Influence of bank slope on sinuosity-driven hyporheic exchange flow and residence time distribution during a dynamic flood event

Supporting Information

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This supporting information contains additional information on how the model used in our simulations was set up in COMSOL. Our modeling approach builds on the work of Gomez-Velez et al. (2017), which serves as a baseline case. Due to their large file size the COMSOL model files and the raw data to the figures in the manuscript are available upon request and we are delighted to share them directly. For this please contact Yiming Li (liym@cug.edu.cn) or Zhang Wen (wenz@cug.edu.cn).

S1 Water flow model

The water flow model is based on that of Gomez-Velez et al. (2017), comprising an alluvial valley with a sinusoidal meandering river that overlies non-permeable river deposits, as shown in Fig. S1. To simplify the model, aquifer properties are assumed to be spatially homogeneous and isotropic. This means they can be modeled by the commonly used vertical-integrated approach which can reduce a 3-D groundwater flow to a two-dimensional (2-D) problem, as shown in Fig. S2a. The model is bounded by hillslopes and a two-period fully penetrating sinusoidal river. By neglecting the compression of groundwater, the unsteady, 2-D transient groundwater flow through the deformable aquifer is described by the Boussinesq's equation:

$$S_y \frac{\partial h}{\partial t} = \nabla[K(h - z_b)\nabla h] \quad \text{(S1a)}$$

$$h(\mathbf{x}, t = 0) = h_0(\mathbf{x}) \quad \text{(S1b)}$$

$$\mathbf{n} \cdot \nabla \left[K(h - z_b)\nabla h\right] = 0 \quad \text{for } \Omega_v \quad \text{(S1c)}$$

$$h(x_u, y, t) = h(x_d, y, t) + 2\,\lambda J_x \quad \text{for } \Omega_u \text{ and } \Omega_d \quad \text{(S1d)}$$

$$h(\mathbf{x}, t) = \left(\frac{J_x}{\sigma}\right)s(x) + H_s(t) + 2\,\lambda J_x \quad \text{for } \Omega_{in} \cup \Omega_{out} \quad \text{(S1e)}$$

where $\mathbf{x} = (x, y)$ [L] is the spatial coordinate with x positive in the upstream direction, t [T] is simulation time, S_y [-] is specific yield, K [LT^{-1}] is the hydraulic conductivity, ∇ is the Laplace operator, $h(\mathbf{x}, t)$ and $h_0(\mathbf{x})$ [L] represent the hydraulic head at t and $t = 0$, while $z_b(\mathbf{x})$ [L] is the elevation of the underlying impermeable layer with respect to the reference datum $z = 0$ (see Fig. S2b and S2c), respectively. $H(\mathbf{x}, t) = h(\mathbf{x}, t) - z_b(\mathbf{x})$ [L] is the thickness of the saturated aquifer, $\mathbf{n}$ is the outward normal vector along the model boundary, Ω_v, Ω_u and Ω_d are the valley, upstream and downstream boundaries, respectively, while Ω_{in} and Ω_{out} are the inlet and outlet boundaries along the river. The fluxes are calculated by Darcy's law via: $\mathbf{q} = -K\nabla h$ [LT^{-1}]. Here, $\mathbf{q}$ is the specific discharge or Darcy flux, $\mathbf{q}/\theta$ [LT^{-1}] is the pore water velocity with θ [-] as effective porosity, and $\mathbf{Q} = \mathbf{q}(h - z_b)$ [L^2T^{-1}] is the aquifer-integrated discharge in our 2-D model. The valley boundary (Ω_v) is assigned as a no-flow boundary and located at $y = n\lambda$, with the scaling number $n = 4.5$, which has proven to be sufficiently large for this simulation based on a series of pre-simulation tests while λ [L] is the wavelength of the river sinusoid. The river has been assigned the known transient hydraulic head, $h_r(x, t) = (J_x/\sigma)s(x) + H_r(t)$ [m], where J_x [-] is the base head gradient of ambient flow along the valley in positive x direction, $H_r(t)$ [L] is the elevation of river stage above the impermeable deposit at the downstream end.

