Supplementary information for

**A mathematical model to improve water storage of glacial lakes prediction towards addressing glacial lake outburst floods**

Miaomiao Qia,b, Shiyin Liua,b,c\*, Yongpeng Gaod,e, Fuming Xiea,b, Georg Vehf, Letian Xiaoa,b, Jinlong Jingg, Yu Zhua,b, Kunpeng Wua,b

a *Yunnan Key Laboratory of International Rivers and Transboundary Eco-Security, 650091 Yunnan University, Kunming, China;*

b *Institute of International Rivers and Eco-Security, Yunnan University, 650091, Kunming, China;*

c *Yunnan International Joint Laboratory of China-Laos-Bangladesh-Myanmar Natural Resources Remote Sensing Monitoring, Kunming 650091, China;*

*d Faculty of Geography, Yunnan Normal University, Kunming, 650500, China;*

*e Key Laboratory of Resources and Environmental Remote Sensing for Universities in Yunnan, Kunming 650500, China;*

*f  Institute of Environmental Science and Geography, University of Potsdam, Potsdam, Germany*

*g School of Mathematics and Statistics, Yunnan University, 650091, Kunming, China;*

\*Corresponding author: Shiyin Liu, [shiyin.liu@ynu.edu.cn](mailto:shiyin.liu@ynu.edu.cn);

\*Corresponding author: Shiyin Liu, [shiyin.liu@ynu.edu.cn](mailto:shiyin.liu@ynu.edu.cn);

First author: Miaomiao Qi, qmm@mail.ynu.edu.cn

## 1. Model Procedure

### 1.1 Volume calculation of elliptic paraboloid

Figure 1 illustrates schematic diagrams of the MDL outlet (a) and inlet (b) as elliptical paraboloids, with their respective volumes labeled as V1 and V3. In this section, we derive the expressions for V1 and V3 using the definite integral rule. We denote the semi-major and semi-minor axes of the ellipse in Figure 4a as *a1* and *b1*, and those of the ellipse in Figure 4b as *a2* and *b2*. As depicted in Figure 4, points A (*w*·2-1, 0, *h*) and B (0, *m*, *h*) reside on the ellipse V1, while points C (*w*·2-1, 0, *h*) and D (0, *n*, *h*) lie on the ellipse V3. By utilizing formulas (1) and (2), we can derive the following expressions:

, ,  （1）

The volume of the elliptical paraboloid can be determined by integrating the cross-sectional area with respect to the height variable, c (0≤c≤h). The cross-sectional equation for the ellipsoid paraboloid is expressed as follows:

