



- 1 Process-based three-layer synergistic optimal allocation
- 2 model for complex water resource systems considering
- 3 reclaimed water
- 4 Jing Liu¹ Yue-Ping Xu^{1*} Wei Zhang² Shiwu Wang³ Siwei Chen¹
- 5 ¹ Institute of Water Science and Engineering, College of Civil Engineering and
- 6 Architecture, Zhejiang University, Hangzhou 310058, China
- 7 ² College of Computer Science and Technology, Zhejiang University, Hangzhou
- 8 310058, China
- 9 ³ Zhejiang Institute of Hydraulics & Estuary, Hangzhou 310020, China
- *Corresponding author: Yue-Ping Xu, yuepingxu@zju.edu.cn

11 Abstract

- 12 The increasing water demand due to human activities has aggravated water scarcity,
- and conflicts among stakeholders have increased the risk of unsustainable development.
- 14 Ignoring the effects of trade-offs leads to misguided policy recommendations. This
- 15 study highlights the concept of synergy among different aspects of water allocation
- 16 process. A process-based three-layer synergistic optimal allocation (PTSOA) model is
- 17 established to integrate the interests of stakeholders across subregions, decision levels
- 18 and time steps while simultaneously coupling reclaimed water to establish

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19 environmentally friendly solutions. A synergy degree index is constructed by applying

20 network analysis for optimization. PTSOA is applied in Yiwu City, Southeast China,

and is shown to improve the contradictions among different dimensionalities in a

complex system. Overall, 2.43×10⁷~3.95×10⁷ m³ of conventional water is saved, and

notable improvements in management are achieved. The application demonstrates the

24 efficiency and excellent performance of the PTSOA.

25 **Keywords** Three-layer optimization, water allocation, process, synergy, reclaimed

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1. Introduction

28 Water scarcity has become one of the major impediments to the sustainable

29 development of cities (Yue et al., 2020). Emerging water scarcity concerns in cities are

associated with limited available water, severe water pollution and the relentlessly

growing demand for water as driven by industrial growth, population growth and higher

32 living standards; these factors have lead to intense competition for freshwater among

33 stakeholders of interest (Dai et al., 2018; Wu et al., 2023). However, the heterogeneous

34 distribution of water resources at both spatial and temporal scales is common in many

cities and results in water shortage risks and conflicts, which often require the

36 optimization of water resource allocation (Friesen et al. 2017). Moreover, some

satisfactory alternatives for individual stakeholders may result in negative externalities

38 on others. Therefore, it is critical to develop a synergistic optimal allocation model to

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39 alleviate conflicts and ensure the security, efficiency, equality, eco-environmental 40 sustainability, and sustainable development of water systems simultaneously.

As equitable access to water resources is closely related to social stability, several 42 qualitative and indirect methods have been developed to assess water allocation 43 equality (D'Exelle et al. 2012). In cases with limited water resources, more water would 44 be allocated to users with better economic conditions to achieve more economic benefits. Thus, stakeholders with poor economic status are ignored, resulting in 45 imbalanced development. Consequently, actions are often needed by local government 46 47 managers to avoid such situations. The Gini coefficient has been widely used to 48 evaluate equality and enhance the optimization of water allocation in water use sectors 49 (Xu et al. 2019; Hu et al. 2016; D'Exelle et al. 2012). However, it is unable to reflect 50 the dynamic interactions among objectives, i.e., how objectives interact with each other 51 and impact the equity of a system in cases with diverse alternative decisions. While, in 52 the perspective of coordinated alloaction, multiple goals are simultaneously considered 53 to avoid negative effects as much as possible. Therefore, in addition to equity, 54 coordination should be considered in water allocation systems, and these two concepts 55 can be combined to promote systemic synergy. By identifying the dynamic interactions among objectives, the internal mechanisms of a water system can be clarified, and 56 synergy can be achieved in cases with different potential decisions. It is also helpful to 58 identify the hurdles and opportunities associated with sustainable development for 59 cities and to establish specific action priorities for cities based on a comprehensive

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analysis of this study.





understanding of the interactions among objectives. To address this knowledge gap, a correlational network approach is applied in this study, and a synergy degree index is presented to consider both the equity and coordination of water systems. Moreover, systemic analysis is used to assess the level of coordination of complex objective interactions in city water systems. Network analysis, which has been widely used in studies of complex systems (Ball et al., 2000; Saavedra et al., 2011; Bond, 2017), is a holistic approach for exploring the characteristics of interactions among objectives. It provides clear visualization and conceptualization of the interactions among variables to fully characterize those interactions (Swain and Ranganathan, 2021). An array of network metrics (for example, degree centrality, betweenness centrality, eigenvector centrality, closeness centrality, and community) can be applied to quantify the importance of objectives or targets in an interaction network (Zhou and Moinuddin, 2017) and reveal the strongly connected pairs of goals or targets in the network (Allen et al., 2019). A key network metric in such analysis is connectivity, which reflects the degree of coordination among different objectives in a system; in synergy networks, high connectivity indicates that many objectives can be achieved simultaneously and that the negative effects of interactions are mild (Wu et al., 2022). Thus, to facilitate the discovery of high-quality decision alternatives, alleviate negative conflicts among multiple utilities and inform decision making, a synergy degree evaluation index is established and applied to the network