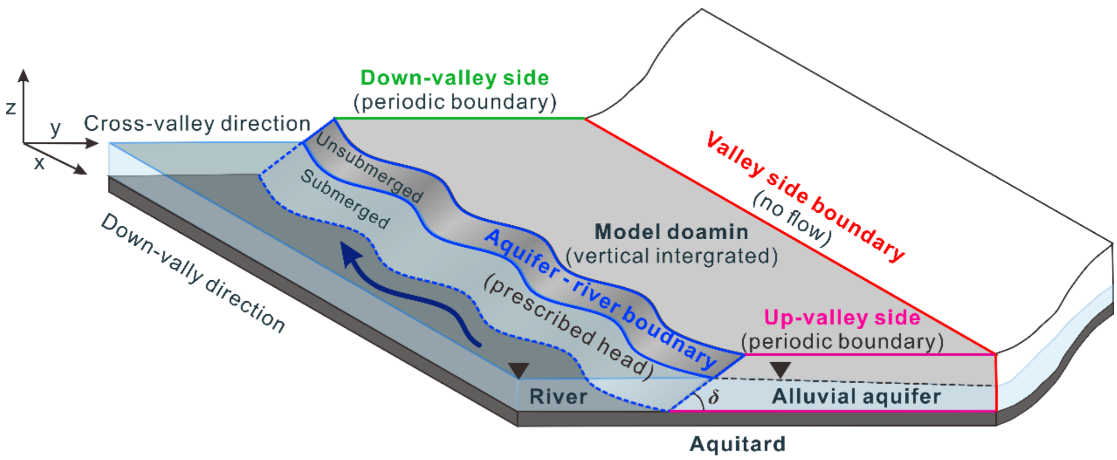

Figure S1. Conceptual model of the study area. Colored lines represent the river, up-valley, down-valley and valley side boundary conditions set in the model. Modified from Schmadel et al. (2016)

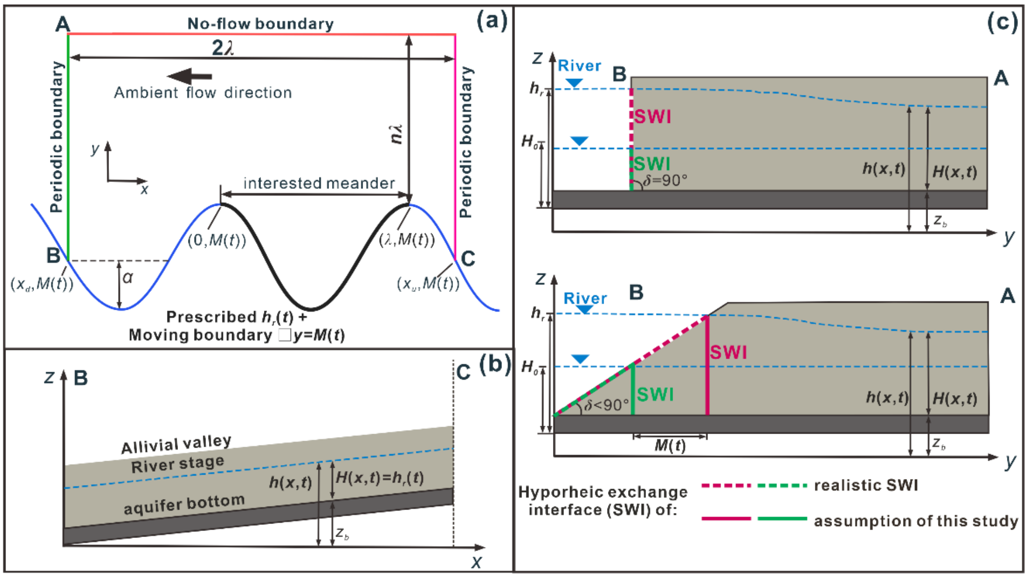

Figure S2. Modified after Gomez-Velez et al. (2017): (a) Schematic representation of the boundary conditions for the non-submerged alluvial system. The colors of the boundaries correspond to those in Fig. S1. (b) Representation of the stream stage variation along the channel thalweg. (c) Cross-section of unconfined aquifer and floodplain of vertical (δ = 90°) and sloping riverbank (δ < 90°). Green and red lines refer to the sediment-water interface (SWI) during base flow condition and flood event, respectively; the dashed lines on riverbank surface and the vertical bold lines in Fig. S2c indicate the realistic SWIs and SWIs of this study, respectively.

