

1 **A mathematical model to improve water storage of glacial lakes prediction**  
2 **towards addressing glacial lake outburst floods**

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20  
21 **Abstract:** Moraine-dammed glacial lakes are vital sources of freshwater but also pose a hazard to  
22 mountain communities if they drain in sudden glacial lake outburst floods. Accurately measuring  
23 the water storage of these lakes is crucial to ensure sustainable use and safeguard mountain  
24 communities downstream. However, thousands of glacial lakes still lack a robust estimate of their  
25 water storages because bathymetric surveys in remote regions are difficult and expensive. Here we  
26 geometrically approximate the shape and depths of moraine-dammed lakes and provide a cost-  
27 effective model to improve lake water storage estimation. Our model uses the outline and the terrain  
28 surrounding a glacier lake as input data, assuming a parabolic lake bottom and constant hillslope  
29 angles. We validate our model using ten new bathymetrically surveyed glacial lakes on the Qinghai-  
30 Tibet Plateau, and compiled data from 34 recently measured lakes. Our model overcomes the  
31 autocorrelation issue inherent in earlier area/depth-water storage relationships and incorporates an  
32 automated calculation process based on the topography and geometrical parameters specific to  
33 moraine-dammed lakes. Compared to other models, our model achieved the lowest average relative  
34 error of approximately 14% when analyzing 44 observed data, surpassing the >44% average relative  
35 error from alternative models. Finally, the model is used to calculate the water storage change of

36 moraine-dammed lakes in the past 30 years in High Mountain Asia. The model has been proven to  
37 be robust and can be utilized to update the water storage of lake water for conducting further  
38 management of glacial lakes with the potential for outburst floods in the world.

## 39 **1. Introduction**

40 Moraine-dammed glacial lakes (MDLs) trap meltwater from snow, ice and liquid precipitation  
41 within basins behind dams at or near the termini of glaciers (Westoby et al., 2014; Yao et al., 2018;  
42 Veh et al., 2019). As glaciers have been retreating in past decades in most mountain regions  
43 worldwide, new MDLs have been forming, and existing ones have been growing in size and water  
44 storage (Bolch et al., 2012; Carrivick and Tweed, 2013; Cook et al., 2018; Shugar et al., 2020; Zhang  
45 et al., 2023). During the period from 1990 to 2018, High Mountain Asia witnessed a remarkable 52%  
46 and 54% increase in the number and area of MDLs, respectively (Wang et al., 2020). Notably, the  
47 Eastern Himalayas experienced the most significant growth, leading in both the number and area of  
48 MDLs during this period. MDLs are vital water reservoirs for communities in glaciated high  
49 mountains, but were also repeatedly sources for Glacial Lake Outburst Floods (GLOFs) (Westoby  
50 et al., 2014; Wu et al., 2019; Gao et al., 2021; Fischer et al., 2021; Zheng et al., 2021a). According  
51 to a report by Lützwow et al. (2023), a total of 630 GLOFs have been linked to MDLs occurring in  
52 27 countries between 850 and 2022 CE. A recent study indicates that multiple GLOFs documented  
53 from 1964 to 2022 have caused damage to infrastructure in High Mountain Asia (Nie et al., 2023).

54 MDLs are prone to sudden failure due to the instability of the dam structure, releasing parts of  
55 the impounded water storage in catastrophic floods (Westoby et al., 2014; Zheng et al., 2021b; [Duan  
56 et al., 2023](#)). MDLs can grow towards steep slopes, where debris or ice could fall into the lakes,  
57 causing the barriers to overflow (Emmer et al., 2014; Carrivick and Tweed, 2013; Liu et al., 2020).  
58 Due to their high altitude and potential energy, these flood waves can attain runout distances of  
59 many tens of kilometers, transporting and entraining large amounts of sediments from moraines and  
60 riverbanks (Westoby et al., 2014). Many GLOFs have transformed into debris flows and their coarse  
61 debris rapidly filled hydropower reservoirs and further destroyed infrastructure along the flow path  
62 (Westoby et al., 2014; Zheng et al., 2021b). For example, GLOFs descending from the mountains  
63 with high kinetic energy have recently damaged transport and power infrastructure such as the  
64 Upper Bhote Koshi hydropower plant, with a reconstruction cost of 57 million USD (United States  
65 dollar) (Cook et al., 2018). Future flash floods are a potential threat to major new infrastructure, for

66 example, hundreds more hydropower projects (Nie et al., 2023). GLOFs may also undercut  
67 hillslopes along mountain rivers, which may fail, impound river runoff, and form potentially  
68 unstable lakes (Zheng et al., 2021a). Thus, MDLs have become a major glacier-related hazard in  
69 high mountains, and will likely remain so as glaciers could lose more than a third of their mass by  
70 the end of the 21st century (Rounce et al., 2023). Appraising the water storage of glacial lakes is  
71 key to allowing for sustainable development along river channels originating in glaciated  
72 headwaters (Yao et al., 2018; Harrison et al., 2021; Shugar et al., 2020; Liu et al., 2020).

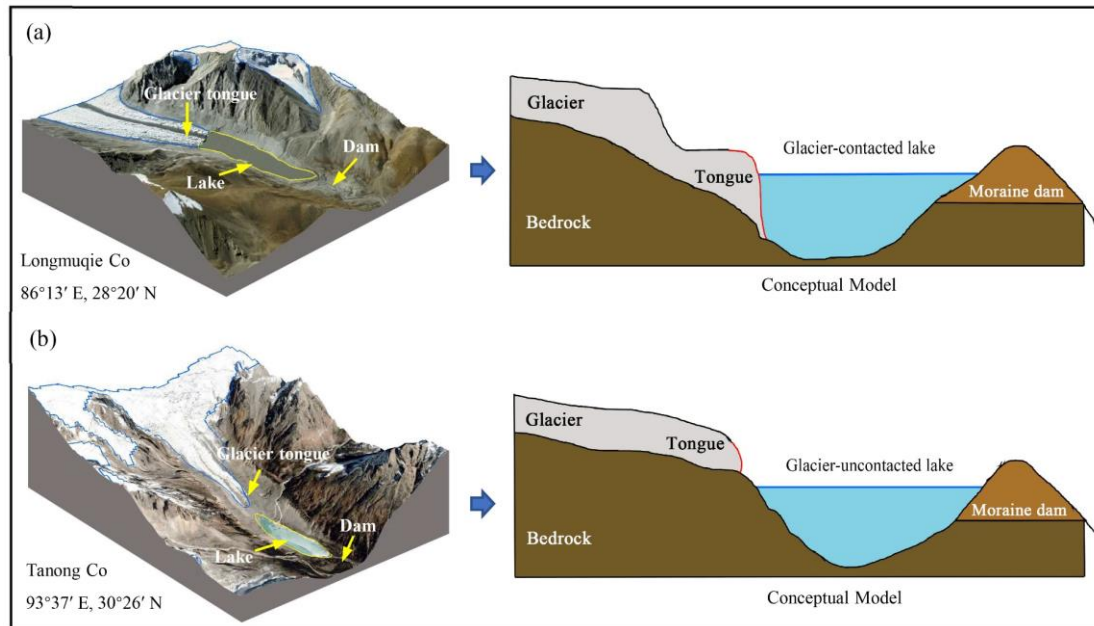
73 Effective management of GLOF hazards hinges on the ability to assess both the likelihood and  
74 magnitude of such events (Clague et al., 2000). This typically requires understanding several critical  
75 factors, including the water storage of MDL, the structural integrity and stability of the dam,  
76 potential external triggers, and the flood's anticipated flow path (e.g., Richardson and Reynolds,  
77 2000; Westoby et al., 2014; Mergili et al., 2020; Sattar et al., 2021; Qi et al., 2023). Estimating  
78 glacial lake volume, however, presents significant challenges. Many glacial lakes are situated in  
79 remote, physically demanding, and hazardous environments, complicating bathymetric surveys of  
80 the lake basins (Cook and Quincey, 2015; Qi et al., 2022).~~The peak discharge during GLOFs is a~~  
81 ~~commonly used parameter for assessing flood hazards and can be derived from empirical formulas~~  
82 ~~related to the lake volume (Clague et al., 2000; Westoby et al., 2014; Sattar et al., 2021; Nie et al.,~~  
83 ~~2023). The failure of the MDLs with the largest water storage has sustained high discharges for~~  
84 ~~many hours, causing widespread inundation in mountain valleys (Mergili et al., 2020). The~~  
85 ~~Sangwang Tsho experienced disastrous outbursts in July 16, 1954, featuring one of the highest~~  
86 ~~reported flood water storages ( $71.6 \times 10^6 \text{ m}^3$ ) and discharges ( $\sim 10,000 \text{ m}^3 \cdot \text{s}^{-1}$ ) (Patel et al., 2017;~~  
87 ~~Veh et al., 2019). Researchers therefore developed numerous empirical regression equations to~~  
88 ~~predict the potential peak discharge during an outburst from a given lake water storage (Wang et al.,~~  
89 ~~2018; Veh et al., 2019; Duan et al., 2023). In any case, these predictions and simulations of peak~~  
90 ~~discharge depend on accurate estimates of lake water storage, ideally obtained through bathymetric~~  
91 ~~surveys. However, measurements of lake depth are expensive and difficult to conduct in high-~~  
92 ~~altitude regions with limited access (Cook and Quincey, 2015; Qi et al., 2022).~~ Therefore, in situ  
93 measurements of lake depth are available only for a few dozen cases in the Himalayas, while the  
94 water storage remains unknown for the other thousands of lakes in this region. Current optical or  
95 radar-based satellite missions, while useful for mapping lakes, are limited in measuring lake

106 bathymetry due to the strong attenuation of electromagnetic waves in glacial lakes (Zhu et al., 2019).  
107 As such, there has been an ongoing effort to refine empirical scaling relationships from the few  
108 available worldwide samples that relate glacial lake depth and/or area to lake water storage (Fujita  
109 et al., 2013; Loriaux and Casassa, 2013; Carrivick and Quincey, 2014; Cook and Quincey, 2015;  
110 Veh et al., 2019; Shugar et al., 2020; Qi et al., 2022). However, these equations may yield significant  
111 errors in orders of magnitude for a given lake area due to the the autocorrelation issue inherent in  
112 earlier area/depth-volume relationships. Although there are models considering the specific  
113 geometric shapes and topography surrounding lakes, they limited to estimating the water storage of  
114 larger size plateau tectonic lake (Zhou et al., 2020; Zhu et al., 2019). After numerous experiments,  
115 we have found that the aforementioned models do not apply to estimating the water storage of  
116 glacier lakes due to the lack of consideration for glacial lake and related parameters. Given the  
117 critical role of glacial lake water storage in assessing hazard risk and providing early warning  
118 information, the development of a mathematically robust yet cost-effective model is urgently needed.