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Due to the negative externalities of individual decisions, conflicts occur not only across different users or objectives but also across hierarchical decision levels. Water use contradictions and inconsistent decision making by multiple managers inevitably results in trade-offs, including positive and negative water resource feedback in cases with limited water availability (Wang et al., 2022). In practice, district administrators allocate water to each sector in each subregion, and subregion managers then make use decisions based on the allocated amount of water resources (Safari et al., 2014). Since each decision maker places emphasis on different targets, feedback and coordination among different decision makers are of great importance. Therefore, synergistic hierarchical water allocation that achieves coordination among different decision makers is imperative to avoid conflicts, save water and maintain social stability. To address these hierarchical problems, bilevel programming (BLP) has been widely used, wherein objectives at two hierarchical levels, namely, an upper level and a lower level, are co-optimized (Zhang and Vesselinov, 2016; Jin et al., 2018). The upper-level decision may be affected by the actions of the lower-level decision makers (Arora and Gupta, 2009). Yue et al. (2020) formulated a bilevel programming (BLP) framework to gain insight into the whole water allocation process with district administrators and subregional farmers. Li et al. (2022) built a two-level model with the overall interests of system managers at the top and the individual interests of water supply departments at the bottom. The multilevel programming problem (MLPP) was derived from the bilevel programming problem (BLPP) and is more applicable to real





103 (more than two levels) for water resource allocation, especially in cases with 104 unconventional water supplies. 105 To satisfy both long-term and short-term water needs and avoid unnecessary 106 administration costs and water resource use caused by a lack of coordination among 107 different allocation steps, temporally synergistic allocation and optimization are needed 108 (Haguma and Leconte, 2018). In annual water resource planning, the monthly 109 variability of hydrologic regimes and nonstationarity of the daily water demand must 110 be considered. As an alternative example of synergistic allocation at different time steps, 111 Vicuna et al. (2010) used a monthly nonlinear programming model and an annual 112 sampling stochastic dynamic programming (SSDP) model to establish a monthly 113 operating policy. Haguma et al. (2015) proposed an optimization approach with two 114 separate time steps following the nested model approach. Haguma and Leconte (2018) 115 constructed deterministic and stochastic optimization models with two time steps (intra-116 annual and interannual) and two levels of inflow variability: seasonal and interannual. 117 The purpose of their short-time-step model was to derive aggregate performance 118 functions associated with potential long-time-step decisions in these studies. However, 119 short-term benefits should not be overlooked due to their appreciable impact on long-120 term effects. Accordingly, synergistic allocation that enhances both long-term and 121 short-term allocations is of great importance for water resource management in cities. 122 However, optimizing the structure of a model to achieve maximized benefits and

world practices (Baky, 2014). However, limited studies have explored applying MLPP

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123 balancing the trade-offs among time steps are tasks that have rarely been studied. The 124 synergy among different time steps is addressed with a new innovative framework and 125 a corresponding algorithm in our study. 126 Most of the abovementioned traditional models are based on a benefit-oriented 127 mechanism, which leads to a high degree of satisfaction in high-benefit regions and 128 large water shortages in other regions. The existence of high-benefit regions in a city 129 during the allocation process often exacerbates regional disparities and heterogeneous 130 development. Moreover, spatial factors influence allocation results, especially when 131 there is spatial hierarchical heterogeneity among water resource allocation elements (Li 132 et al., 2022). It is thus appropriate to conceptualize water allocation problems in a 133 multistage framework that fully considers the interests of not only the regional authority 134 but also subregional managers (Yao et al., 2019). Hence, the synergy among subregions 135 must be considered to optimally allocate water resources. Ideally, the benefits of all 136 subregions should be integrated equally in the model, and the weights of 137 hyperparameters should be adjusted to best support flexible policies. 138 The optimal allocation of conventional and unconventional water resources also 139 significantly impacts water security and aquatic ecosystems. The reuse of reclaimed 140 water is beneficial for alleviating high water supply pressure on conventional water 141 resources and reducing the emission of pollutants. To effectively integrate conventional 142 and unconventional water resources, Yang et al. (2008) and Han et al. (2008) introduced 143 unconventional water resources as critical factors in water management. Avni et al.

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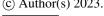
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(2013) investigated the mixing of unconventional water resources with other conventional water sources to meet the magnesium requirements for drinking water and irrigation water. Yu et al. (2017) developed a cost-benefit analysis-based utilization model for externally transferred water and desalinated water. The allocation of both conventional and unconventional water has been widely studied, but there remains a lack of methods to guide the synergistic allocation of conventional and unconventional water resources and embed reclaimed water supply systems in allocation schemes. The overexploitation of conventional water resources is not conducive to the sustainable development, while the extensive use of unconventional water could ultimately result in high economic burden. To synergistically integrate conventional and unconventional water resources and guide the coordinated allocation of these two types of water resources, corresponding mechanisms must be implemented. As a result, our study aims to couple the allocation of conventional water resources and unconventional water resources to establish synergistic solutions. In summary, as insufficient water supplies and increasing water demands intensify competition for water resources and lead to conflicts among different stakeholders in different dimensions, water allocation must be optimized in cities and regions to achieve synergistic decision-making at various levels and time steps considering the value of reclaimed water. Therefore, a new process-based three-layer synergistic optimal allocation (PTSOA) model is developed here to generate numerous candidates or Pareto solutions and identify several desirable decision alternatives. The synergy of



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time and space optimization is achieved in the new model to avoid waste and promote balanced spatial development. Furthermore, in the PTSOA model, reclaimed water is used to replenish conventional water resources in water-scarce areas. The remainder of this paper is organized as follows. The mathematical model is formulated in Section 2. Section 3 gives a numerical example for Yiwu city to

demonstrate the effectiveness and efficiency of the proposed methods. The results are shown in Section 4; different water allocation strategies under varying inflow

conditions are explored, and some policy implications are discussed. Section 5 presents 172

173 conclusions.