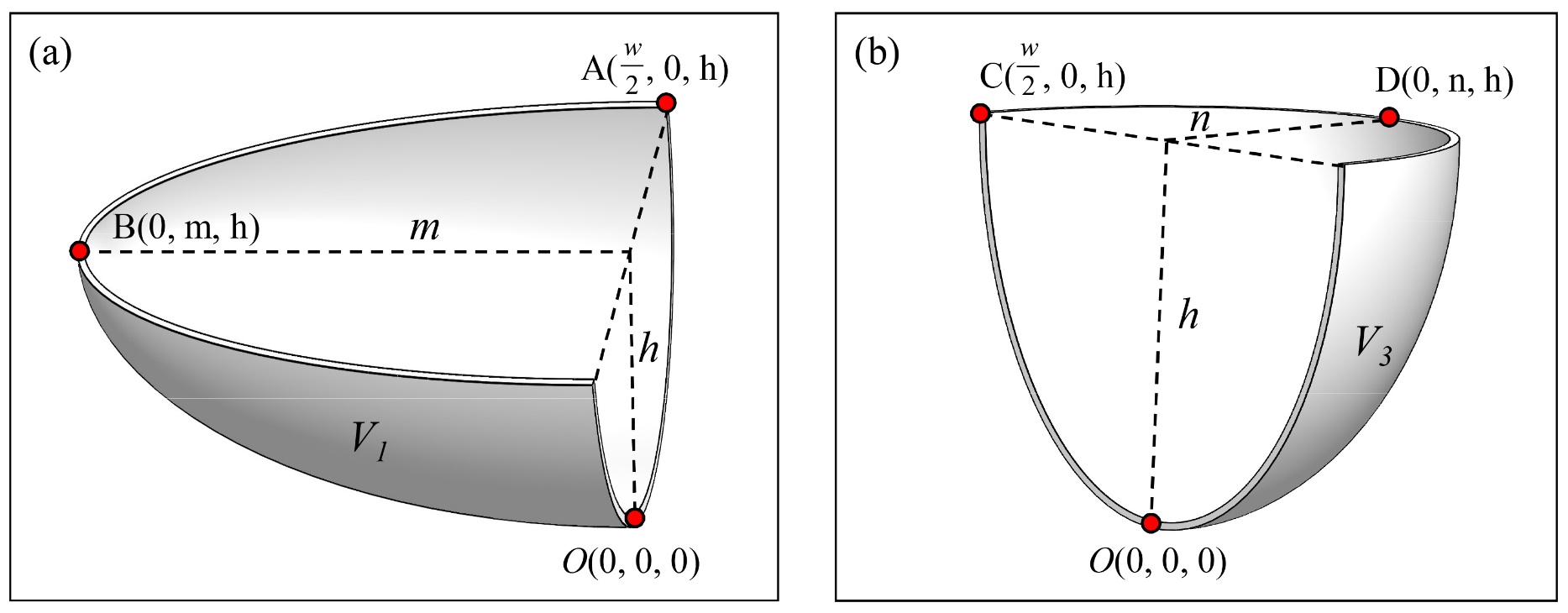
 （2）

Thus，

 （3）

Similarly,

 （4）



**Figure S1.** Schematic illustration of the ellipsoidal paraboloid at outlet (a) and inlet (b) of the MDL.

### 1.2 Volume calculation of parabolic cylinder

As illustrated in Figure 1a, the central section of the MDL is represented as a parabolic cylinder to calculate the volume V2, with Ps representing its cross-section. Figure 5b illustrates that Ps takes the shape of a paraboloid, defined by the equation y=k*x*2. Assuming that point F (*w*·2-1, *h*) lies on the parabola, we can express the parameter *x* as follows:

，， （5）

The area of *Ps* can be calculated using the following formula:

 （6）

Therefore, the volume of the parabolic cylinder is given by:

 （7）

In the volume equations provided for *V1*, *V2* and *V3* of the MDL, the variables involved are are *w*, *h*, *m*, *n* and *r*. With the exception of the unknown parameter *h*, all other parameters can be automatically determined based on the boundary data of the MDL. Subsequently, we can derive the expression for the variable *h* using point F. As point F (*w*·2-1, *h*) lies on the parabola shown in Figure 3.5b, we can employ the geometric interpretation of the derivative to obtain the derivative y'(*w*·2-1), which represents the slope of the tangent line at point F on the parabola. In this context, we denote the average slope around the MDL as *a*.

Thus,

 （8）

 （9）

Finally, considering the four distinct types of MDL, we can focus on the case where *r*=0 and *n*=0, which corresponds to GCL-1. In this scenario, the volume of the GCL-1 can be represented by the following expression:

 （10）

When *n*=0, the model of MDL corresponds to GCL-2, and its volume can be represented by the following expression:

