

Referee #2

In what follows, we respond in turn to each of the comments (included verbatim, in **bold**).

Present work tries to develop a semi-analytical solution for the so-called reactive Lauwerier Problem (a thermos-hydro-chemical exercise with several simplification assumptions) aiming to evaluate the related porosity evolutions of the confined rock. The manuscript presents the development of a method that integrates various ideas, assumptions, and approaches, many of which have been previously presented. I believe that the manuscript is much better suited to possible publication as a 'Method' or 'Short Communications' paper than a 'Research Article'.

We respectfully disagree with the assessment provided by the Referee suggesting that the contribution of the present paper only “integrates various ideas, assumptions, and approaches that have been previously presented” and welcome direction to any literature we may have overlooked. First and foremost, our work represents an original, analytical contribution. To the best of our knowledge, there are no existing previously published analytical solutions for thermally-driven reactive flow in this type of geometry, i.e. injection of thermal fluid from a point source into a confined aquifer. In 1955, Lauwerier derived an analytical solution for the injection of thermal fluid into a confined reservoir, famously known as "the Lauwerier Problem" [Lauwerier, 1955]. Now, 70 years later, we have leveraged Lawrier's solution to describe temperature-dependent solubility and develop a thermally-driven reactive transport solution. We have termed this problem "The Reactive Lauwerier Problem" in our work. Our contribution predicts the coupling among temperature, solute concentration, flow rate, and porosity providing a novel understanding of thermally-driven reactive flow systems, which are relevant to natural, geothermal, and CO₂ applications.

Secondly, we emphasize that the solutions presented are fully analytical, without any associated approximations that would classify them as "semi-analytical". Furthermore, the manuscript presents also the solutions for the evolution of solute profiles in the aquifer (Eqs. 15 and 20), not solely porosity evolution (Eqs. 17 and 21). Additionally, we both present the solutions and use them to investigate common important case studies, including (I) hot CO₂-rich water injection into carbonate aquifers and (II) hot silica-rich water injection, leading to mineral dissolution and precipitation, respectively.

Lastly, we argue the derivation of analytical solutions is more than a technical advancement or exercise. Innovative analytical solutions are key to developing a basic theoretical description of physical processes, offer an understanding of the underlying mechanisms of a problem, and reveal relationships between variables and the fundamental properties of the system. As described in the Aims & Scope section, HESS encourages submissions of both fundamental and theoretical research. Therefore, we believe our work, which involves both theoretical and important case study investigations, is well-suited for publication as a Research Article in HESS.

- 1. The primary question would be the effectiveness of the method. There is no validation for the implementation of the methodology presented in this work (it is clear that I am not taking about validation of the assumption $t' \approx t$).**

We do not fully understand what the Reviewer means when referring to the effectiveness of the method. The article presents analytical solutions, not a numerical method, numerical computation is used in few places only to prevent the integer overflow, when one needs to deal with exponentiation of very large negative numbers. We are happy to address the Reviewer's comment with additional insight.

2. Authors may decide to convert and perform analysis in an dimensionless conditions.

As the Referee noted, in this work, we have chosen dimensional presentation over dimensionless presentation. This decision is based on the investigation of case studies presented in Section 3, which requires the use of explicit parameters with prescribed values. Consequently, we believe that dimensional presentation is more suitable in this case, in order to facilitate comparison with real-world case studies.

3. Please clearly state the programming environment that Authors scripted. Do Authors develop a toolkit to perform simulations or use an available commercial/open-source software?

Following the Reviewer's comment, the manuscript will be revised to state that Matlab was the programming software used. However, let us reiterate again that almost all of the curves presented in the manuscript represent analytical, closed-form solutions, which can be plotted using any graphical software. In specific instances, an approximate numerical solution was used to avoid integer overflow (Eq. D.2 in Appendix D). The manuscript currently states clearly when Eq. D.2 is used (lines 378, 395, and 496). In particular, to obtain the porosity profiles (Figs. 2b-c and 3) at specific times, an iterative numerical solution of Eq. D.2 was employed to avoid integer overflow. The manuscript will be revised to state more precisely when this iterative numerical procedure is used to attain the porosity profiles.

4. Authors suggested that present work can stand as a benchmark for complex coupled numerical codes. Then I recommend making the Code & Data availability to "Publicly Available" (not on request).