2. Modelling

With water resources becoming increasingly scarce, multidimensional synergistic optimal allocation in a hierarchical system is crucial for ensuring sustainable development in water-scarce cities. There are three dimensions of synergy in the established allocation model, as shown in Fig. 1: process synergy, decision-level synergy and time-scale synergy. The synergy of the process refers to synergistic water allocation among the three stages throughout the whole allocation process to reduce waste in bridging processes, which has rarely been considered. In the three stages, first, the original water is released from reservoirs or diverted from external water transfer projects to water works; then, the water stored in water works infrastructure is supplied to different departments that need different types of water, including both conventional

https://doi.org/10.5194/hess-2023-160 Preprint. Discussion started: 15 September 2023



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and reclaimed water. Finally, the water is supplied to different users. Decision-level synergy refers to synergistic water allocation considering the interests of decision makers at different levels, namely, the city, water department and regional levels, to coordinate solutions and avoid conflicts among decision makers. The city level represents the overall interests of a city from the perspective of government, the water supply department level represents the interests of water supply corporations, and the regional level focuses on the comprehensive benefits of each region in the city and mitigate development imbalance among regions. Optimal decision making at the department level is constrained by the allocation results at the city level, and so on, and the final solution should satisfy the needs of decision makers across all levels. The time scale synergy involves the coordination of the daily configuration goal with the monthly goal, the monthly goal with the yearly goal, and so on. Synergistic temporal allocation can largely alleviate time conflicts during configuration operations, ensuring that all configuration periods serve the same final configuration objectives to save water resources and improve efficiency. However, time scale synergy mainly depends on artificial operations rather than automated intelligent operations in practice. In-depth exploration has yet to be demonstrated. Consequently, the PTSOA model is constructed here to fully consider these three dimensionalities of synergy. The dimensionalities are coupled this model to achieve the efficient maximization of comprehensive benefits at all levels under the premise of saving water resources.

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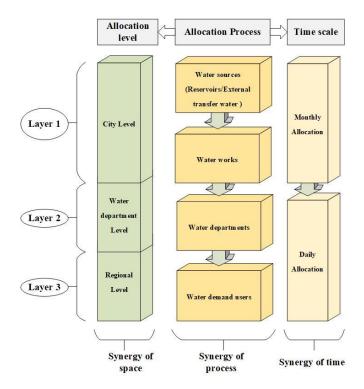


Fig. 1. Conceptual map of the PTSOA model

In water-scarce cities, using reclaimed water as an alternative water source is a proven approach to efficiently improve the environment by reducing sewage discharge. The quality of inland tributaries has deteriorated in many water-scarce cities due to limited consideration of the water environment and the large-scale emission of pollutants. Transferring reclaimed water and main river water to urban inland tributaries for ecological water replenishment is a promising approach for improving the quality of urban water environments and areas with water shortages. However, there has been a lack of studies on the integration of reclaimed water reuse systems and inland water

https://doi.org/10.5194/hess-2023-160
Preprint. Discussion started: 15 September 2023



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distribution systems in allocation modelling. Therefore, in addition to saving water resources and improving efficiency through multidimensional synergistic allocation, the model encompasses reclaimed water reuse systems and ecological water distribution systems for inland tributaries. Finally, the PTSOA model is constructed to solve the multidimensional synergistic allocation problem involving complex water resource networks that couple reclaimed water reuse systems and inland ecological water distribution systems with multiple sources, processes and regions to guarantee the sustainable development of waterscarce cities. To select the most synergistic solution of the PTSOA model, a new evaluation index named the total synergy index (TSI) is proposed to assess the synergy degree of different decision alternatives. Furthermore, the network analysis method is applied for the first time to analyse dynamic interactions in water optimal allocation. This method visually depicts the dynamic interactions and conflicts among different subareas in a city, which is helpful for system managers to realize how the water allocation scheme in one region influences that of other areas; consequently, more reasonable and flexible measures are established based on dynamic regional development targets. The detailed framework developed in this study is shown in Fig. 2.

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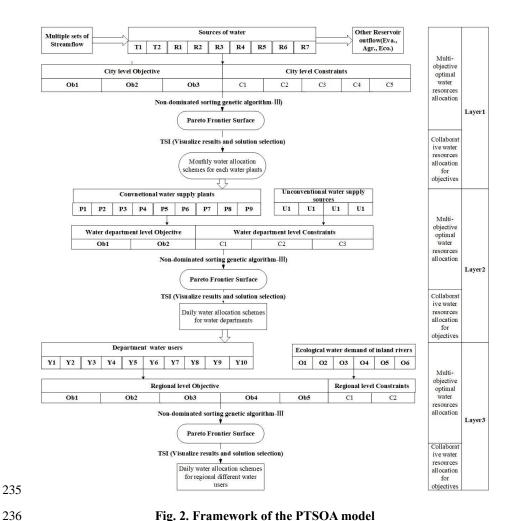


Fig. 2. Framework of the PTSOA model

2.1 First layer of the PTSOA decision-making process

Three dimensionalities of synergistic water resource allocation are coupled in the first layer of the PTSOA model. The first stage of the process (original water is released from reservoirs or external water transfer projects to water works) is optimized in the first layer. This stage demonstrates a strong constraint effect on the later stages. To





satisfy the overall development goals of the city, the first-layer processes involve city-level decision-making. The city manager focuses on the overall goal of the water resource system in the city, which is the first and most important phase of the decision-making process. The established allocation scheme highly influences decision makers at other levels, and optimal allocation schemes at other levels must align with this overall goal. Additionally, since water resource planning in most Chinese cities is based on an annual planning period and monthly planning unit, the time step of the first layer is set as months. Finally, the monthly decision alternatives for the volume of water allocated from reservoirs to water works is obtained at the city decision level.