119 Our goal is to introduce a novel approach for accurately estimating water storage by  
120 incorporating its geometry and surrounding terrain. To this end, we propose a three-dimensional  
121 model to approximate the basin morphology of MDLs and derive its analytical equation. We assess  
122 the performance of this model against field-measured underwater topography data and further  
123 compare the model error against other available empirical scaling relationships. Finally, we discuss  
124 the uncertainty and rationality of the new model and apply the model to estimate the water storage  
125 of the MDLs in High Mountain Asia.

## 116 **2. MDLs types and their geometric approximation**

117 MDLs can be classified into glacier-contacted lakes (GCL) and glacier-uncontacted lakes  
118 (GUL). GCLs are supraglacial ponds on top of debris-covered glaciers or lakes at the termini of  
119 glaciers (Richardson 2000; Bennett et al., 2012). We term GCL as MDL in direct contact with the  
120 glacier terminus (Figure 1a). By contrast, GULs are separated from the present glaciers, but  
121 impound substantial parts of the meltwater from the glacier upstream (Figure 1b). The bottom of an  
122 MDL may be a sediment-covered bedrock depression that was eroded and deepened by the parent  
123 glacier during earlier advances. As glaciers retreat, they provide space for lakes to grow between  
124 the glacier terminus, with the abandoned moraine trapping excess meltwater from the parent glacier  
125 (Nie et al., 2023).



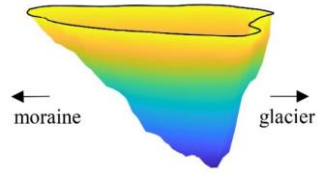
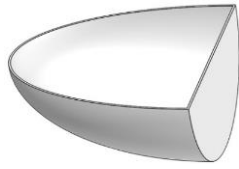
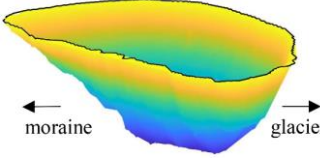
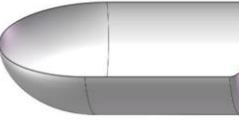
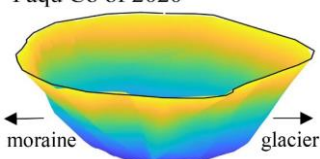
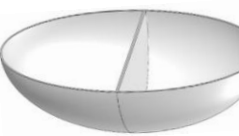
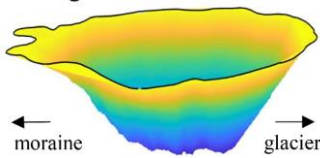
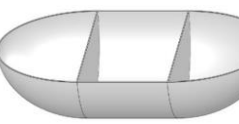
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127 **Figure 1.** Longitudinal cross-sections along a glacier-contacted (a) and glacier-uncontacted lake (b) (The base  
 128 images are from Google Earth imagery) (©Google Earth). Sketches are idealized and do not represent measured  
 129 elevations.

130 We use the glacial lake inventory of High Mountain Asia by Wang et al. (2020) to differentiate  
 131 these two types of MDLs. In general, glacial lakes grow in area largely because they become longer.  
 132 Lower values of the ratio ( $R$ ) between the maximum width and maximum length indicate that the  
 133 shape of the lake is elongated;  $R$  equals 1 if the lake is perfectly circular or square (Qi et al., 2022).  
 134 According to the glacial lake inventory, the  $R$  value for glacial lakes in High Mountain Asia ranges  
 135 from 0.1 to 1.0. If  $R$  is less than 0.1, it may indicate the presence of glacial lakes with lengths  
 136 exceeding 10 meters but widths of approximately 1 meter. However, in reality, glacial lakes with  
 137 such dimensions are practically non-existent. Therefore, thresholds of  $R$  allow us to distinguish  
 138 glacial lakes into four subclasses (Table 1). We find that newly formed GCLs typically have small  
 139 surface areas and high values of  $R$ . We classified GCLs with  $R$  between 0.70 ~ 1.0 as GCL-1, and  
 140 those with  $R$  less than 0.69 as GCL-2. Examples of these two types are Poiqu No.1 Lake (85.92°E,  
 141 28.14°N) and Bienong Co (93°26'E, 30°31'N) (Table 1). With ongoing glacier recession, lakes  
 142 might become decoupled from their parent glacier, switching from a lake-terminating to a land-  
 143 terminating glacier. We termed lakes as GUL-1, if  $R$  ranged between 0.5 and 1.0, and GUL-2 if  $R$  <  
 144 0.49. Paqu Co (86°15'E, 28°30'N) and Jialong Co in 2020 are the examples of these two classes  
 145 (Table 1). It is noteworthy that the establishment of the  $R$  threshold in this study is grounded in the  
 146 glacial lake catalog dataset developed by Wang et al, (2020). Initially, the glacial lakes were divided

147 into two major categories, GCL and GUL. Subsequently,  $R$  values for each glacial lake were  
 148 calculated, and all co-authors classified the geometric shapes based on different types and sizes of  
 149 glacial lakes. Ultimately, through statistical analysis of glacial lake sizes for different types, we  
 150 defined the threshold for  $R$ . This allows the model to automatically categorize glacial lakes based  
 151 on this value.

152 **Table 1** Examples of glacier-contacted lake and glacier-uncontacted lake. The ratio  $R$  represents the maximum width  
 153 (m) divided by the maximum length (m) of the glacial lake. The vertical scale is exaggerated.

Type	Lake bathymetry	Model	Features	$R$
GCL-1	PoiquNo.1 of 2021 		A newly formed MDL typically has a small scale and is located at the glacier tongue.	$0.70 \leq R \leq 1.0$
GCL-2	Bienong Co of 2021 		The MDL gradually grows in the area but has not yet reached the maximum range determined by the surrounding terrain.	$0.10 \leq R \leq 0.69$
GUL-1	Paqu Co of 2020 		As the glacier continues to retreat, the distance between the glacier tongue and the MDL gradually increases.	$0.50 \leq R \leq 1.0$
GUL-2	Jialong Co of 2020 		The length of the MDL increases with time due to the continuous supply with glacier meltwater.	$0.10 \leq R \leq 0.49$

154

### 155 3. Model Development

#### 156 3.1. Input data

157 We suggest specific geometric models for the four subclasses (Table 1) to approximate the  
 158 water storages of MDLs. Our models are fed with data from a digital elevation model (DEM) and  
 159 from the outline of a glacial lake. We used a 12.5-meter ALOS PALSAR DEM, which is freely

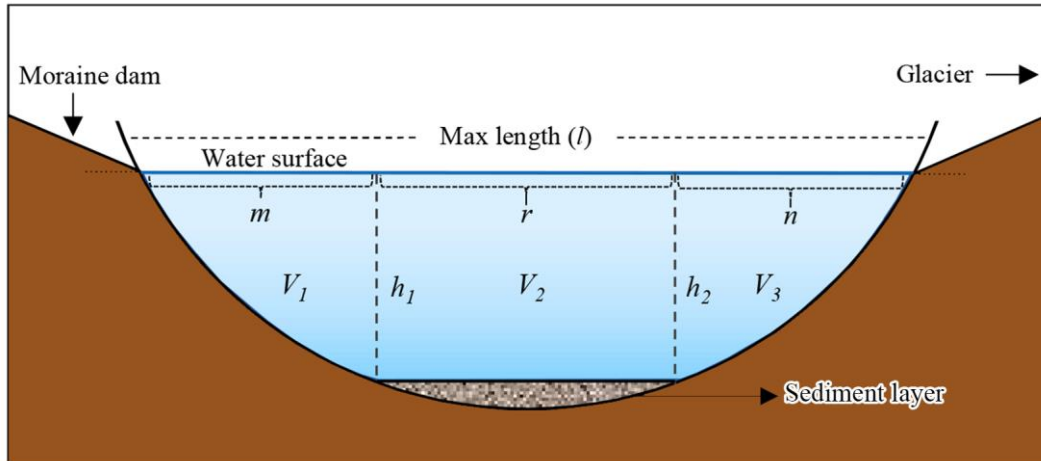
160 available from the Japan Aerospace Exploration Agency (JAXA, <https://www.eorc.jaxa.jp>).

### 161 3.2. Analytical equations

162 We surmise that an ideal cross-section of a MDL (Figure 2) can be partitioned into three distinct  
163 portions,  $V_1$ ,  $V_2$ , and  $V_3$ , representing the water storage of the lake stored adjacent to the moraine  
164 dam, at the center of the lake, and near the glacier (or bedrock if the lake is disconnected from the  
165 glacier). The corresponding lengths of these three portions along the maximum length of the lake  
166 are denoted by  $m$ ,  $r$ , and  $n$ . The lake has its maximum depth,  $h_1$  and  $h_2$ , on either side of  $r$ . Points  $g$   
167 and  $f$  represent the positions of a sediment layer at the lake bottom, and  $a$  and  $\beta$  are the slopes of  
168 near the water surface.

169 The core assumptions of our geometric model can be summarized such that: 1) an MDL has a  
170 parabolic longitudinal bottom profile with a uniform sediment layer at the bottom of the lake to keep  
171  $h_1 = h_2$ , and a parabolic cross-section  $P_S$  (Figs. 2; 3); (2) the lake surface shape can be approximated  
172 by ellipses at both ends and a rectangle in between; (3) The glacier surface and the moraine dam dip  
173 towards the lake with the same slope.