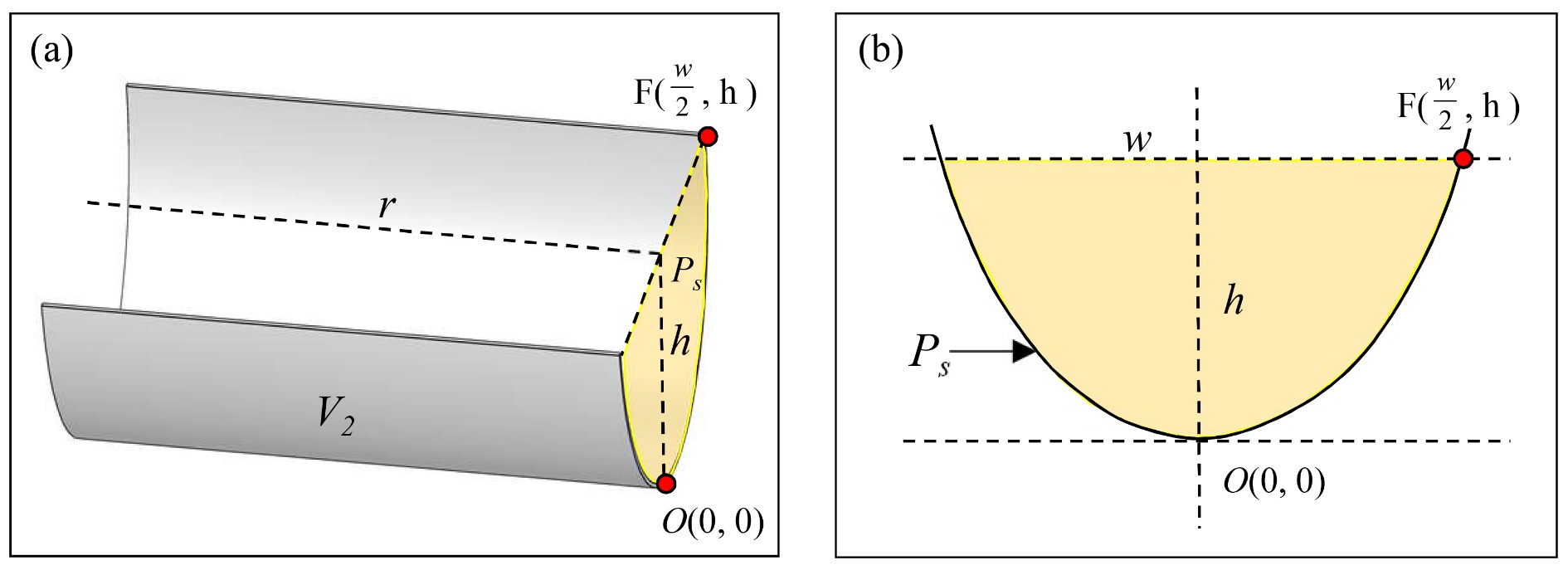
 （11）

When *r*=0, the model of MDL conforms to GUL-1, and its volume can be expressed as:

 （12）

When the type of MDL corresponds to GUL-2, its volume can be expressed as:

 （13）



**Figure S2**. Parabolic cylinder (a) in the middle of the MDL and its schematic of the cross-section (b)

**Table S1** Details of all empirical formulas used in this study

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Formulas | Code | N | Dam material | Relationship | R2 |
| Qi et al., 2022 | Eq. 1 | 27(>0.1 km2) | Moraine (>0.1km2) | V=40.67A1.184-3.218Rmxw/mxl | 0.96 |
| 70(<0.1 km2) | Moraine (<0.1km2) | V=557.4A2.454+0.2005Rmxw/mxl | 0.80 |
| Eq. 2 | 35(>0.5 km2) | Moraine | V=0.0126A2+0.0056A+0.0132 | 0.98 |
| 227(<0.5 km2) | Moraine | V=0.0235A1.4083 | 0.90 |
| Wang et al, (2012) | Eq. 3 | 20 | Moraine | V=0.0354A1.3724 | 0.92 |
| Eq. 4 | 20 | Moraine | V=0.087A1.434 | 0.50 |
| Evans (1986) | Eq. 5 | / | Ice | V=0.035A1.5 | / |
| Huggel et al, (2002) | Eq. 6 | 15 | Moraine, ice and bedrock | V=0.104A1.42 | 0.92 |
| Cook and Quincey (2015) | Eq. 7 | 15 | Moraine, ice and bedrock | Vstere=0.1217Asquare1.4129 | 0.95 |
| Eq. 8 | 42 | Moraine, ice and bedrock | Vstere=0.5057Asquare1.2884 | 0.38 |
| Eq. 9 | 30 | Moraine, ice and bedrock | Vstere=0.1746Asquare1.3725 | 0.60 |
| Eq. 10 | 57 | Moraine, ice and bedrock | Vstere=0.3211Asquare1.324 | 0.57 |
| Eq. 11 | 45 | Moraine, ice and bedrock | Vstere=0.1697Asqkm1.3778 | 0.75 |
| Fujita et al, (2013) | Eq. 12 | / | Moraine, ice and bedrock | V=0.055A1.25 | / |
| Loriaux and Casassa (2013) | Eq. 13 | 31 | Moraine and ice | V=0.2933A1.3324 | 0.96 |
| Emmer and Vilimek (2013) | Eq. 14 | 35 | Moraine and bedrock | Vstere =0.054393Asquare1.483009 | 0.92 |
| Kapitsa et al, (2017) | Eq. 15 | 32 | Moraine and bedrock | V=0.036A1.49 | 0.91 |
| Muñoz et al., (2020) | Eq. 16 | 69 | Moraine and ice | V=A(0.041Lw+2), Lw-lake width | 0.85 |
| O’Connor et al, (2001) | Eq. 17 | / | Moraine | V=0.003114A+0.1685A2 | / |
| Khanal et al, (2015) | Eq. 18 | 33 | Moraine | V=0.0578A1.5 | 0.93 |
| Sakai (2012) | Eq. 19 | 15 | Moraine | V(×106 km3)=43.24A1.5307 | / |
| Yao (2012) | Eq. 20 | 15 | Moraine | V=0.0493A0.9304 | 0.99 |