2.1.1 Objective functions

252 Social objective function: Minimization of total water supply shortages

The social objective function is established by the city manager to minimize the total water supply shortages in a water system. The objective is established to sufficiently meet the water demands of users in a water resourcees system. The water deficit is considered, and this objective can reflect the ability of the water supply to meet the water demand, as shown in Eqs. (1-3):

$$\min f_{11}(x) = D - S \tag{1}$$

$$D = \sum_{r=1}^{R} \sum_{t=1}^{T} D_r^t$$
 (2)

$$S = \sum_{t=1}^{T} \sum_{i=1}^{J} \sum_{j=1}^{J} x_{ij}^{t} \alpha_{ij} + \sum_{t=1}^{T} \sum_{e=1}^{E} \sum_{j=1}^{J} x_{ej}^{t} \beta_{ej}$$
(3)





261 where $D(10^4 \text{ m}^3)$ is the total water demand of the system, $D_s^t(10^4 \text{ m}^3)$ is the water demand of the rth subregion at tth time step, r = 1, 2, ..., R, R is the total number of 262 subregions in the area, t=1,2,...,T, T is the total number of months in the period, S (10⁴) 263 264 m³) is the total water supply of the water system for all waterworks, x'_{ij} (10⁴ m³) is the 265 amount of water supplied from ith reservoir to the jth waterworks in the tth month of 266 the configuration period, i=1,2,...I, I refers to the total number of reservoirs, j=1,2,...J, J is the number of total water works, $x_a^t (10^4 \text{ m}^3)$ is the amount of water supplied from 267 the eth external transfer water source to the ith water works in the ith month of the 268 configuration period, e=1,2,...,E, E is the total number of external transfer water 269 270 sources in the city, α_{ij} is the water supply relationship coefficient between the ith 271 reservoir and the *j*th water works, where 0 indicates no supply and 1 indicates a water supply, and β_{ej} is the water supply relationship coefficient between the eth external 272 273 transfer water source and the jth water works, where 0 indicates no supply and 1 274 indicates a water supply.

275 Economic objective function: Maximization of the total water supply benefit

- 276 A city manager operates a water allocation system to maximize the overall economic
- benefit by establishing an economic objective function, as shown in Eqs. (4-7):

278
$$\max f_{12}(x) = B - C_{rs} - C_{es}$$
 (4)

279
$$C_{rs} = k \times \sum_{t=1}^{T} \sum_{i=1}^{J} \sum_{j=1}^{J} x_{ij}^{t} \alpha_{ij} + \sum_{t=1}^{T} \sum_{i=1}^{J} \sum_{j=1}^{J} \left(x_{ij}^{t} \alpha_{ij} \times c_{i} \right)$$
 (5)

280
$$C_{es} = m \times \sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{e=1}^{E} x_{ej}^{t} \beta_{ej} + \sum_{t=1}^{T} \sum_{e=1}^{E} \left(n_{e} \times \sum_{j=1}^{J} x_{ej}^{t} \beta_{ej} \right)$$
 (6)





281
$$B = \sum_{j=1}^{J} b_{j} \times \left(\sum_{t=1}^{T} \sum_{i=1}^{I} x_{ij}^{t} \alpha_{ij} + \sum_{t=1}^{T} \sum_{e=1}^{E} x_{ej}^{t} \beta_{ej} \right)$$
 (7)

282 The overall economic benefit is the difference between the total benefit and total cost at the city level. In the equations, B (Chinese yuan, shortened to yuan in the 283 284 following text) is the total direct water supply benefit (mainly considering the income 285 from water charges for the city). The total water supply cost consists of the reservoir water supply cost $\frac{C_{rs}}{c}$ and the external water supply cost $\frac{C_{es}}{c}$; k (yuan/m³) denotes 286 287 the water resources fees paid to the government; c_i (yuan/m³) denotes the water fees 288 paid to the *i*th reservoir authority; m (yuan/m³) is the charge to an external 289 administrative district per unit of externally transferred water; n_e (yuan/m³) is the 290 charge associated with the eth external water source per unit of transferred water; and 291 b_i (yuan/m³) is the unit price of water supply revenue for the *j*th user. Sustainable development objective function: Maximization of the total amount of 292 293 reserved water in reservoirs 294 In water-scarce cities, the problem of water scarcity is a serious challenge that prevents 295 sustainable allocation of water resources. A prominent feature of most water-scarce cities is that water inflows are limited, and the fluctuations in water availability are 296 large. Therefore, to reduce the risk that the inflows in the next configuration period are 297 298 too short to meet the basic demand of the city, such that a sustainable development 299 objective function is developed. The sustainable development objective function seeks

to maximize the amount of water remaining in the reservoir at the end of a configuration





- 301 period to hedge against drought risk and guarantee water use in the next period, as
- 302 shown in Eqs. (8-10):

303
$$f_{13}(x) = \max \sum_{i=1}^{N} (V_i^{\text{max}} - V_i) \times p(V_i^{\text{max}} - V_i)$$
 (8)