174

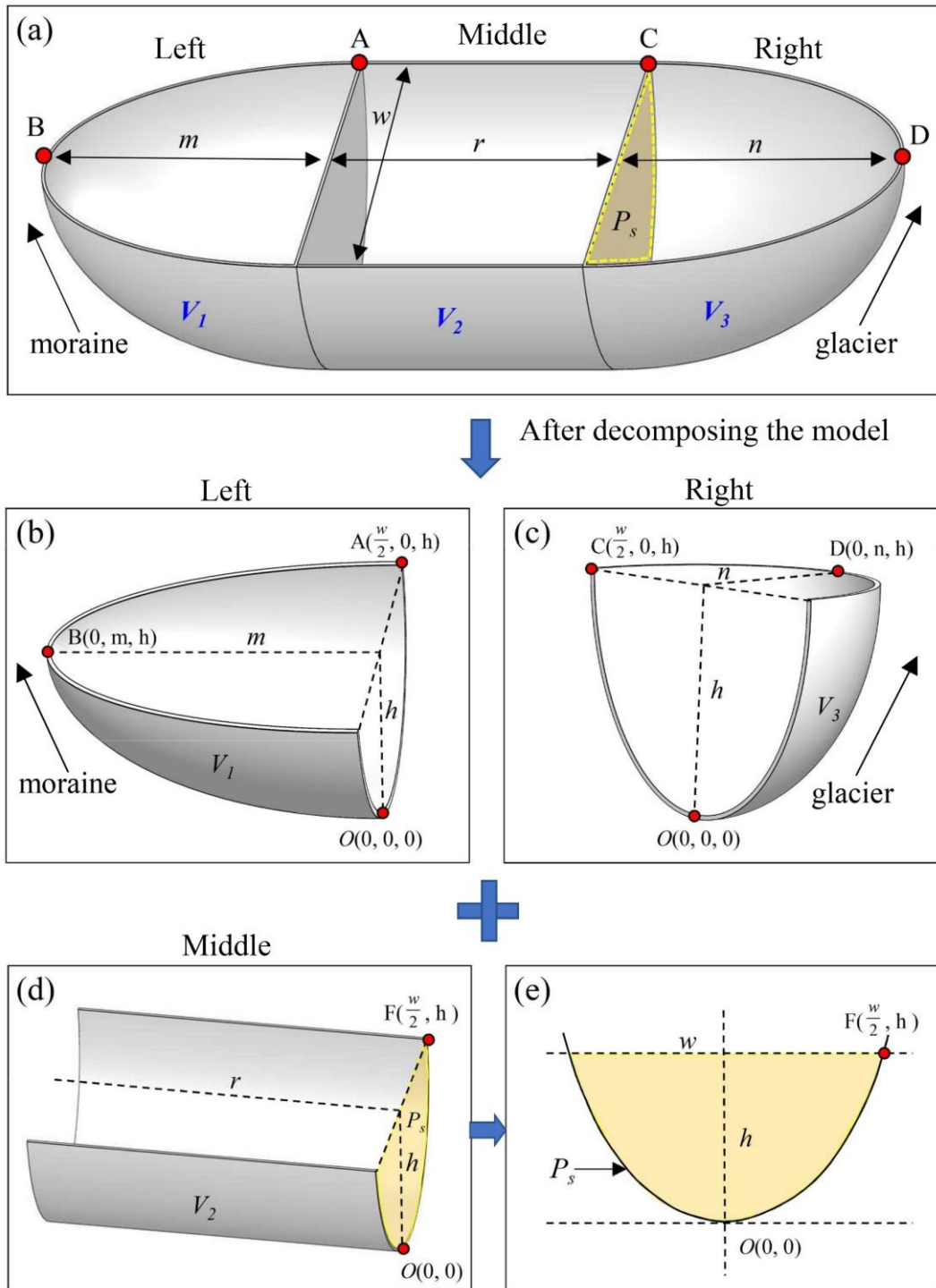


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176 **Figure 2.** Longitudinal cross-section through a MDL. The blue horizontal line ( $l$ ) is the maximum length on the lake  
177 surface, subdivided by  $m$ ,  $r$ , and  $n$ . The solid black line is the hypothetical bottom of the lake, and the gray texture  
178 area represents a sediment layer covering the lake bottom. The maximum water depth is  $h=h_1=h_2$ , and points  $g$  and  
179  $f$  are at equal depths.

180 In three-dimensional form, the MDL basin can be divided into three parts with each having a  
181 water storage of  $V_1$ ,  $V_2$ , and  $V_3$  (Figure 3a).  $V_1$  and  $V_3$  can be considered as the water storages of  
182 elliptical semi-paraboloids controlled by the water depth  $h$  (Figure 3b and c). Significantly,  $V_1$  and  
183  $V_3$  may or may not be equal, depending on the values of  $m$  and  $n$ .  $V_2$  is a semi-parabolic cylinder

184 (Figure 3d) that has height  $r$ , diameter  $w$ , and a parabolic cross-section  $P_s$  (Figure 3e). Thus, the total  
 185 water storage of the MDL is  $V=V_1+V_2+V_3$ .



186  
 187 **Figure 3.** Definition diagram for the geometry of a MDL. a, hypothetical three-dimensional model of a  
 188 MDL. b, Model for  $V_1$  describing the lake water storage adjacent to the moraine dam. c, Model for  $V_1$   
 189 describing the lake water storage adjacent to the glacier. d, Model for  $V_3$  describing the lake water storage  
 190 stored in the center part of the lake. e, Cross section of the column  $P_s$ . The parameters  $m$  and  $n$  are the  
 191 semi-major axis of the elliptical paraboloid near the MDL inlet and outlet, respectively;  $r$  is the length of  
 192 the parabolic cylinder in the middle of MDL;  $w$  and  $l$  represent the largest width and length of the MDL,



193 respectively;  $h$  is the lake depth.

194 To obtain the individual lake water storages, we define the elliptical paraboloids for  $V_1$  and  $V_2$   
195 (equations 1-2) in a Cartesian coordinate system  $(x, y, z)$  as

$$196 \quad V_1 = \left\{ (x, y, z) \mid \frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} \leq z, y \geq 0, 0 \leq z \leq h \right\} \quad (1)$$

$$197 \quad V_3 = \left\{ (x, y, z) \mid \frac{x^2}{a_2^2} + \frac{y^2}{b_2^2} \leq z, y \geq 0, 0 \leq z \leq h \right\} \quad (2)$$

198 and the parabolic cylinder for  $V_2$  (equation 3) as

$$199 \quad V_2 = \left\{ (x, y, z) \mid kx^2 \leq z \leq h, 0 \leq y \leq r \right\} \quad (3)$$

200 where  $a_1 > 0, b_1 > 0, a_2 > 0, b_2 > 0$  are length of the semi-axes of upper surfaces of  $V_1$  and  $V_3$ ;  $h >$   
201  $0$  is the height of  $V_1, V_2$  and  $V_3$ ;  $r > 0$  is the length of  $V_2$ .

202 Considering the four types of MDLs, GCL-1 corresponds to the case where  $r=0$  and  $n=0$ . In  
203 this study,  $m$  represents the part of the lake area closer to the moraine dam, and in most cases,  $m$  is  
204 not equal to zero. However, in certain special cases, such as the Lake Zhasuo Co (93.25°E, 30.31°N)  
205 in southeastern Tibet,  $m=n=0$ , because the surface morphology of this lake is rectangular. In most  
206 scenarios, the water storage of the GCL-1 can be represented as:

$$207 \quad V_{\text{GCL1}} = \frac{\pi w m h}{8}. \quad (4)$$

208 When  $n=0$ , the model of MDL corresponds to GCL-2, and its water storage can be  
209 represented as

$$210 \quad V_{\text{GCL2}} = \frac{\pi w m h}{8} + \frac{2}{3} w h r. \quad (5)$$

211 When  $r=0$ , the model of MDL conforms to GUL-1, and its water storage can be expressed as:

$$212 \quad V_{\text{GUL1}} = \frac{\pi w h l}{4}. \quad (6)$$

213 When the type of MDL corresponds to GUL-2, its water storage can be expressed as:

$$214 \quad V_{\text{GUL2}} = \frac{\pi w h (l - r)}{4} + \frac{2}{3} w h r. \quad (7)$$

215 Finally, the water depth ( $h$ ) can be derived from the  $w$  and slope angles ( $\alpha$ ) of the glacial lake:

$$216 \quad h = \frac{w \tan(\alpha)}{4}. \quad (8)$$

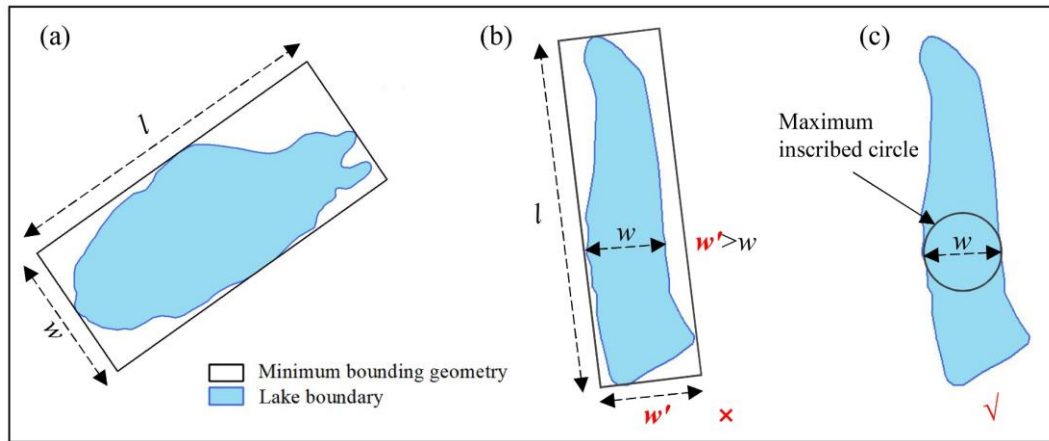
217 Section 1 in the Supplementary file elaborates more on the derivation of these analytical  
218 equations, Table 2 shows the definition of the abbreviations in the model procedure.

**Table 2.** The definition of the abbreviations in the geometric model.

Abbreviation	Description and definition
MDL	The moraine-dammed lake
GUL	The glacier-uncontacted lake
GCL	The glacier-contacted lake
$R$	The ratio of the maximum width to the maximum length of the MDL
$m$	The semi-major axis of the elliptical paraboloid of the MDL outlet
$n$	The semi-major axis of the elliptical paraboloid at the MDL inlet
$c$	The arbitrary height of the cross-section of an elliptic paraboloid
$r$	The length of the parabolic cylinder in the middle of MDL
$h$	The maximum water depth of MDL
$w$	The diameter of the largest inscribed circle of the MDL
$l$	The length of the minimum bounding rectangle of MDL
$P_s$	The cross-section of the middle of MDL
$S_{P_s}$	The area of the cross-section in the middle of MDL
$a$	The median slope of the 80 m buffer zone around the MDL

### 220 3.3. Determination of model parameters

221 We determined the parameters in Eq. 4 - 8, namely  $w$ ,  $l$ ,  $a$ ,  $m$ ,  $n$  and  $r$ , using the lake boundary  
222 and the DEM. We measured  $w$  and  $l$  by drawing a minimum rectangle bounding box with length  $l$   
223 encompassing the MDL (Figure 4a). If the width  $w'$  of the bounding box of the MDL exceeds the  
224 actual width ( $w$ ) of the lake, as in the case of the tortuous boundary of Lake Longmuqie Co (86.23°E,  
225 28.35°N) (Figure 4b), we assign the diameter of the maximum inscribed circle within the MDL as  
226  $w$  in Figure 4c.