The river ($\Omega_{in} \cup \Omega_{out}$) is implemented as a sinusoid, following the conceptualizations of Boano et al. (2016), Cardenas (2009a, 2009b), and Gomez-Velez et al. (2017). The initial condition is represented as: $y_0(x) = \alpha\cos(2\pi x/\lambda) - \alpha$, where α [L] is the amplitude of the river boundary. Left- and right-bottom vertices in initial condition are located at $x_d = -3\lambda/4$ and $x_u = 5\lambda/4$, respectively.

The impermeable bottom deposit $z_b = J_x(x - x_d)$ [L] is assumed to be parallel to the alluvial valley. Ω_u and Ω_d are periodic with a known variable hydraulic head drop $h(x = x_u, y, t) = h(x = x_d, y, t) + 2\lambda J_x$ [L] to eliminate any boundary effects. Thus, the model domain can represent two periodic parts in horizontal direction of the infinite aquifer. The river stage fluctuates during the dynamic flood event following (Cooper and Rorabaugh, 1963):

$$H_s = \begin{cases} H_0 + H_p \exp\left\{ -\eta (t - t_p) \dfrac{[1 - \cos(wt)]}{[1 - \cos(wt_p)]} \right\} & \text{if } 0 < t < t_p \\ H_0 & \text{if } t_p < t \end{cases} \tag{S2}$$

where $H_0(\mathbf{x})$ [L] is the initial river stage, H_p [L] is the maximum (peak) river stage during the flood event, while t_d and t_p [T] are the duration of flood event and the time-to-peak river stage, respectively. $\omega = 2\pi/t_d$ [T^{-1}] is the flood event frequency, $\eta = \omega\cot(\omega t_p/2)$ [T^{-1}] represents the degree of flood event asymmetry. The peak river stage and time-to-peak are assumed to be linearly correlated with the base flow stage ($H_p = n_0 H_0$) and the duration of the event ($t_p = n_d t_d$), respectively. Constants n_0 [-] and n_d [-] represent river stage hydrograph intensity and skewness.

S2 Conservative solute transport model and calculation of HZ area (extent)

In this work, we adopt the mathematical model used by Gomez-Velez et al. (2017), where the transport of a conservative solute within the vertically integrated system is given by:

$$\frac{\partial (H\theta C)}{\partial t} = \nabla \cdot (\mathbf{D}\nabla C - \mathbf{Q}C) \tag{S3a}$$

$$C(\mathbf{x}, t=0) = C_0(\mathbf{x}) \tag{S3b}$$

$$\mathbf{n} \cdot (\mathbf{Q}C - \mathbf{D}\nabla C) = 0 \text{ for } \Omega_v \tag{S3c}$$

$$C(x_u, y, t) = C(x_d, y, t) \text{ for } \Omega_u \text{ and } \Omega_d \tag{S3d}$$

$$C(\mathbf{x}, t) = C_s(\mathbf{x}, t) \text{ for } \Omega_{in} \tag{S3e}$$

$$\mathbf{n} \cdot (\mathbf{Q}C - \mathbf{D}\nabla C) = 0 \text{ for } \Omega_{out} \tag{S3f}$$

where $C(\mathbf{x}, t)$, $C_0(\mathbf{x})$, and $C_S(\mathbf{x}, t)$ are the solute concentrations [ML^{-3}] in the aquifer, initial concentration and concentration in the river, respectively. The dispersion-diffusion tensor $\mathbf{D} = \{D_{ij}\}$ [L^2T^{-1}] is defined according to Bear and Cheng (2010) as:

$$D_{ij} = \alpha_T |\mathbf{Q}| \delta_{ij} + (\alpha_L - \alpha_T)\frac{Q_i Q_j}{|\mathbf{Q}|} + H\theta\epsilon D_L \qquad \text{(S4)}$$

where α_T and α_L [L] are the transverse and longitudinal dispersivity, respectively, D_L [L^2T^{-1}] is the water diffusivity, $\epsilon = \theta^{1/3}$ [-] represents tortuosity (Millington and Quirk, 1961), and δ_{ij} [-] is the Kronecker delta function.