304
$$p(V_i^{\max} - V_i) = \begin{cases} 2 \times V_i / V_i^{\max} & 0 < V_i < V_i^{\max} / 2 \\ -2 \times V_i / V_i^{\max} + 2 & V_i \ge V_i^{\max} / 2 \end{cases}$$
 (9)

$$V_{i} = \sum_{t=1}^{T} \left(R_{i,initial} + I_{i}^{t} + P_{i}^{t} - A_{i}^{t} - E_{i}^{t} - EP_{i}^{t} \right) - \sum_{t=1}^{T} \sum_{j=1}^{J} x_{ij}^{t} \alpha_{ij}$$
(10)

where $V_i^{\max}(10^4\,\mathrm{m}^3)$ is the maximum allowable storage capacity of the *i*th water source, which is expressed based on the limited storage capacity of a reservoir in the flood season, and $V_i(10^4\,\mathrm{m}^3)$ is the water storage capacity of the *i*th water source at the end of the scheduling period. As much water as possible but less than V_i^{\max} is reserved. However, a reserved water volume in the reservoir that is too high at the end of the scheduling period may lead to considerable pressure on reservoirs to urgently release water if a flood event is forecasted. The reserved water volume should be neither too large nor too small. Thus, the benefits of reservoir retainment must be thoroughly evaluated. Based on the characteristic of the benefit of residual water, we propose a boundary benefit function $p(V_i^{\max}-V_i)$ for different reserved water volumes in a reservoir. The benefit function is a piecewise function, and when V_i is less than $V_i^{\max}/2$, p increases as V_i increases. When V_i is equal to or greater than $V_i^{\max}/2$, p decreases as V_i increases. When $V_i = V_i^{\max}$, p decreases to 0. $R_{i,initial}$ (10⁴ m³) is the initial storage of the *i*th water source, $I_i^i(10^4\,\mathrm{m}^3)$ is the inflow of the *i*th water source





- 320 at the tth time step, P_i^t (10⁴ m³) is the precipitation associated with the ith water source
- 321 at the *t*th time step, A_i^t (10⁴ m³) and E_i^t (10⁴ m³) are the agricultural and ecological
- 322 water supplies associated with the *i*th water source at the *t*th time step, respectively, and
- 323 EP_i^t (10⁴ m³) is the evaporation from the *i*th water source at the *t*th time step.

2.1.2 Constraints

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- 325 The layer includes six main constraints: the reservoir water supply constraint, water
- 326 demand constraint, reservoir storage constraint, water balance constraint, external water
- 327 transfer constraint, and nonnegative constraint.

328 Reservoir water supply constraint

- 329 The maximum water available to supply from an individual reservoir is determined by
- 330 the difference between the total input and total reservoir output. The inputs include
- 331 inflow and precipitation, and the outputs mainly involve agricultural and environmental
- water supplies, evaporation, water supplied for waterworks and reservoir leakage loss.
- 333 All these factors directly affect the decision-making process and are incorporated into
- the model building process as shown in Eqs. (11-15):

$$V_i^t \le V_{i,\max}^t \tag{11}$$

$$V_{i}^{t} = \sum_{j=1}^{J} x_{ij}^{t} \alpha_{ij}$$
 (12)

337
$$V_{i,\text{max}}^{t} = \sum_{t=1}^{t-1} \left(R_{i,\text{initial}} + I_{i}^{t} + P_{i}^{t} - A_{i}^{t} - E_{i}^{t} - EP_{i}^{t} - \sum_{j=1}^{J} x_{ij}^{t} \alpha_{ij} - L_{i}^{t} \right) - V_{i,d}$$
 (13)

$$EP_i^t = ep_i^t \times s_i^t / 1000 \tag{14}$$





$$V_{i}^{t} = \xi_{i}^{t} \times \left(R_{i}^{t-1} + R_{i}^{t}\right) \tag{15}$$

where V_i^t (10⁴ m³) denotes the total water supply from the *i*th reservoir at the *t*th time step; $V_{i,\max}^t$ (10⁴ m³) is the maximum water available to be supplied from the *i*th reservoir at the *t*th time step; ep_i^t (mm) is the water surface evaporation from the *i*th reservoir in the *t*th month; s_i^t (m²) is the monthly average surface area of the *i*th reservoir in the *t*th month; $V_{i,d}$ (10⁴ m³) is the dead storage of the *i*th reservoir; L_i^t (10⁴ m³) is the reservoir leakage loss from the *i*th reservoir at the *t*th time step; R_i^{t-1} (10⁴ m³) is the storage of the *i*th reservoir at the *t*-1th time step; R_i^t (10⁴ m³) is the storage of the *i*th reservoir at the *t*-1th time step; R_i^t (10⁴ m³) is the storage of the *i*th reservoir at the *t*th time step; and ξ_i^t is the *t*th monthly leakage coefficient for the *i*th reservoir.

Water demand constraint

The high-quality water demand of each subarea in a city should be satisfied in the water allocation process. High-quality water in this model refers to water that satisfies the relevant primary (surface water can be used for drinking after simple purification treatment, such as filtration and disinfection) and secondary water quality requirements (water is slightly polluted and can be used for drinking after routine purification treatment, such as flocculation, precipitation, filtration, disinfection, and other processes) according to the Chinese Standard (GB5749), as shown in Eq. (16):

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$$0.8 \times D_{r} \leq \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{j=1}^{J_{r}} x_{ij}^{t} \alpha_{ij} + \sum_{t=1}^{T} \sum_{e=1}^{E} \sum_{j=1}^{J_{r}} x_{ej}^{t} \beta_{ej} \leq 1.2 \times D_{r}, r = 1, 2, ..., R$$
 (16)





- where D (10⁴ m³) is the high-quality water demand in the rth subregion and there are
- a total of R subregions in the city. Jr is the number of waterworks in the rth subregion.
- To ensure that the water supply guarantee in each area is greater than 80%, the total
- water supplied to every subarea is greater than 80% of its demand.

363 Reservoir storage constraint

$$R_i^T \le V_{i,f} \tag{17}$$

$$R_{i}^{T} = \sum_{t=1}^{T} \left(R_{i,initial} + I_{i}^{t} + P_{i}^{t} - A_{i}^{t} - E_{i}^{t} - EP_{i}^{t1} - \sum_{j=1}^{J} x_{ij}^{t} \alpha_{ij} - V_{i}^{t} \right)$$
(18)

- 366 where R_i^T (10⁴ m³) is the storage of the *i*th reservoir at the end of the configuration
- period and $V_{i,f}$ (10⁴ m³) is the flood-limit storage capacity.