227

228 **Figure 4** Schematic illustration of the method for extracting the maximum length ( $l$ ) and width ( $w$ ) of the MDL. The  
 229 outline in Figure a represents the geometric boundary of Lake Jialong Co (86.85°E, 28.21°N), while the outlines in  
 230 Figures b and c depict the geometric boundaries of Lake Longmuqie Co (86.23°E, 28.35°N).

231 To determine the slope  $a$ -value surrounding the MDL, we use a DEM with a spatial resolution  
 232 of 12.5 m in the model computation. We tested buffer sizes of 30 m, 50 m, 80 m, and 100 m width  
 233 beyond the MDL boundary, and extracted the mean and median value of  $a$  within each buffer. By  
 234 comparing the simulated results with the measured data ([lakes Bienong Co, Maqiong Co, Tanong](#)  
 235 [Co, and Jialong Co](#)), we found that the water storage estimation using the median value of  $a$  within  
 236 80 m external buffer zone had a lower relative error and higher overall accuracy. Therefore, we  
 237 defined  $a$ -value as the median slope within the 80 m buffer zone surrounding the MDL boundary.  
 238 The choice of buffer zone distance can be adjusted based on the specific terrain characteristics of  
 239 the research area, allowing researchers to adapt the methodology to their data accuracy.

240 Determining the appropriate thresholds for  $m$ ,  $n$ , and  $r$  of different MDL types is challenging  
 241 as methods for extracting these parameters vary depending on the MDL types. In other words, due  
 242 to the different types of glacial lakes, the values of  $m$ ,  $n$ , and  $r$  vary. Additionally, these values change  
 243 with the size of the glacial lake. To enable the model to automatically identify and calculate the  
 244 corresponding  $m$ ,  $n$ , and  $r$  for each glacial lake, we need to define a threshold. Based on the geometry  
 245 of the glacial lake, we established a proportional relationship between  $m$ ,  $n$ ,  $r$ , and the glacier lake  
 246 length ( $l$ ). This proportional relationship is empirically defined and essentially represents a  
 247 geometric segmentation of the glacial lake. The lake is divided into three sections, and the volume  
 248 of each section is calculated separately. The total water storage of the lake is then obtained by  
 249 summing the volumes of these three sections. Relying on  $R$ , lake boundary from Wang et al. (2020)

250 as well as DEM,  $m$  and  $n$  were estimated for GUL-1 and GUL-2 as shown in Table 3. In the case of  
 251 GCL-1,  $l = m$  due to its small area of water surface. For GCL-2,  $m$  was determined as 35% of  $l$  for  
 252 lakes with  $0.50 < R < 0.69$ , 30% of  $l$  for lakes with  $0.30 < R < 0.49$  and 20% of  $l$  for lakes with  $R < 0.30$   
 253 (Table 3).

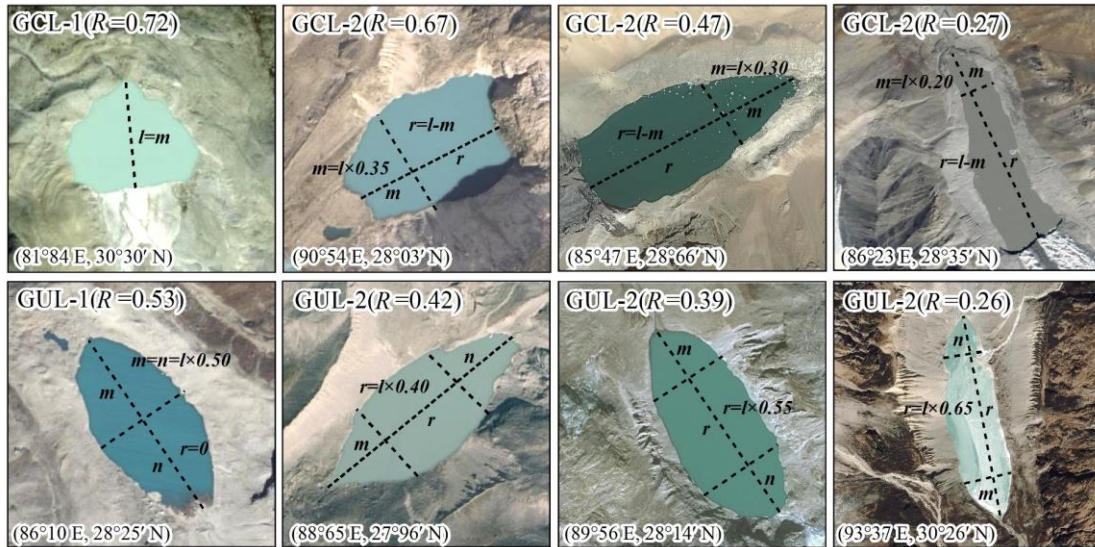
254 For GUL-1,  $R$  ranges from 0.50 to 1.0, both  $m$  and  $n$  are considered equal to half of  $l$ . On the  
 255 other hand, for GUL-2, it is possible to estimate the MDL water storage solely based on  $r$ , as  
 256 described in Equation 7. Accordingly,  $r$  values were statistically set up as  $0.4l$ ,  $0.55l$ , and  $0.65l$ ,  
 257 respectively with three  $R$  levels (Table 3). Figure 5 illustrates several representative cases of MDLs.

258 The above quantitative question about  $m$ ,  $n$  and  $r$  is not based on subjective judgment. First,  
 259 we computed the  $R$  values for all glacial lakes utilizing catalog data, then categorized them by glacial  
 260 lake type, and finally, we provided a definition by statistically assessing the shape of glacial lakes.  
 261 This definition pertains to the proportionality of  $m$ ,  $n$ , and  $r$  concerning the  $l$  of the glacial lake.  
 262 Consequently, our model is capable of autonomously classifying each glacial lake type through  
 263 boundary data analysis. It further computes various parameters for each lake, encompassing  $m$ ,  $n$ ,  $r$ ,  
 264 and  $h$ , ultimately culminating in the determination of the water storage for each lake.

265 **Table 3** Quantification of model input parameters.

Lake type	Calculation rules of model input parameters					
	$a$	$w, l$	$R$	$m$	$n$	$r$
GCL-1			$0.70 \leq R \leq 1.0$	$l$	0	0
			$0.50 \leq R \leq 0.69$	$l \times 0.35$	0	$l - m$
GCL-2	Median slope within the 80 m buffer zone outside the lake boundary	$w$ is the diameter of the largest inscribed circle and $l$ is the maximum length of the minimum bounding geometry	$0.30 \leq R \leq 0.49$	$l \times 0.30$	0	$l - m$
			$0.10 \leq R \leq 0.29$	$l \times 0.20$	0	$l - m$
GUL-1			$0.50 \leq R \leq 1.0$	$l \times 0.50$	$l \times 0.50$	0
			$0.40 \leq R \leq 0.49$			$l \times 0.40$
GUL-2			$0.30 \leq R \leq 0.39$	$l - r$		$l \times 0.55$
			$0.10 \leq R \leq 0.29$			$l \times 0.65$

266



267

268

**Figure 5.** Example for the extraction of input parameters for different types of MDLs. The base map is a Google Earth image (©Google Earth).

269

270

We executed our workflow (Figure 6) on 44 MDLs in High Mountain Asia that have known depths and water storages. For each lake, we checked whether its outline was in contact to the parent glacier. We automatically fitted a rectangular bounding box to calculate  $R$ , and then automatically assigned each lake to one of the four types of MDL based on  $R$  thresholds (Table 1). Finally, we estimated their water storages using our and traditional empirical relationships. Our model requires MDL boundary and DEM data as inputs, and it automatically quantifies each parameter while selecting the optimal model for water storage estimation.

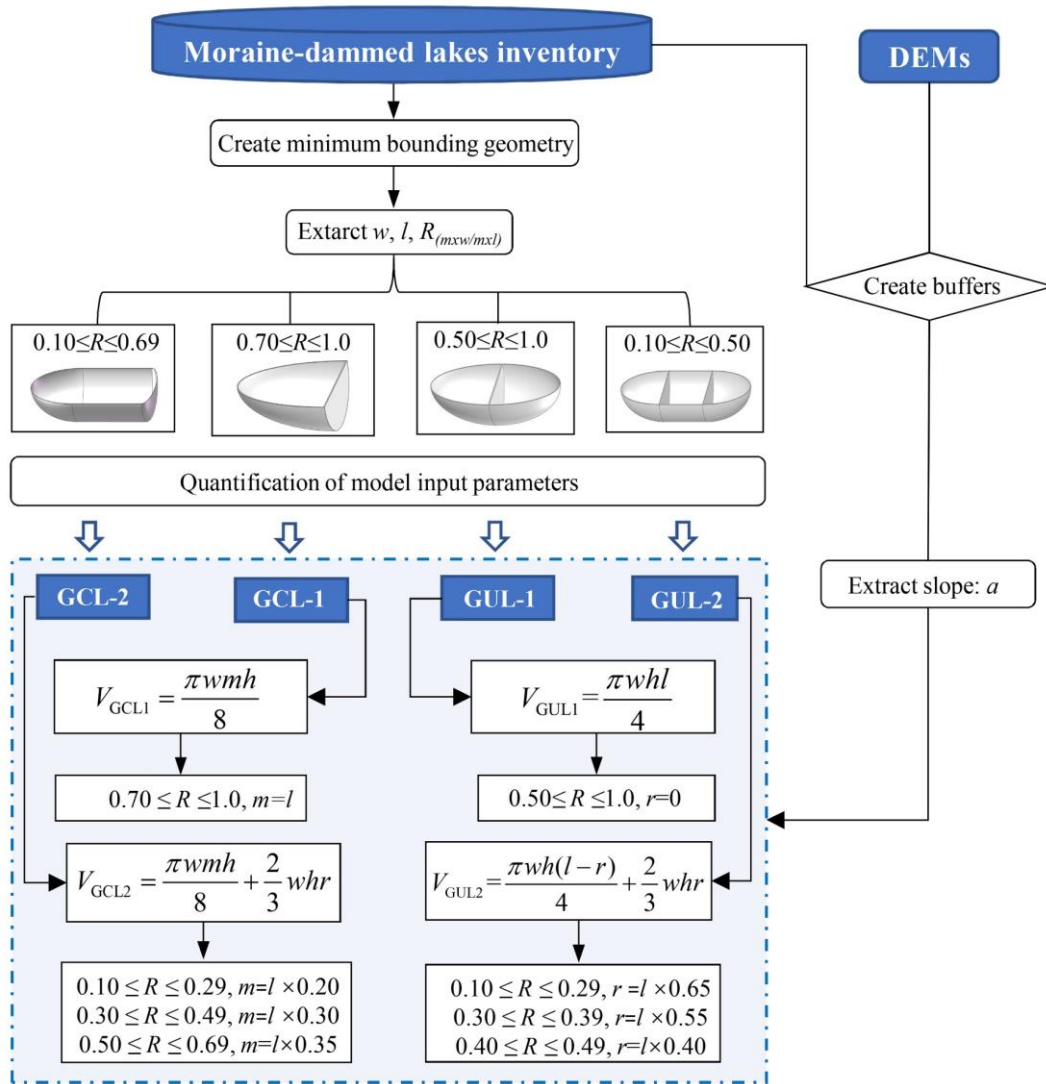
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277

