Response to comments of Anonymous Referee 2

We would like to thank the referee for the valuable comments and suggestions, which improved the quality of the paper. Below you will find our response in regard to the comments and suggestions.

Comments to the Authors:

This technical note provides a solution for flow field variability in the stochastic sense when the hydraulic parameters are not second order stationary, but rather can be seen as intrinsic random functions. The work develops all the equations for depth-averaged heads and flow mean and variogram. Non-stationarity is here given by variability in thickness, which is only one of the many possibilities to have such non-stationarity. As a Technical Note, everything is correct: equations, text, figures are balanced, and they provide exactly what they say.

I am not sure about the significance of this work, compared for example to another one by the same group of authors published just one year ago: Stochastic Environmental Research and Risk Assessment (2022) 36:2503-2518 https://doi.org/10.1007/s00477-021-02125-7, where the authors write specifically that they do the following: "A spectrally based perturbation approach is used to arrive at the general results for the statistics of the flow fields in the Fourier domain, such as the variance of the depth-averaged head, and the mean and variance of integrated discharge". So, the work is indeed different, but I am not sure about the significance of this additional step in the integration.

Response

1. This work differs considerably from that of Chang et al. (2022):
   (a) In earlier work, published in Stochastic Environmental Research and Risk Assessment (SERRA, 2022), it was assumed that the confined aquifer with varying thickness can be approximated by an exponentially varying aquifer. This means that the variation in the thickness of the aquifer is considered **deterministic**.

   However, natural variations, such as variations in aquifer thickness caused by complex natural events, cannot be accurately predicted. Therefore, the variation in aquifer thickness is considered **random** in this work and characterized by a **nonstationary random process** with stationary increments (i.e. an intrinsic random process). That is, the stochastic theories developed here generalize the existing stochastic theories presented in SERRA (2022) to the case of nonstationary random inputs for the logarithmic thickness of the aquifer.

To our knowledge, the consideration of the thickness of the aquifer as a random variable for the evaluation of statistics of the flow fields in a heterogeneous confined aquifers of variable thickness has not yet been presented in the literature.

(b) In earlier work (SERRA, 2022), it was assumed that the random input parameter, such as the logarithmic hydraulic conductivity field, is stationary of the second order and can be described by a covariance function. The variances of the depth-averaged head and the integrated discharge can then be obtained using the Fourier-Stieltjes representation. This means that the stochastic theories developed by Chang et al. (2022) are only valid if **the associated covariance function for the input parameter exists and is second-order stationary**.

In many practical applications, however, the covariance function of the input parameter may not exist or the second-order stationary covariance function cannot be identified from the available data. Therefore, this work develops a new approach to evaluate the flow field semivariograms using the **intrinsic (nonstationary) hypothesis for the input parameter processes** instead of the second-order stationarity hypothesis. This means the stochastic theories developed here for quantifying the flow field variability apply to nonstationary random processes of input parameters. In other words, the semivariograms of flow fields proposed here are valid even if the corresponding covariance function for the input parameter does not exist.

In summary, Chang et al. (2022) propose variances of depth-averaged head and integrated specific discharge to quantify the variability of flow fields in heterogeneous confined aquifers, where hydraulic conductivity is treated as a random variable while the variation of aquifer thickness is deterministic. In the present work, the semivariograms of depth-averaged head and integrated specific discharge are developed to quantify the variability of the flow fields, treating both hydraulic conductivity and aquifer thickness as intrinsic (nonstationary) random processes.

2. Much of the literature on quantifying the variability of the flow fields in heterogeneous aquifers assumes that the random input parameter processes are second-order stationary and can be characterized by spatial covariance functions. However, the stationarity in the spatial variation of soil properties in heterogeneous aquifers is very rare in nature, and the covariance functions for the input parameter fields may not exist because the available data do not exhibit finite a priori variance.

   The stochastic theories proposed here for quantifying the variability of flow fields generalize the existing stochastic theory to intrinsic (nonstationary) random processes of input parameters. It is clear that the intrinsic hypothesis is weaker than the second-order stationarity hypothesis. The stochastic theories developed here improve the quantification of the variability of flow fields in natural confined aquifers of variable thickness.

   The significance of the present work is added on page 12 (Line 181) as follows:

   > "Based on the fact that the stationarity in the spatial variation of soil properties is very rare in nature, the proposed stochastic theories for quantifying the variability of flow fields generalize the existing stochastic theory, which applies to random input parameter processes with second-order stationarity, to intrinsic (nonstationary) random processes of input parameters. It is clear that the intrinsic hypothesis is weaker than the second-order stationarity hypothesis. The stochastic theories developed here improve the quantification of the variability of flow fields in natural confined aquifers of variable thickness."