368 Water balance constraint

$$R_i^{t+1} = R_i^t + I_i^t + P_i^t - A_i^t - EP_i^t - E_i^t - V_i^{t-1} - \sum_{j=1}^J x_{ij}^t$$
(19)

370 External transfer water constraint

371
$$\sum_{t=1}^{T} \sum_{j=1}^{J} x_{ej}^{t} \beta_{ej} \le E_{e,\text{max}}$$
 (20)

- 372 where $E_{e,max}$ refers to the maximum water supply capacity of an external water source
- over the whole configuration period.

374 Nonnegative constraint

$$x_{ii} \ge 0 \tag{21}$$

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2.2 Second layer of the PTSOA decision-making model

Similarly, the second layer of the PTSOA model fuses all three dimensions of synergistic water resource allocation mentioned previously. The second stage of the process (the water stored in water works is supplied to different departments needing water volumes of different quality) is optimized in the second layer. After city-level decision-making, a conflict of interest inevitably occurs between traditional water supply departments and unconventional water supply departments. Because conventional and unconventional water supply departments compete for limited water demand market shares, the stability of the water allocation system may be jeopardized if excessive competition is not controlled. Thus, the second layer is implemented at the department level. Decision-making at the department level seeks to guide the two water supply departments to partake in benign competition and avoid conflicts to realize synergy. In this case, the decision plan of the first layer in the hierarchy is followed. Temporally, short-term allocation changes are needed as mentioned above; hence, the time scale of the second layer is daily. Thus, the daily decision alternatives for the volume of water allocated from water works to different water departments are obtained to make relevant decisions.

2.2.1 Objective functions

Conventional water supply department objective function: Minimization of the





total amount of water retained in water works

The managers of conventional water supply departments strive to operate conventional water systems efficiently and achieve the most equitable water share possible. The amount of conventional water (of high quality) retained in a water works system is a crucial factor affecting the efficiency and benefits of conventional water supply departments. Therefore, the benefit of conventional water departments is established by minimizing the total amount of water retained in water works at the end of a configuration period, as shown in Eq. (22):

403
$$\min f_{21}(x) = W_L = \sum_{i=1}^{T} \sum_{i=1}^{I} \sum_{j=1}^{J} x_{ij}^t \alpha_{ij} + \sum_{i=1}^{T} \sum_{e=1}^{E} \sum_{j=1}^{J} x_{ej}^t \beta_{ej} - \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{z=1}^{Z} q_{jz}^{t,m} \chi_{jz}$$
 (22)

where W_L (10⁴ m1) is the total amount of water retained in a water works system at the end of a configuration period; $q_{jz}^{t,m}$ (10⁴ m³) is the water supply from the jth water works system to the zth water user on the mth day in the tth month in the period of configuration; m=1,2,...,M; and M is the total number of days in the tth month (28, 29, 30 or 31). Additionally, z=1,2,...Z, and Z is the total number of water users. χ_{jz} is the water supply relationship coefficient between the jth water work and the zth water user, where 0 indicates no supply and 1 indicates a water supply.

411 Unconventional water supply objective function: Maximization of the amount of

412 unconventional water supplied

The reclaimed water reuse system and ecological water distribution system for inland tributaries are incorporated into the PTSOA model which are associated with

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415 unconventional water supply departments. The managers of unconventional water 416 supply departments seek to supply as much unconventional water as possible to 417 promote their interests. Thus, the objective of unconventional water departments is 418 established to maximize the amount of unconventional water supplied. Unconventional 419 water mainly includes reclaimed water and river water, which is of low quality (i.e., not 420 meeting the quality standard mentioned in Sect. 2.1.2) and is mainly used for industrial production, ecological water replenishment for inland rivers and municipal road 421 422 sprinkling.

Unconventional water departments operate reclaimed water reuse systems and ecological water distribution systems to supply unconventional water, and the associated equations are as follows in Eqs. (23-26):

426
$$\max f_{22}(x) = W_r + EW_r \tag{23}$$

427
$$W_{r} = \sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{j=1}^{J} r_{nj}^{t} p(b_{c}, b_{u}) \theta_{nj}$$
 (24)

428
$$p(b_c, b_u) = \frac{1}{3} \times \frac{b_c}{b_u} - \frac{2}{3}$$
 (25)

$$EW_{r} = \sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{z=1}^{Z} r_{nz} \theta_{nz}$$
 (26)

where W_r (10⁴ m³) is the total amount of reclaimed water supplied for all water users; EW_r (10⁴ m³) is the total amount of river water supplied to maintain ecological flows in inland tributaries; r'_{nj} (10⁴ m³) is the amount of water supplied to the *j*th user from the *n*th reclaimed water source at the *t*th time step; n = 1, ..., N; N is the total number of





reclaimed water sources; $p(b_c, b_u)$ is a function expressing the willingness of residents 434 435 to use reclaimed water, b_c (yuan/ 10^4 m³) is the price per unit of conventional water; 436 b_u (yuan/10⁴ m³) is the price per unit of unconventional water; and θ_{nj} is the water 437 supply relationship between the nth reclaimed water source and the jth user. In this case, 438 $\theta_{ni} = 1$ indicates a water supply relationship, and $\theta_{ni} = 0$ indicates no water supply 439 relationship. r_{m} (10⁴ m³) is the amount of water supplied from the *n*th reclaimed water source to the zth inland tributary; z = 1, 2, ..., Z; Z is the total number of inland tributaries 440 441 requiring ecological flow compensation; and θ_{nz} is the water supply relationship between the *n*th reclaimed water source and the *z*th inland tributary. 442

2.2.2 Constraints

444 Conventional water supply constraint

According to conservation of mass, the total daily amount of conventional water allocated in the second layer should be less than the total monthly amount of conventional water allocated in the first layer, as described in Eq. (27):

448
$$\sum_{t=1}^{T} \sum_{i=1}^{J} \sum_{j=1}^{J} x'_{ij} \alpha_{ij} + \sum_{t=1}^{T} \sum_{e=1}^{E} \sum_{j=1}^{J} x'_{ej} \beta_{ej} \ge \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{z=1}^{Z} q^{t,m}_{jz} \chi_{jz}, t = 1, ..., T$$
 (27)

Unconventional water constraints

449

The two types of unconventional water have separate constraints. For reclaimed water supplied to water users, the amount should satisfy the relevant water recycling standard.