Finally, we applied our model to more than 10,000 glacial lakes with unknown bathymetry in High Mountain Asia. This region had one of the highest rates of MDLs growth in the world in past decades.

278

279



280

281 **Figure 6.** The flow chart of the model procedure derivation.

282 **3.4. Model validation and application**

283 In this study, we initially validated our parameterization using bathymetric measurements from  
 284 four representative glacial lakes surveyed between 2020 and 2021. Subsequently, we combined the  
 285 data from these four lakes with the remaining six glacier lakes we measured, along with water  
 286 storage data from 34 MDLs obtained from relevant literature sources (see Appendix A for details).  
 287 This resulted in a dataset of 44 lakes, which was used to compare and validate the performance of  
 288 our model against other existing methods.

289 A glacier lake inventory of the High Mountain Asia region, published by Wang et al, 2020 was  
 290 used as input data for the model application to assess the water storage of moraine-dammed lakes  
 291 in this region. Notably, Wang's glacier lake inventory provides a detailed classification of GCL and  
 292 GUL, which has been internationally recognized. It is important to note that in his dataset, GUL

293 refers specifically to glacier lakes that do not contact glaciers, which may not necessarily all be  
294 moraine-dammed lakes. ~~We conducted a thorough review and made revision to ensure that we~~  
295 ~~retained only those GULs classified as moraine-dammed lake.~~ To ensure the accuracy of our analysis,  
296 we conducted a thorough review based on the classification criteria proposed by Yao et al., (2018)  
297 which identify three types of moraine-dammed lakes: (1) lakes situated between the end moraine  
298 ridge and the glacier terminus, (2) lakes beside the lateral moraine ridge, and (3) lakes on the  
299 moraine ridge. Each GUL in the dataset was individually assessed against these criteria, and only  
300 those meeting the classification as moraine-dammed lakes were retained for further analysis.

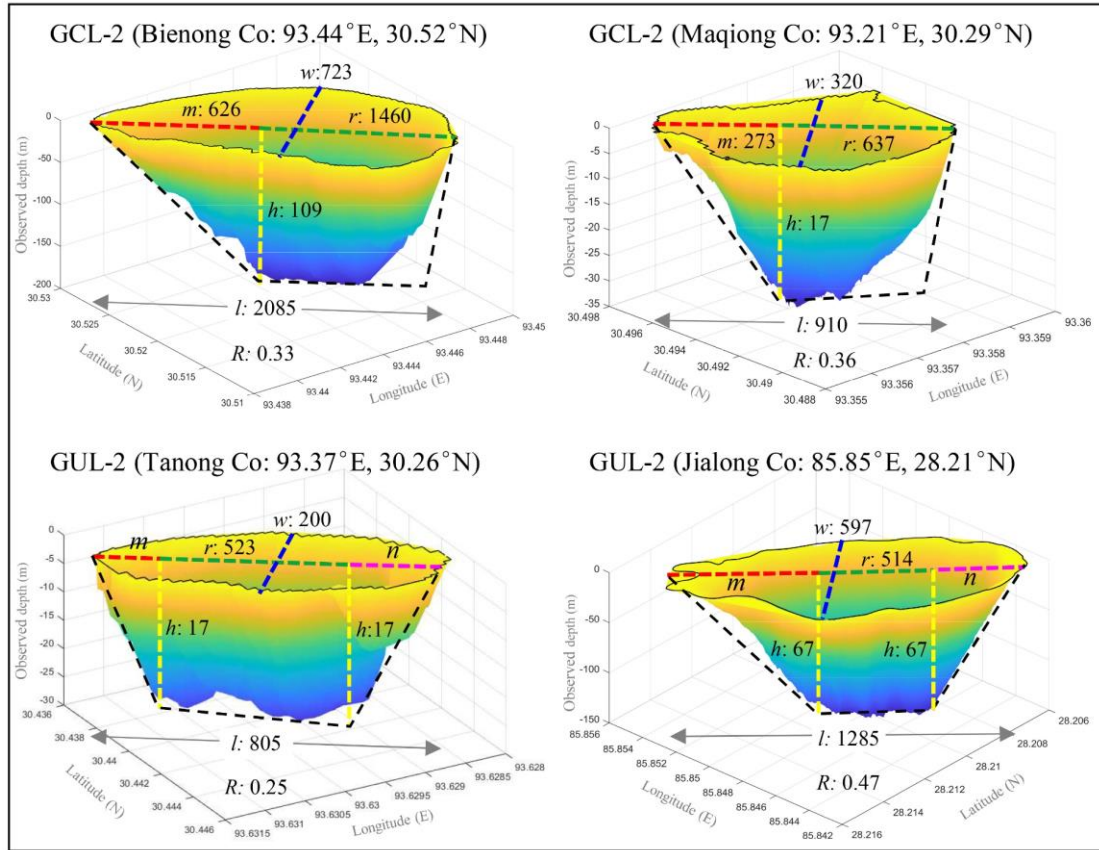
## 301 **4. Results**

### 302 **4.1. Model validation**

303 We validated our parameterization using bathymetry measurements from four representative  
304 glacial lakes, namely, Bienong Co, Maqiong Co, Tanong Co, and Jialong Co, located in the Qinghai-  
305 Tibet Plateau. These lakes represent the four types of glacier lakes, with depths measured through  
306 bathymetric surveying (Figure 7). In comparing estimated with measured water storages (Table 4),  
307 we find that Jialong Co has the highest accuracy with a relative error of only 1%. Maqiong Co and  
308 Tanong Co are overestimated by approximately 5% and 7%, respectively. The largest lake, Bienong  
309 Co, had an underestimated water storage of 6%.

310 In addition, our model is designed to approximate the mean depth of MDLs and therefore  
311 underestimates the maximum measured lake depth by about 50% (Table 4). Modeled mean water  
312 depths only deviate by 18% (mean) from the measured mean water depths. Except for a notable  
313 prediction error for Bienong Co (+47%), errors for Jialong Co, Tanong Co, and Maqiong Co range  
314 from 6% to 13% relative to the measured values.

315 In summary, our model has a high degree of concordance with observed glacial lake water  
316 storages and provides better estimations of water depth compared to the measured average depths.  
317 This suggests that our proposed model can be used in glacial lake water storage estimation and the  
318 management of GLOF hazards.



319

320 **Figure 7.** Subaqueous glacial lake morphology based on bathymetric surveys. The black dashed line represents the  
 321 hypothetical longitudinal profile of the glacial lake;  $l$  and  $w$  are measured from the lake boundary,  $h$  is simulated  
 322 lake depth and the remaining parameters ( $m$ ,  $n$ ,  $r$ ) are calculated by rule in Table 3. Lake depth is exaggerated.

323 **Table 4** Validation results of the mathematical model.

Name	Year of survey	Type	Area (km <sup>2</sup> )	Lake depth (m)			Water storage (10 <sup>6</sup> m <sup>3</sup> )		
				Observed (max/mean)	Simulated (mean)	Relative error	Observed	Simulated	Error
Bienong Co	2021	GCL2	1.16	181/74	109	+47%	102.00	95.689	-6%
Maqiong Co	2021	GCL2	0.22	34/16	17	+6%	3.325	3.581	+7%
Tanong Co	2021	GUL2	0.13	29/15	17	+13%	1.821	1.915	+5%
Jialong Co	2020	GUL2	0.55	135/62	67	+8%	37.530	37.952	+1%

324

## 325 4.2. Comparison with other methods

326 Table 5 displays the dataset of glacial lake bathymetry used in this study to validate the model.  
 327 We compared our model with another model that employed the lake geometry (Zhou et al., 2020),  
 328 and also with 20 additional formulas (EqS1-EqS20) collated by Qi et al. (2022) in Table S1. In the  
 329 estimation of a single MDL, formulas EqS4, EqS6, EqS13, EqS17, and EqS20 displayed significant  
 330 inaccuracies (132% - 853%). For instance, EqS13 shows an average error of 853%. Consequently,



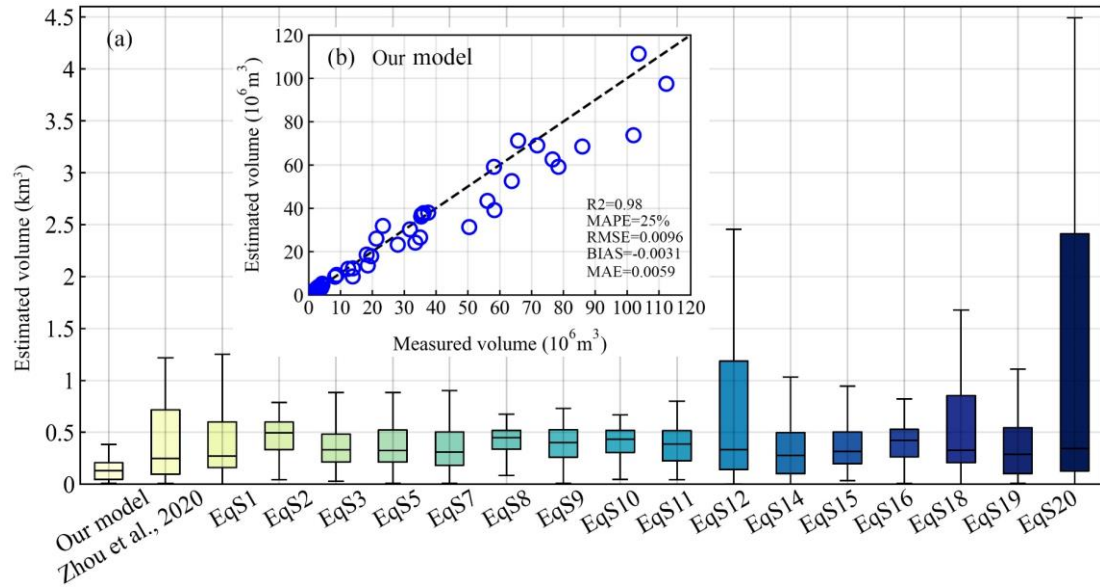
331 we have refrained from conducting a comparative analysis of these five formulas in the subsequent  
 332 discussions.

