori Prefecture; (b) Nissho**

266

Pass.

267 **7. Discussions and Conclusions**

268 Based on a conceptual slope model, numerical simulations of the rainfall-runoff process are performed
 269 by using the diffusion wave (DW) approximation of SWEs. A “plume” shaped nonlinear relationship
 270 between water storage and outflow, defined as the water storage ratio, is found between the inside average
 271 water depth and the outlet water depth in a catchment. The water storage ratio is controlled by three limits,
 272 namely upper limit, steady limit, and lower limit with the value of approximately 1.0, 0.625, and 0.4125,
 273 respectively. Under the control of the three limits, meteorological, vegetation, and terrain conditions only
 274 affect the size of the “plume” without changing its shape. The regular curve shape of the water storage ratio
 275 provides the possibility to construct a correlation between the water storage in the catchment area and the
 276 outlet discharge.

277 Based on the water storage ratio, a hydrological-hydrodynamic integrated model-DRM, is established,
 278 which shows high calculation accuracy and computational efficiency. This is because the governing
 279 equations of DRM are ordinary differential equations (ODEs), which are much easier to solve than nonlinear

280 partial differential equations (PDEs). However, the calculations of DRM and DW only involve the
281 confluence part of surface water and infiltration. While the interbasin groundwater flow as inputs to the
282 watershed (exfiltration) and evaporation are not considered, this is inconsistent with the real rainfall-runoff
283 process in the watershed and may lead to deviations in the calculation results. Therefore, the flow exchange
284 between surface water and groundwater during the existence and extinction of runoff also needs to be further
285 realized by establishing a dynamic coupling model of surface water and groundwater.

286 In addition, the water storage and discharge are limited to envelope lines, and the discharge/water depth
287 process lines during water rising and falling showed a grid-shaped distribution, which might be the cause of
288 the looped rating curve, i.e., higher discharges for the rising limb than for the recession limb at the same
289 stage. Rainfall, especially weak rainfall (i.e. rainfall intensity is less than 5.0 mm h^{-1}) significantly affects the
290 fluctuations of water storage ratio. The fluctuations of water storage ratio during a real rainfall event can be
291 divided into three modes, that is Mode I identified as inverse S-shape type during the rainfall beginning stage,
292 Mode II identified as Wave type during weak rainfall duration stage, and Mode III identified as checkmark
293 type during rainfall end stage. It is worth noting that a qualitative determination of the three fluctuation
294 modes of water storage ratio during rainfall events is obtained, but the quantitative analysis still needs to be
295 further carried out in the future.

296 The findings in this study provide a key to establishing a simpler prediction model for flash floods. The
297 water storage ratio has been proven to be effective in improving the effectiveness and efficiency of flood
298 forecasting. Therefore, the determination of the nonlinear relationship of the water storage ratio curve under
299 different geographical scenarios will provide new ideas for simulation and early warning of flash floods.

300 **Authors' contributions**

301 **Yulong Zhu:** Conceptualization, Methodology, Software, Validation, Formal Analysis, Investigation, Data

302 Curation, Writing-Original draft, Writing - Review & Editing.

303 **Yang Zhou:** Methodology, Validation, Investigation, Resources, Data Curation.

304 **Xiaorong Xu:** Methodology, Investigation, Data Curation.

305 **Changqing Meng:** Validation, Investigation, Data Curation.

306 **Yuankun Wang:** Conceptualization, Methodology, Writing-Original draft, Writing - Review & Editing,

307 Supervision, Project administration, Funding acquisition.

308

309 **Availability of data and materials**

310 The datasets used and/or analyzed during the current study are available from the corresponding author on

311 reasonable request.

312

313 **Competing interests**

314 The authors declare that they have no conflict of interest.

315

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