In order to mimic a periodical repetition of the meanders in x direction and eliminate potential boundary effects, a periodic boundary condition (Eq. (S3d)) is used at Ω_u and Ω_d. This type of boundary condition can produce the periodic nature of the model domain, flow field as well as the HZ that repeats for each meander bend (Gomez-Velez et al., 2017). However, in order to explore the local HZ that is caused by the HEFs at the studied meander: $0 < x < \lambda$ (i.e., bold black line along the meander in Fig. S2a), the conservative in-stream concentration is given by

$$C_s(t) = \begin{cases} 1 & \text{if } x \in [0, \lambda] \cap \Omega_{in} \\ 0 & \text{else} \end{cases} \qquad \text{(S5)}$$

According to Eq. (S5), the river concentrations are assigned as an open boundary condition along the studied meander with the external concentration mimicking the concentration of the tracer (100% of stream water). Then the concentration of the pore water within the aquifer represents the fraction of water inflow from river at any given location and time.

S3 Model of residence time distribution

The residence time distribution (RTD) in the HZ describes the characteristic time scale over which water or solutes are exposed to the biogeochemical conditions within the hyporheic sediment. RTD is controlled by the advective and dispersive characteristics of the system. Similar to Gomez-Velez et al. (2017), here we focus on the orders of moment of RTD that represent the mean residence time distribution:

$$\mu_n(\mathbf{x}, t) = \int_0^\infty \tau^n \rho(\mathbf{x}, t, \tau)\,\mathrm{d}\tau \qquad n = 0, 1, \ldots \qquad \text{(S6)}$$

Where $\mu_n(\mathbf{x}, t)$ [T^n] is the n-th moment, $\rho(\mathbf{x}, t, \tau)$ [T^{-1}] is the residence time distribution,

τ [T] is residence time, and $\mu_0 = 1$. The first moment of RTD ($\mu_1(\mathbf{x}, t)$) is the mean residence time distribution at a given location and time, which can be used to evaluate the transient variation of RTD. Here, we used the approach provided by Gomez-Velez et al. (2017) where the moments of RTD are calculated by a form of the advection-dispersion equation following

$$\frac{\partial(H\theta\mu_n)}{\partial t} = \nabla\cdot(\mathbf{D}\nabla\mu_n - \mathbf{Q}\mu_n) + nH\theta\mu_{n-1} \tag{S6a}$$

$$\mu_n(\mathbf{x}, t=0) = \mu_{n,0}(\mathbf{x}) \tag{S6b}$$

$$\mathbf{n}\cdot(\mathbf{Q}\mu_n - \mathbf{D}\nabla\mu_n) = 0 \text{ for } \Omega_v \tag{S6c}$$

$$\mu_n(x_u, y, t) = \mu_n(x_d, y, t) \text{ for } \Omega_u \text{ and } \Omega_d \tag{S6d}$$

$$\mu_n(\mathbf{x}, t) = 0 \text{ for } \Omega_{in} \tag{S6e}$$

$$\mathbf{n}\cdot(\mathbf{Q}\mu_n - \mathbf{D}\nabla\mu_n) = 0 \text{ for } \Omega_{out} \tag{S6f}$$

where $\mu_{n,0}(\mathbf{x}, t)$ is the initial condition of the n-th RTD that is calculated by the base flow condition (steady forcing before the arrival of flood event), while the upstream and downstream boundaries are assigned periodic boundary conditions (Eq. (S6d). As we ignore the vadose zone, the RT is defined as the time since the water entered the model domain from the river. Thus, n-th RTD at the inflow river boundary is zero (Eq. (S6e). A flow boundary is used for the region where the water exits the model domain (Eq. (S6f)).

S4 Metrics

We used the following dimensionless metrics to quantify the effects of bank slope on the response of the dynamic hyporheic zone: (i) hyporheic exchange flux along the river, (ii) in-valley penetration distance (i.e., the distance the river water penetrates into the aquifer), (iii) the area of the HZ (i.e., the area of the aquifer exposed to river water), and (iv) RTD and flux-weighted relative RT of HZ water discharging into the river. In this section, we briefly define and describe each of these terms.