333 **Table 5** The glacial lake bathymetry data set used in this study. The lake bathymetry data are shown in bold provided  
 334 by this study, and the rest are obtained from references, see Appendix A for details.

Lake Name	Type	Area (km <sup>2</sup> )	Water storage(10 <sup>6</sup> m <sup>3</sup> )		Measurements based on remote sensing images						
			Measured	Estimated	<i>l</i>	<i>w</i>	<i>R</i>	<i>a</i>	<i>m</i>	<i>r</i>	<i>h</i>
<b>Kajiaqu</b>	<b>GCL2</b>	<b>0.29</b>	<b>3.45</b>	<b>3.00</b>	<b>1436</b>	<b>230</b>	<b>0.13</b>	<b>14</b>	<b>287</b>	<b>1149</b>	<b>15</b>
<b>Bienong Co</b>	<b>GCL2</b>	<b>1.17</b>	<b>102.00</b>	<b>95.69</b>	<b>2085</b>	<b>723</b>	<b>0.33</b>	<b>31</b>	<b>626</b>	<b>1460</b>	<b>109</b>
<b>Longmuqie Co</b>	<b>GCL2</b>	<b>0.58</b>	<b>8.28</b>	<b>8.47</b>	<b>1775</b>	<b>380</b>	<b>0.21</b>	<b>12</b>	<b>355</b>	<b>1420</b>	<b>21</b>
<b>Tanong Co</b>	<b>GUL2</b>	<b>0.13</b>	<b>1.82</b>	<b>1.92</b>	<b>805</b>	<b>200</b>	<b>0.25</b>	<b>19</b>	<b>0</b>	<b>523</b>	<b>17</b>
<b>Maqiong Co</b>	<b>GCL2</b>	<b>0.22</b>	<b>3.32</b>	<b>3.58</b>	<b>910</b>	<b>320</b>	<b>0.36</b>	<b>12</b>	<b>273</b>	<b>673</b>	<b>17</b>
<b>Zhasuo Co</b>	<b>GUL2</b>	<b>0.33</b>	<b>4.28</b>	<b>5.18</b>	<b>890</b>	<b>380</b>	<b>0.4</b>	<b>12</b>	<b>0</b>	<b>356</b>	<b>21</b>
<b>Jialong Co</b>	<b>GUL2</b>	<b>0.55</b>	<b>37.53</b>	<b>37.95</b>	<b>1285</b>	<b>597</b>	<b>0.46</b>	<b>24</b>	<b>0</b>	<b>514</b>	<b>67</b>
<b>Paqu Co</b>	<b>GUL2</b>	<b>0.58</b>	<b>8.80</b>	<b>9.22</b>	<b>2134</b>	<b>314</b>	<b>0.15</b>	<b>14</b>	<b>0</b>	<b>1387</b>	<b>19</b>
<b>Chmaqudan Co</b>	<b>GUL2</b>	<b>0.56</b>	<b>19.61</b>	<b>17.91</b>	<b>1459</b>	<b>450</b>	<b>0.31</b>	<b>19</b>	<b>0</b>	<b>802</b>	<b>38</b>
<b>Tara Co</b>	<b>GUL2</b>	<b>0.23</b>	<b>2.64</b>	<b>3.19</b>	<b>1024</b>	<b>255</b>	<b>0.26</b>	<b>15</b>	<b>0</b>	<b>666</b>	<b>17</b>
Jialong Co	GUL2	0.46	18.20	18.59	1133	537	0.47	17	0	453	41
Rewuco	GCL1	0.42	13.85	8.52	839	613	0.73	15	839	0	42
PoiqNo.1	GCL2	0.09	2.53	2.21	428	300	0.64	22	150	278	30
Ranzeria Co	GCL2	0.29	3.88	3.16	1181	288	0.23	12	236	945	15
BethungTsho	GCL2	0.45	4.28	4.51	1355	373	0.28	9	271	1084	15
Guangxie Co	GCL2	0.41	2.61	2.71	1032	390	0.3	7	310	722	12
Shishapangma	GCL2	0.6	18.59	13.61	1721	500	0.29	12	344	1377	26
Lugge	GCL2	1.63	71.76	69.02	3163	578	0.18	23	633	2531	62
Raphstreng2	GCL2	1.31	58.19	59.13	2117	816	0.39	16	635	1482	59
Galong Co	GCL2	5.49	377.39	403.18	4284	1500	0.35	16	1285	2999	107
Bnecuoguo Co	GUL1	0.11	1.69	1.98	490	288	0.59	14	0	0	18
Cirenma Co	GUL2	0.33	12.43	12.03	1276	367	0.29	22	0	829	36
Longbasaba	GCL2	1.15	56.16	43.47	2114	680	0.3	17	634	1479	52
Midui	GCL2	0.22	1.13	1.34	968	280	0.31	7	290	678	8
Lugge	GCL2	1.18	58.30	39.18	2520	545	0.2	19	504	2016	47
Thulagi	GCL2	0.76	31.80	30.33	1991	437	0.22	28	398	1593	57
Tsho Rolpa	GCL2	1.39	76.60	62.59	2942	590	0.2	22	588	2353	59
Imja Tsho	GCL2	0.6	28.00	23.18	1341	543	0.38	22	402	939	54
Cirenma Co	GUL2	0.33	13.90	12.23	1276	370	0.29	22	0	829	37
Pidahu	GCL2	0.89	50.44	31.37	2071	500	0.21	22	414	1657	50
Imja Tsho	GCL2	1.14	63.80	52.55	2191	605	0.24	23	438	1753	65
South Lhonak	GCL2	1.31	65.80	71.22	2328	715	0.31	22	699	1630	73
Tam Pokhari	GCL2	0.45	21.25	26.02	1178	470	0.41	34	353	825	80
Thulagi	GCL2	0.91	23.30	31.83	2522	417	0.17	25	504	2017	49
Imja Tsho	GCL2	1.03	35.50	37.03	2028	556	0.27	21	406	1622	54
Thulagi	GCL2	0.94	35.37	36.19	2541	430	0.17	27	508	2033	54

Tsho Rolpa	GCL2	1.54	85.94	68.58	3304	566	0.17	23	661	2643	60
Thulagi	GCL2	0.92	36.10	37.75	2504	439	0.18	27	501	2003	56
Lower Barun	GCL2	2.14	103.60	111.38	3297	730	0.22	23	659	2638	76
Lower Barun	GCL2	1.77	112.30	97.45	3091	717	0.23	22	618	2473	72
Imja Tsho	GCL2	1.15	78.40	59.12	2208	610	0.24	25	442	1767	72
Amphulapche	GUL1	0.12	3.20	3.79	404	369	0.99	19	0	0	32
Chamlang Tsho	GCL2	0.76	35.00	26.53	1627	588	0.32	18	488	1139	47
Imja Tsho	GCL2	0.75	33.48	24.13	1557	550	0.32	19	467	1090	48

335 Our assessment (Table 6) involves the relative error (RE, absolute value), bias, root mean  
336 square error (RMSE), mean absolute percentage error (MAPE) and mean absolute error (MAE) to  
337 quantify the uncertainty of new model. We use the coefficient of determination  $R^2$  to describe the  
338 goodness of fit between the model-derived data series and the measured data. Accordingly, our  
339 model had an  $R^2$  value of approximately 0.98, indicating a strong correlation between observed and  
340 predicted lake water storages (Figure 8). Moreover, our model has the lowest variance, according  
341 to a bias (-0.0031 km<sup>3</sup>), MAE (0.0059 km<sup>3</sup>), RMSE (0.0096 km<sup>3</sup>), and MAPE(25%). Also, our  
342 model has the lowest average relative error, at around 14%. The average relative error of EqS2,  
343 EqS3, EqS5, EqS7, EqS9, EqS11, EqS15 and EqS16 ranged from 44% to 50%, while the remaining  
344 formulas display average relative errors exceeding 50%. Although all equations achieved  $R^2 > 0.93$ ,  
345 the predicted values have a high variance and tend to either overestimate or underestimate the water  
346 storage of glacial lakes. Compared with our method, their bias, MAE, RMSE, and MAPE were all  
347 55%, 64%, 52% and 64%, respectively, and thus higher than ours. EqS7 had a better prediction  
348 accuracy. However, its bias, MAE and RMSE values are 82%, 64% and 52% higher than those of  
349 our model, respectively. This indicates a significant estimation error for specific glacial lakes, and  
350 both RMSE and MAE are sensitive to outliers. Overall, most of the equations tend to underestimate  
351 glacial lake water storages, with the underestimation becoming more pronounced for larger water  
352 storages. Nevertheless, we consider the accuracy level of our method to be acceptable due to the  
353 lower uncertainty compared to other models, providing an alternative for predicting the water  
354 storage of MDLs.



355

356 **Figure 8.** Comparison of the overall performance in glacial lake water storage estimation between our and

357 previous models (a) and comparison of measured and estimated water storage by our model (b).

358

359 **Table 6** Comparison of all empirical scaling relationships (EqS1-EqS20) in terms of bias, mean absolute error (MAE)

360 and root mean square error (RMSE) are measured in cubic kilometers. See Appendix B for details.