We agree with the Referee that the code and data should be publicly available, and therefore, we will make all code and data publicly accessible (and not just to Referees). However, it should be noted that the solutions (Eqs. 15, 17, D.2, 20, and 21) can be easily implemented in short scripts.

5. It is hard to follow the storytelling of the manuscript. I expect that the manuscript (by itself) must be enough to understand "What the Authors did?", instead supplementary materials can support some details on "How they technically implement the analysis?". Indeed, I see that the provided information is mixed among the manuscript and supplementary materials. I would strongly recommend Authors to rearrange the structure of the work.

We agree with the Referee that the manuscript itself should be sufficient to understand the work, with supplementary materials supporting technical details. This is also the philosophy that we were trying to follow when writing this manuscript. We kindly ask the Referee for specific comments on where it would be beneficial to move material from the Appendix to the main text. In our view, overloading the text with derivations could disrupt the flow of the manuscript, but we are open improving clarity in certain sections that may lack sufficient detail.

This work includes supportive appendices A-E. Their precise functionality is described below,

a) [Appendix A: An Extended Form of the Conservation Equations](#)

This appendix provides the expanded versions of the conservation equations and the assumptions that lead to the final equations appearing in the main text (section 2.3) and used to develop the solutions. Consequently, it supports the main work and is appropriate to appear in its place.

b) [Appendix B: Timescales Analysis to Validate the Quasi-static Assumption](#)

This appendix provides an in-depth analysis and discussion related to the quasi-static assumption. This section also supports the main work, and adding it to the main text would make the manuscript unnecessarily long and tedious. To make this section clearer, it will be revised to include separate equations instead of inline equations.

c) [Appendix C: Lauwerier Solution Validity Assuming \$t' \approx t\$](#)

This appendix provide support to the main text by showing the validity of the assumption $t' \approx t$.

d) [Appendix D: Asymptotic Expansion for the Disequilibrium Solutions](#)

This appendix presents an approximation for Eqs. 15 and 20, which are useful for efficient computation and to avoid computational overflow. Therefore, it should not be part of the main text.

e) [Appendix E: Permeability of an Aquifer with Nonuniform Porosity Profile](#)

This section provides technical details for calculation of the effective permeability and hence appears as an appendix.

6. - Line 152: I cannot see “numerical simulations” in present work, please specify.

Please see reply to comment 8.

7. - Please avoid discussion of the results of the Figures in captions (instead of the text body).

All the interpretations that appear in the caption also appear in the main text. We believe that non-purely technical captions are more informative to the reader. However, we are open to revising them.

8. - Line 190: last sentence “Thermal variations in the...” is incomplete.

Will be corrected.

9. - Table 1: Please move it to the end of paper, as “Nomenclature”.

We placed Table 1 early in the manuscript for easy reference to the list of parameters, but we are open to revise the location of Table 1.

10. - Please rearrange Eq. (4) not to start with zero value.

The presentation of the equation in its current form emphasizes the transient term is equal to zero. Typically, transient terms appear on the left-hand-side of the equation (see e.g., Battiato et al., 2009; Steefel et al., 2005; Steefel & Maher, 2009). Staying consistent with the presentation in past literature, the current presentation emphasizes that this is a steady-state equation, and the study adopts a quasi-static approach.

11. - Please avoid discussion of the literature in supplementary materials.

Please refer to the reply to comment 5.

References

- Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., & Scheibe, T. D. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. *Adv. Water Resour.*, 32(11), 1664–1673.
- Lauwerier, H. (1955). The transport of heat in an oil layer caused by the injection of hot fluid. *Applied Scientific Research, Section A*, 5(2), 145–150.
- Steeffel, C., Depaolo, D., & Lichtner, P. (2005). Reactive transport modeling: An essential tool and a new research approach for the Earth sciences. *Earth and Planetary Science Letters*, 240, 539–558. <https://doi.org/10.1016/j.epsl.2005.09.017>
- Steeffel, C. I., & Maher, K. (2009). Fluid-Rock Interaction: A Reactive Transport Approach. *Reviews in Mineralogy and Geochemistry*, 70(1988), 485–532. <https://doi.org/10.2138/rmg.2009.70.11>