S4.1 Hyporheic exchange flux

Exchange flux from the river to the HZ ($Q_{in,\,HZ}$) and from the aquifer to the river ($Q_{out,\,HZ}$) was defined as:

$$Q_{in,HZ}(t) = -\int_{\partial\Omega_{in,HZ}(t)} \mathbf{Q}(\mathbf{x},t)\cdot\mathbf{n}\,\mathrm{d}s \tag{S7a}$$

$$Q_{out,HZ}(t) = -\int_{\partial\Omega_{out,HZ}(t)} \mathbf{Q}(\mathbf{x},t)\cdot\mathbf{n}\,\mathrm{d}s \tag{S7b}$$

where $\Omega_{in,\,HZ}\,(t)$ and $\Omega_{out,\,HZ}\,(t)$ correspond to the inflow and outflow boundaries along the meander of interest (black line along the river boundary in Fig. S2a). The net flux from the aquifer into the river ($Q_{net,\,HZ} = Q_{out,\,HZ} - Q_{in,\,HZ}$) can be expressed in dimensionless terms following Gomez-Velez et al. (2017) using $Q^{*}_{in,\,HZ}\,(t) = Q_{in,\,HZ}\,(t)/(K\overline{H}_s^2)$, $Q^{*}_{out,\,HZ}\,(t) = Q_{out,\,HZ}\,(t)/(K\overline{H}_s^2)$, and $Q^{*}_{net,\,HZ}\,(t) = Q_{net,\,HZ}\,(t)/(K\overline{H}_s^2)$. Note that these dimensionless fluxes are proportional to the integrated head gradient between the river stage and the adjacent aquifer along the river boundary.

S4.2 Hyporheic zone area

Dynamic changes of the river-aquifer interface and pressure distribution along the SWI induce variations of the flow field and changes to the HZ as represented by area (i.e., the aquifer area exposed to river water) and penetration distance (i.e., how far river water infiltrates into the aquifer) during the flood event. These are useful metrics for assessing the opportunity for biogeochemical and geochemical reactions induced by hyporheic exchange. Here we use a geochemical definition of HZ proposed by Triska et al. (1989), that defines the HZ as the area within the alluvial valley that contains more than 50% stream water ($C(\mathbf{x},\,t) > 0.5$). It can be calculated using

$$A(t) = \iint a(\mathbf{x},t)\,\mathrm{d}x\mathrm{d}y \tag{S8a}$$

$$a(\mathbf{x},t) = \begin{cases} 1 & \text{if } C(\mathbf{x},t) \geq 0.5 \\ 0 & \text{if } C(\mathbf{x},t) < 0.5 \end{cases} \tag{S8b}$$

where $A(t)$ [L^2] is the area of the HZ. The dimensionless area is then defined similar to

Gomez-Velez et al. (2017) as $A^*(t) = A(t)/\lambda^2$ and the dimensionless variation of HZ area relative to base flow conditions can be calculated by $A^{**}(t) = A^*(t)$ - $A^*(0)$, where $A^*(0)$ is the initial area of HZ in baseflow condition.

S4.3 Penetration distance of the hyporheic zone

The maximum penetration distance $d(t)$ of river water into the HZ in the direction perpendicular to the axis of the river can be calculated by the maximum y coordinate of the HZ. Similar to Gomez-Velez et al. (2017), we focus on the evolution of the dimensionless term of $d^{**}(t) = d^*(t)$ - $d^*(0)$, where $d^*(t) = d(t)/\lambda$.

S4.4 Residence time

The difference in mean residence time distribution between a sloping and a vertical riverbank model was calculated by $\mu_r^*(\mathbf{x}, t) = \log_{10}(\mu_{\tau\text{-}S}(\mathbf{x}, t)/\mu_{\tau\text{-}V}(\mathbf{x}, 0)$). $\mu_r^* < 0$ indicating that RT was overestimated in these areas when ignoring the bank slope while $\mu_r^* > 0$ indicating the contrary. Furthermore, a representative value of the flux-weighted ratio of mean RT to mean RT under baseflow conditions along the river boundary is given by: $\mu^*_{\text{out}}(x, t) = \mathbf{n}\cdot Q^*_{out}(x, \text{t})\log_{10}(\mu_\tau(x, t)/\mu_\tau(x, 0))$, which indicates aquifer discharge of younger water with relatively short travel times (values smaller than zero) or older water with longer travel times within the alluvial aquifer as compared to the baseflow conditions.

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