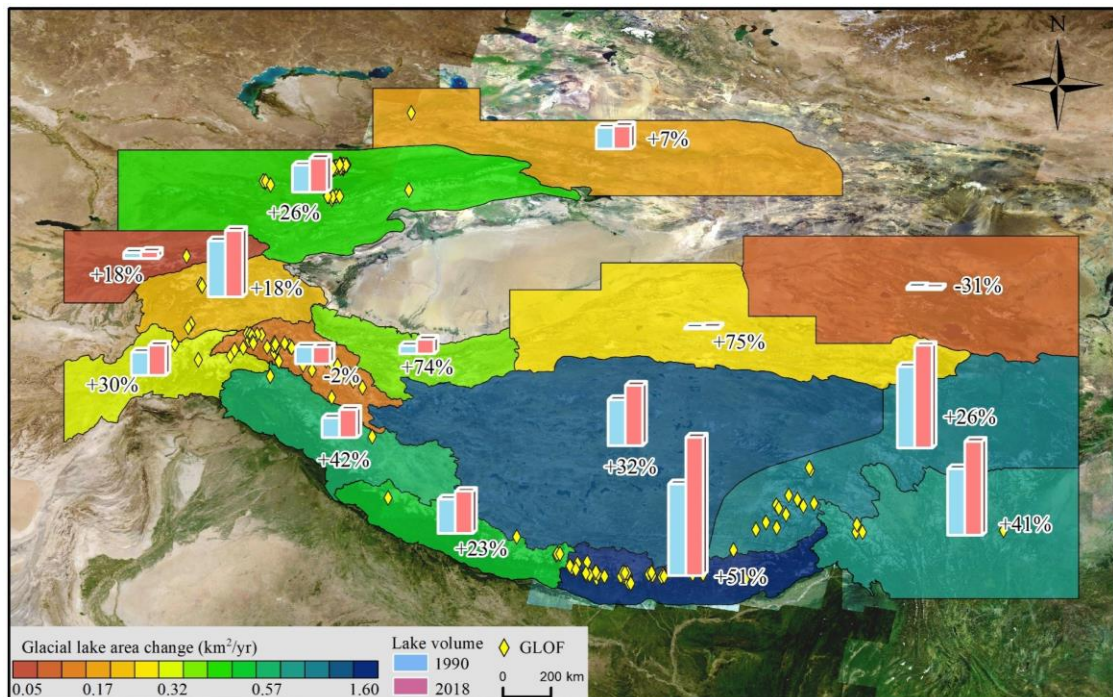
Equation	RE	BIAS	MAE	MAPE	R <sup>2</sup>	RMSE
<b>Our model</b>	<b>14%</b>	<b>-0.0031</b>	<b>0.0059</b>	<b>25%</b>	<b>0.9793</b>	<b>0.0096</b>
Zhou et al., 2021	53%	0.0097	0.0142	95%	0.9289	0.0485
Eq1	63%	-0.0060	0.0104	49%	0.9654	0.0174
Eq2	49%	-0.0185	0.0192	130%	0.9521	0.0299
Eq3	50%	-0.0074	0.0100	44%	0.9556	0.0150
Eq4	164%	0.0448	0.0448	120%	0.9494	0.1035
Eq5	45%	-0.0056	0.0112	51%	0.9418	0.0182
Eq6	219%	0.0609	0.0609	130%	0.9509	0.1331
Eq7	48%	-0.0056	0.0097	41%	0.9516	0.0146
Eq8	52%	-0.0162	0.0177	117%	0.9621	0.0295
Eq9	49%	-0.0126	0.0143	74%	0.9556	0.0213
Eq10	50%	-0.0149	0.0164	98%	0.9596	0.0262
Eq11	49%	-0.0112	0.0131	63%	0.9551	0.0192
Eq12	94%	0.0089	0.0118	37%	0.9642	0.0186
Eq13	853%	0.2362	0.2362	159%	0.9590	0.4404
Eq14	51%	0.0022	0.0113	61%	0.9438	0.0268
Eq15	46%	-0.0048	0.0110	50%	0.9430	0.0182
Eq16	44%	-0.0153	0.0160	88%	0.9288	0.0230
Eq17	316%	0.2088	0.2089	292%	0.8736	0.7300
Eq18	77%	0.0178	0.0207	98%	0.9418	0.0582
Eq19	50%	0.0036	0.0124	74%	0.9379	0.0336

Eq20	132%	0.000238	0.0132	59%	0.9501	0.0245
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361 **4.3. Application of the new model**

362 Considering the frequent occurrence of GLOF events in High Mountain Asia, posing threats to  
 363 downstream infrastructure and the safety of the lives and properties of the local communities,  
 364 assessing the water storage of glacial lakes is crucial for management potentially hazardous ones  
 365 (Nie et al., 2023). Therefore, this study employs our model to provide preliminary estimates of  
 366 glacial lake water storages in the study area.

367 A glacial lake inventory data (Wang et al., 2020) reveals that in 2018, there were a total of  
 368 13,166 glacial lakes ( $\geq 0.01 \text{ km}^2$ ) distributed in High Mountain Asia. The dataset highlights a  
 369 significant increase in both the number and area of GCLs from 1990 to 2018, experiencing a  
 370 remarkable growth of 52% and 54%, respectively. Model estimation results indicate that the total  
 371 glacial lake water storage in the study area was  $37.18 \text{ km}^3$  in 2018. Over the past three decades, the  
 372 overall MDL's water storage increased by  $8.94 \text{ km}^3$  from  $28.24 \text{ km}^3$  in 1990, representing a growth  
 373 of approximately 32%. The expansion rates of glacial lakes varied significantly across different  
 374 regions (Figure 9). Notably, the Hindu Kush-Karakoram and the central and eastern of the  
 375 Himalayas to the Hengduan Mountains witnessed the fastest increases in both glacial lake area and  
 376 water storage.



378 **Figure 9** Changes in the area and water storage of glacial lakes from 1990 to 2018 in High Mountain Asia. The base

379 map is a Google Earth image (©Google Earth).

380 The Eastern Himalayas had the largest gain in both the area and water storage of glacial lakes,  
381 concurrently establishing it as a hotspot for frequent GLOFs (Figure 9). The results indicate that the  
382 water storage of 1,410 MDLs ( $\geq 0.01 \text{ km}^2$ ) within the study area was  $9,337 \pm 990 \times 10^6 \text{ m}^3$  in 2022.  
383 Among these, GCLs and GULs account for 70% and 30% of the total water storage, respectively.  
384 Between 1990 and 2022, the total water storage in glacial lakes representing a substantial growth of  
385 162%. Notably, GCLs contributed 134% with an average annual growth rate of  $8.8\% \text{ a}^{-1}$ , indicating  
386 an overall increase of 280%. In contrast, the change in the water storage of unconnected lakes  
387 remained relatively stable, experiencing a modest growth of 52% over the past 32 years,  
388 considerably lower than that of GCLs.

