Technical Note: revisiting the general calibration of cosmic-ray neutron sensors to estimate soil water content (SUPPLEMENT)

Maik Heistermann[1], Till Francke[1], Martin Schrön[2], and Sascha E. Oswald[1]

[1]Institute of Environmental Science and Geography, University of Potsdam, Karl-Liebknecht-Straße 24–25, 14476 Potsdam, Germany

[2]UFZ - Helmholtz Centre for Environmental Research GmbH, Dep. Monitoring and Exploration Technologies, Permoserstr. 15, 04318, Leipzig, Germany

Correspondence: Maik Heistermann (maik.heistermann@uni-potsdam.de)

1 Partial derivatives

In this section, we provide the partial derivatives of θ^G (Eq. 2 in the technical note) and θ^L (Eq. 10 in the technical note). For the sake of comprehensibility, we will reproduce the underlying equations for θ^G and θ^L at the beginning of the following sub-sections.

1.1 Partial derivatives of θ^G

The following is the equation for the general calibration function:

$$\theta^G(N) = \left(\frac{a_0}{f_p \cdot f_h \cdot f_{\text{in}} \cdot f_b \cdot f_s \cdot \frac{N}{N_0} - a_1} - a_2 - \theta_g^{\text{OM}} - \theta_g^{\text{LW}}\right) \cdot \frac{\rho_b}{\rho_w} \tag{1}$$

Assuming $f_p = f_h = f_{\text{in}} = 1$ and including Eqs. 6-9 from the technical note, the partial derivatives required for the error propagation result to the following:

$$\frac{\partial \theta}{\partial N} = \frac{0.009^{-1} \cdot a_0 \cdot \rho_b \cdot (\text{AGB} - 0.009^{-1}) \cdot f_s \cdot N_0}{\rho_w \cdot (a_1 \cdot (\text{AGB} - 0.009^{-1}) \cdot N_0 + 0.009^{-1} \cdot f_s \cdot N)^2} \tag{2}$$

$$\frac{\partial \theta}{\partial N_0} = \frac{0.009^{-1} \cdot a_0 \cdot \rho_b \cdot (\text{AGB} - 0.009^{-1}) \cdot f_s \cdot N}{\rho_w \cdot (a_1 \cdot (\text{AGB} - 0.009^{-1}) \cdot N_0 + 0.009^{-1} \cdot f_s \cdot N)^2} \tag{3}$$

$$\frac{\partial \theta}{\partial f_s} = \frac{0.009^{-1} \cdot a_0 \cdot \rho_b \cdot (\text{AGB} - 0.009^{-1}) \cdot N \cdot N_0}{\rho_w \cdot (a_1 \cdot (\text{AGB} - 0.009^{-1}) \cdot N_0 + 0.009^{-1} \cdot f_s \cdot N)^2} \tag{4}$$

$$\frac{\partial \theta}{\partial \text{AGB}} = \frac{0.009 \cdot a_0 \cdot \rho_b \cdot f_s \cdot N \cdot N_0}{(\rho_w \cdot (a_1 \cdot N_0 \cdot (0.009 \cdot \text{AGB} - 1) + f_s \cdot N)^2)} \tag{5}$$

$$\frac{\partial\theta}{\partial\rho_b} = \Big(\frac{a_0}{\frac{f_s \cdot N}{N_0\cdot(1-\mathrm{AGB}\cdot 0.009)} - a_1} - a_2 - s\cdot 0.556 - \theta_g^{LW}\Big)\cdot\frac{1}{\varrho_w} \tag{6}$$

$$\frac{\partial\theta}{\partial OM} = -0.556\cdot\frac{\rho_b}{\rho_w} \tag{7}$$

$$\frac{\partial\theta}{\partial\theta_g^{LW}} = -\frac{\rho_b}{\rho_w} \tag{8}$$

1.2 Partial derivatives of θ^L

The following is the equation for the general calibration function:

$$\theta^L(N) = \left(a_0\cdot\left(\frac{\tau\cdot N}{\tau_{\mathrm{cal}}\cdot N_{\mathrm{cal}}}\cdot\left(\frac{a_0}{\theta_{\mathrm{cal}}\cdot\frac{\rho_w}{\rho_b}+a_2}+a_1\right)-a_1\right)^{-1}-a_2\right)\cdot\frac{\rho_b}{\rho_w} \tag{9}$$

Assuming $\tau = \tau_{\mathrm{cal}} = 1$, the partial derivatives required for the error propagation result to the following:

$$\frac{\partial\theta}{\partial N} = -\frac{a_0\cdot\rho_b\cdot N_{\mathrm{cal}}\cdot(a_2\cdot\rho_b+\theta_{\mathrm{cal}}\cdot\rho_w)\cdot(a_0\cdot\rho_b+a_2\cdot a_1\cdot\rho_b+a_1\cdot\theta_{\mathrm{cal}}\cdot\rho_w)}{\rho_w\cdot(a_2\cdot a_1\cdot\rho_b\cdot(N_{\mathrm{cal}}-N)-a_0\cdot\rho_b\cdot N+a_1\cdot\theta_{\mathrm{cal}}\cdot\rho_w\cdot(N_{\mathrm{cal}}-N))^2} \tag{10}$$

$$\frac{\partial\theta}{\partial N_{\mathrm{cal}}} = \frac{a_0\cdot\rho_b\cdot N\cdot(a_2\cdot\rho_b+\theta_{\mathrm{cal}}\cdot\rho_w)\cdot(a_0\cdot\rho_b+a_2\cdot a_1\cdot\rho_b+a_1\cdot\theta_{\mathrm{cal}}\cdot\rho_w)}{\rho_w\cdot(a_1\cdot(N-N_{\mathrm{cal}})\cdot(a_2\cdot\rho_b+\theta_{\mathrm{cal}}\cdot\rho_w)+a_0\cdot\rho_b\cdot N)^2} \tag{11}$$

$$\frac{\partial\theta}{\partial\theta_{\mathrm{cal}}} = \frac{a_0^2\cdot\rho_b^2\cdot N\cdot N_{\mathrm{cal}}}{(a_1\cdot(N-N_{\mathrm{cal}})\cdot(a_2\cdot\rho_b+\rho_w\cdot\theta_{\mathrm{cal}})+a_0\cdot\rho_b\cdot N)^2} \tag{12}$$

$$\begin{aligned}\frac{\partial\theta}{\partial\rho_b} = {} & \frac{1}{\rho_w\cdot(a_1\cdot(N-N_{\mathrm{cal}})\cdot(a_2\cdot\rho_b+\theta_{\mathrm{cal}}\cdot\rho_w)+a_0\cdot N\cdot\rho_b)^2}\cdot((a_1\cdot(N-N_{\mathrm{cal}})(a_2\cdot\rho_b+\theta_{\mathrm{cal}}\cdot\rho_w)+a_0\cdot N\cdot\rho_b) \\ & \cdot(a_0\cdot N_{\mathrm{cal}}(a_2\cdot\rho_b+\theta_{\mathrm{cal}}\cdot\rho_w)-a_2\cdot(a_1\cdot(N-N_{\mathrm{cal}})\cdot(a_2\cdot\rho_b+\theta_{\mathrm{cal}}\cdot\rho_w)+a_0\cdot N\cdot\rho_b)) \\ & -a_0^2\cdot N_{\mathrm{cal}}\cdot N\cdot\theta_{\mathrm{cal}}\cdot\rho_w\cdot\rho_b)\end{aligned} \tag{13}$$