The ecological water used to replenish inland tributaries is mainly pumped from

- 453 reclaimed water works and main rivers. Therefore, this replenished volume is limited
- 454 by the pumping capacity. The constraints for unconventional water are shown in Eqs.
- 455 (28)-(29):

456
$$\sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{j=1}^{J} r_{nj}^{t} \theta_{nj} + \sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{z=1}^{Z} r_{nz} \theta_{nz} = \sum_{t=1}^{T} \sum_{i=1}^{J} \sum_{j=1}^{J} x_{ij}^{t} \delta_{ij} \eta_{ij} + PU$$
 (28)

457
$$PU = \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{p=1}^{P} Q_{t,m}^{p,s} / 10^4$$
 (29)

- 458 where δ_{ij} is the sewage discharge coefficient, which is the proportion of the water
- 459 supplied from sewage discharge; η_{ij} is the sewage water reuse rate, which is the
- proportion of reused water in the total volume of sewage water; PU (10⁴ m³) is the
- amount of water pumped from the main river; and $Q_{t,m}^{p,s}$ (t/d) is the flow through the
- sth pumping station on the mth day at time step t.

463 **Pumping constraints**

$$Q_{t,s}^{p} \le Q_{\max,s}^{p} \tag{30}$$

$$Q_{t}^{P} = \sum_{s=1}^{Np} r_{t,s}^{P}$$
 (31)

- where $Q_{\max,s}^P$ (t/d) denotes the upper flow boundary of the sth pumping station; r_t^s (t/d)
- 467 is the power of the pth pump installed at the sth pump station; and Np is the number of
- 468 pumps stalled at the sth pump station.

469 Water quality constraint

- 470 To control the impacts of various point and nonpoint sources on receiving water bodies
- 471 in cities, water authorities impose water quality standards for the management of river





- 472 basins. These standards seek to maintain the water quality at a desired target level by
- 473 defining discharge limits for conventional, specific, or priority pollutants. To satisfy the
- 474 relevant standards, the following water quality constraint is established:

475
$$\sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} \left(x_{ij}^{t} \delta_{ij} \psi_{ij} h_{j}^{u} - x_{ij}^{t} \delta_{ij} \eta_{ij} h_{j}^{u} \right) \times 10 \le H^{u}$$
 (32)

- 476 where ψ_{ij} is the sewage water treatment rate, which is the proportion of sewage water
- 477 that is treated; h_i^u (mg/L) is the concentration of the *u*th contaminant per unit treated
- 478 water required by the jth user; and H^u (kg) is the upper limit of the uth contaminant
- allowed to be discharged in the study area.

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2.3 Third layer of the PTSOA decision-making model

After obtaining the results for the former two stages of the allocation process and the two levels of decision-making, the third model layer is constructed to achieve regional synergy s. It refers to the collaborative allocation of water resources in different subregions of a city, and it is intended to balance and maximize the interests of each subregion as much as possible. Additionally, the needs of different kinds of water users in different subregions can be met to the greatest extent possible with this approach. Therefore, the three dimensions of synergy are also fused in this layer. The third stage of the process (the water in different departments is supplied to different kinds of water users, namely, residential users, industrial users and municipal users, in different subregions) is optimized in this layer. After department-level decision-making,

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conflicts of interest inevitably occur among various water users in different subregions of a city. Therefore, the third layer considers regional-level decision-making to to coordinate water needs and avoid conflicts of subregions in the city. Moreover, the various development priorities of subregions are emphasized by adjusting certain hyperparameters in the third layer. This layer is established based on the allocation scheme obtained in the second layer of the hierarchy, and the time scale of this layer is the same as that of the second. Although water pollutants are controlled in the second layer, the detailed spatial distribution of pollutants remains unknown. If one of the subregions emits a greater pollution load than others such that the river pollution limit is exceeded, it constrains sustainable development and undermines the fairness of the allocation. To ensure the coordination of water quality among regions, the representative pollutant concentration of the main reach in each subregion after configuration should meet the relevant environmental capacity requirements. If these requirements are not met, then the objective function for this subregion will call for a punishment, and more environmentally friendly plans will be searched. After sewage with pollutants is transported from outlets to water bodies, advective transport, longitudinal dispersion and transverse mixing will occur. At the same time, physical, chemical and biological interactions will occur in the water body. To objectively describe the degradation of pollutants in water, it is necessary to use mathematical models to simulate physical dynamics. Due to the heterogeneity of pollutants entering water bodies and the

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uncertainty of hydrological processes, it is usually of little practical significance to calculate the change in river water capacity over time. A steady-state model is therefore used to calculate the water capacity of the target water body (Cetintas et al., 2010; Zhang et al., 2019). When water quality changes are studied at the annual scale and complete mixing is assumed, the following equation can be used to describe the water quality change, as shown in Eq. (33):

$$\frac{Vdc}{dt} = Q(Ce - C) + Sc + r(c)V$$
 (33)

where V (m³) is the volume of water; Q (m³/a) is the flow in and out of the system