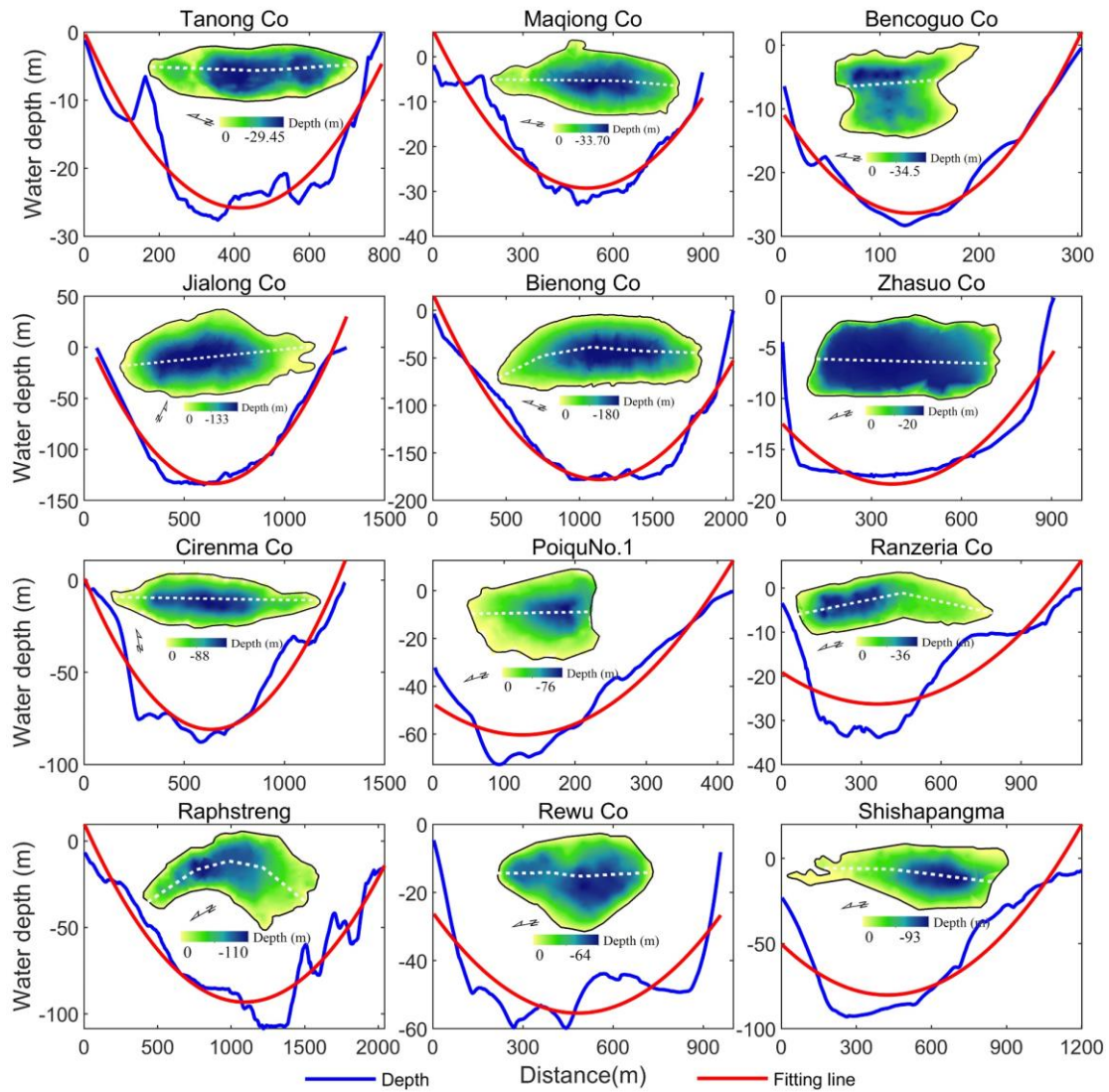
## 389 **5. Discussion**

### 390 **5.1 Justification and uncertainty of model assumptions**

391 In this study, we discuss the rationality and uncertainty of the model from three aspects. We  
392 first assumed that the MDL features a parabolic longitudinal bottom profile and a uniformly  
393 distributed sediment layer. The basin morphology of glacial lakes is a result of glacial erosion during  
394 the glacier retreat process. Glacier erosion involves certain lateral shear stress, leading to the  
395 formation of U-shaped valleys. Glacial lakes develop on these U-shaped valley terrains (Seddik et  
396 al., 2009). Therefore, based on the lake bathymetry and the longitudinal bottom profile of the MDLs  
397 (Figure 10), the variations in the underwater morphology of MDLs can be fitted with a parabolic  
398 curve. However, when observing trends in underwater topography, it is evident that some large and  
399 deep lakes (depth  $> 100 \text{ m}$ ), such as Jialong Co and Bienong Co, exhibit relatively flat underwater  
400 terrain, while others do not (Figure 7). This finding aligns with the research conducted by Carrivick  
401 and Tweed (2013), who proposed that most proglacial lake basins have flat landforms resulting from  
402 extensive sedimentation. These flat terrains, which were previously subdued and smoothed by  
403 glaciation, can become covered and obscured by thin layers of silts and clays. Furthermore, it has  
404 been suggested by some scholars that in large and deep proglacial lakes, the instability of the glacier  
405 margin and the increased likelihood of wave erosion can lead to the erosion of moraine ridges at the  
406 lake bottom (Murton et al., 2012).

407 The underwater landforms of some MDLs are not always a smooth parabolic shape. As  
408 depicted in Figure 11, the bottom topography of most glacial lakes exhibits a fluctuating parabolic

409 trend. Golledge (2008) and Bennett et al. (2000) revealed that subaqueous moraines in glacial lakes  
 410 often have linear or sinuous crests, and their ridges frequently exhibit heavily glactectonized  
 411 sediment structures indicative of compression. Although the presence of subaqueous moraines is  
 412 uncertain, this perspective offers a plausible explanation for the fluctuations in underwater  
 413 topography. In conclusion, concerning the formation process of subglacial geomorphology in MDLs  
 414 and lake bathymetry, both aspects substantiate our postulation that the MDL features a parabolic  
 415 longitudinal bottom profile. Furthermore, we hypothesize the presence of uniform sediment surface  
 416 to keep  $h_1 = h_2$ , although sediment distribution may be non-uniform due to factors such as the  
 417 position of the ice margin and water density (Carrivick and Tweed, 2013). As a result, the uneven  
 418 terrain at the bottom of some glacial lakes or the non-uniform distribution of sediments therein  
 419 constitutes one of the sources of uncertainty in the model.



420  
 421 **Figure 10.** The longitudinal bottom profile underwater topography of the MDLs obtained by bathymetry and the

422 fitting lines of terrain change trend (The white dotted line is the longitudinal profile line of the lake).

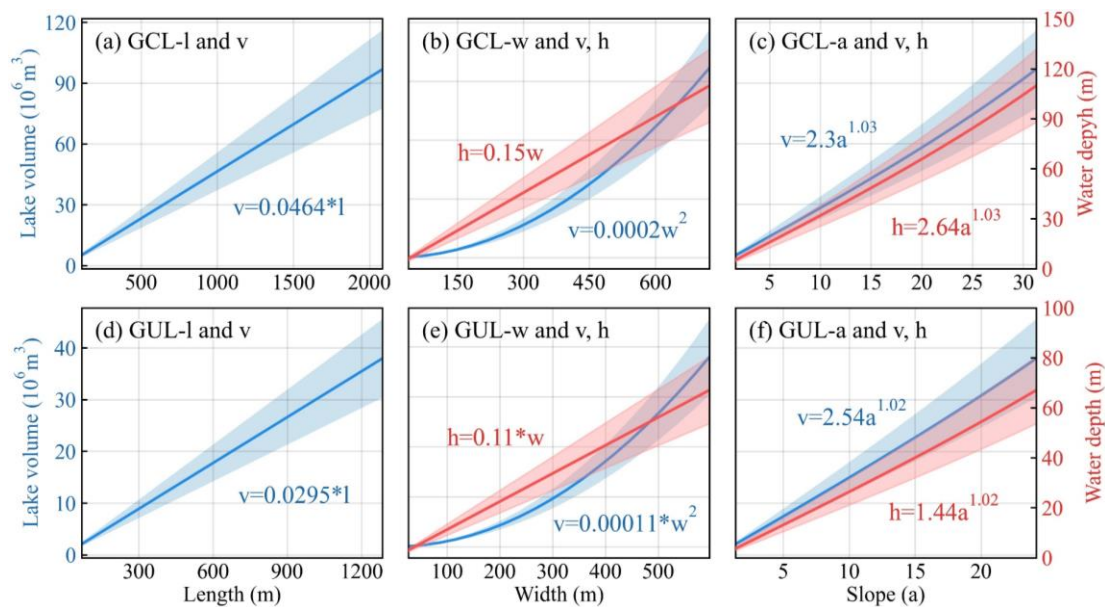
423 The second source of uncertainty in the model arises from the assumption regarding the lake  
424 surface of the MDL. Here, we assumed MDL's surface shape is characterized by an ellipse at both  
425 ends and a rectangle in between. MDLs develop on parabolic or U-shaped glacial troughs. A mature  
426 MDL, characterized by a relatively stable surface morphology, tends to exhibit an elliptical shape  
427 due to its geological characteristics (e.g., GUL lake type in Figure 5). Similar trends in the  
428 boundaries of MDLs are observed in different lake catalog datasets. Furthermore, in this study,  
429 MDLs are classified into four types based on their geometric shapes (see Table 1). Treating the  
430 complete geometric shape of an MDL as an ellipse allows the model to automatically partition the  
431 lake basin structure (e.g.,  $V_1$ ,  $V_2$ ,  $V_3$  in Figure 2) based on the lake's shape coefficient, facilitating  
432 the calculation of the water storage for MDLs with different morphologies. However, in reality, as  
433 suggested by Teller (1987) and Rubensdotter et al. (2009), factors such as the position of the glacier  
434 margin, surrounding landscape elevation and topography, and the location and elevation of lake  
435 overflow channels can affect the basin morphology of MDLs. For instance, Bencoguo Co and  
436 Raphstreng in Figure 10 do not exhibit the characteristic elliptical shape on the lake surface. This  
437 uncertainty in the geometric shape of the lakes may lead to an overestimation of lake water storage  
438 in the model, as the maximum width of the lake significantly influences the model results.

439 Finally, assuming the slope angle near the lake remains constant ( $\alpha=\beta$ ) is another aspect  
440 contributing to the uncertainty in the model. In actuality, the slopes surrounding the lake exhibit  
441 variations influenced by factors like the glacier tongue's position, the surrounding topography, and  
442 the presence of moraine ridges. This variability in slope angles can further contribute to the  
443 uncertainty when estimating the model's maximum water depth and water storage.

## 444 5.2 Sensitivity of model input parameters

445 Additionally, our model requires key parameters, namely,  $w$ ,  $l$ ,  $a$ ,  $m$ ,  $n$ , and  $r$ , with the  
446 relationship between  $m$ ,  $n$ ,  $r$ , and  $l$  defined as  $l = m + n + r$ . Thus, we only investigated the sensitivity  
447 of our model to  $l$ ,  $w$ , and  $a$ . Since water depth is closely related to  $w$  and  $a$  (see equation (13)), we  
448 also conducted parameter sensitivity tests on the estimated water depth using our model. In this  
449 study, we employed Jialong Co and Bienong Co as representatives of GUL and GCL of MDLs,  
450 respectively, to assess the sensitivity of the model to various parameters across different types of  
451 glacial lakes. Figure 11 (a-f) demonstrates the sensitivity of volume ( $v$ ) and water depth ( $h$ ) in our

452 model to variations in the maximum length ( $l$ ), maximum width ( $w$ ), and slope ( $a$ ) of glacial lakes.  
 453 Overall, there was a linear increase in glacial lake volume with changes in length (Figures 11 a and  
 454 d). As shown in Figures 11b and e, variations in maximum width exhibited a consistent power-law  
 455 relationship with volume, where volume increased exponentially with width. The water depth of  
 456 glacial lakes demonstrated a linear increase with changes in width. The slope of the lake's edge  
 457 showed a power-law relationship with both estimated water depth and volume (Figures 11e and f).  
 458 In summary, when estimating volume using our model, glacial lake width and slope were found to  
 459 be the most sensitive parameters, followed by the lake's length. Regarding water depth, the model  
 460 was most sensitive to the slope, followed by the width.



461  
 462 **Figure 11.** Parameter sensitivity analysis for glacial lake volume estimation using new model (note: the shaded  
 463 part represents the confidence interval, and definition of parameters in the figure as shown in Table 2).

## 464 6. Conclusion

465 Water storage plays a crucial role in predicting peak discharge of GLOFs. This study proposed  
 466 a mathematically robust and cost-effective approach for estimating lake water storage in regions  
 467 where field measurements of bathymetry are limited. The new model utilized lake geometry and  
 468 DEMs to estimate lake water storage. By parameterizing the model based on assumptions such as a  
 469 parabolic longitudinal bottom profile and consistent slope angles, it offers a reliable estimation of  
 470 lake water storage.

471 We validated our parameterization using bathymetry measurements from four representative  
 472 glacial lakes, namely, Bienong Co, Maqiong Co, Tanong Co, and Jialong Co, located in the Qinghai-



473 Tibet Plateau. Additionally, we applied the new model to 10 glacial lakes with depth measurements  
474 conducted during 2020-2021, and we included bathymetry data from 34 other glacial lakes sourced  
475 from published literature. Our model overcomes the autocorrelation issue inherent in earlier  
476 area/depth-water storage relationships and incorporates an automated calculation process based on  
477 the topography and geometrical parameters specific to MDLs. Compared to other models, our model  
478 achieved the lowest average relative error of approximately 14% when analyzing 44 observed data,  
479 surpassing the >44% average relative error from alternative models. This study model will allow  
480 researchers and practitioners to better predict potential outburst water storages and peak discharge  
481 of MDLs.

### 482 **Competing interests**

483 The contact author has declared that none of the authors has any competing interests.

### 484 **Data availability**

485 All data used in this study can be found in Table 5 and supplementary files.

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