520 at equilibrium; Ce (g/m³) is the contamination concentration in the inflow (g/m³); C521 is the pollutant concentration; Sc denotes other external pollution sources (m³/a); and 522 r(c) is the reaction rate of pollutants in water (g/m³/a). The above equation can be 523 defined as the basic mass balance of a water body in a completely mixed system. 524 Because the pollutants are evenly mixed in each small interval, the horizontal and 525 vertical concentration gradients of pollutants can be neglected. Therefore, the model of 526 water quality in mixed rivers under steady-state design conditions is adopted (Yue et 527 al., 2021):

528
$$W_{c} = 31.54 * \left[C_{s} \cdot (Q_{p} + Q_{E} + Q_{S}) - Q_{p} \cdot C_{p} \right]$$
 (34)

where W_c represents the water environmental capacity (t/a); Q_p is the flow in the reach (m³/s); C_p is the pollutant concentration in the river (mg/L); Q_E is the sewage discharge (m³/s); Q_S is the total flow of nonpoint sources into the reach above the





- control section (m³/s); and C_s is the target concentration of river pollutants (mg/L).
- 533 The result calculated based on the total hydrological capacity standard is often
- 534 relatively large, which is generally referred to as nonconservative. To conform to real
- conditions, the concept of a nonuniformity coefficient is introduced for correction:

$$W_c = \alpha \cdot W_c = 0.6 \cdot W_c \tag{35}$$

- This coefficient is used to assign a punishment if the water quality exceeds the
- 538 relevant value in a given subregion. Based on the coefficient value, the objective
- 539 functions and constraints are adjusted accordingly. Finally, the daily decision
- alternatives for water allocation from water departments to water users are obtained at
- 541 the regional decision level.

542 **2.3.1 Objective function**

- 543 Regional objective function: Maximization of the comprehensive benefits of each
- 544 subregion

545
$$\max f_3(x) = \sum_{t=1}^{T} \sum_{i=1}^{J} \sum_{j=1}^{Jr} x_{ij}^t b_{ij} - \sum_{i=1}^{Jr} \left(D_j - \sum_{t=1}^{T} \sum_{i=1}^{J} x_{ij}^t \alpha_{ij} \right) \times \omega_j - P_r(r_{nz}) - G_r(x_{ij}^t) \times q \quad (36)$$

$$P_r(r_{nz}) = e_i \times \sum_{p=1}^{Pr} P_p^{pump} \times \nabla t_r + x_{ij}^t \delta_{ij} \psi_{ij} \omega_{ij}$$
(37)

$$\nabla t_{r} = \sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{z=1}^{Zr} \left\{ \left(l_{nz} + \left(r_{nz}^{t} \theta_{nz} / CAS_{nz} \right) \right) / \left(Q_{nz}^{\text{max}} / CAS_{nz} \right) \right\} / 3600$$
 (38)

548
$$G_r(x_{ij}) = \sum_{z=1}^{Zr} \sum_{u=1}^{U} (Q_{z,u,r}^{final} - Q_{z,u,r}^0)$$
 (39)

where b_{ij} (yuan/m³) is benefit per unit of water supply for the jth user; ω_j is the

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penalty coefficient per unit of water deficiency; j=1,2,...,Jr; Jr is the number of water users in rth subregion; r= 1,2,...,R; $P_r(r_{nz})$ is the penalty function for cost in the rth subregion; e_i (yuan/kW·h) is the unit electricity fee; P_p^{pump} (kW·h) is the electrical power consumed by the pth pump at a pump house in each hour; p ranges from 1 to Pr; Pr is the total number of pumps in the rth subregion; ∇t_r (h) is the time required for water transfer to provide support for the inland river flow in the rth subregion; ω_{ii} (yuan) denotes to the fee paid for sewage treatment; l_{nz} (m) is the length of a water diversion pipe from reclaimed water source n to the zth inland river; z ranges from 1 to Zr; Zr denotes the number of inland rivers in the rth subregion; CAS_{nz} (m²) is the cross-sectional area of a pipe from the nth reclaimed water source to the zth inland river; Q_{nz}^{max} (m³) is the maximum overflow capacity of the diversion pipe from the *n*th reclaimed water source to the zth inland river; $G_r(x_{ij})$ is the penalty function for substandard water quality in the rth subregion; $Q_{z,u,r}^{final}$ (mg/L) is the final concentration of the uth pollutant in the control section of the zth inland river in the rth subregion after optimal configuration; $Q_{z,u,r}^0$ (mg/L) is the initial concentration of the uth pollutant in the zth inland river in the rth subregion; and q is the penalty coefficient for substandard water quality in the rth subregion. The number of objective functions in this layer depends on the number of subregions divided in the city, which is based on local conditions.

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2.3.2 Constraints

Water quality constraints

Mathematical models are often developed to help satisfy the water quality standards at

572 monitoring points (Zhang et al., 2019; Pourshahabi et al., 2020; Friesen et al., 2017).

573 However, for some cities with very few monitoring points, such approaches may lead

574 to good water quality in the monitored sections and poor water quality in other sections.

575 In these circumstances, the quality of water bodies in each subregion of a city is not

simultaneously maintained. To maintain the water quality in all subregions of a city at

577 the desired target level, the water quality constraint in Eq. (40) is established:

$$Q_{z,u,r}^{final} \le Q_{z,u,r}^{control} \tag{40}$$

where $Q_{z,u,r}^{control}$ (mg/L) denotes the control standard for the *u*th pollutant in the control

section of the zth inland river in the rth subregion.

581 **2.4 Model solution**

2.4.1 Synergy degree evaluation

Enhancing the understanding of the synergy among water allocation alternatives to achieve broad coordination and equilibrium is crucial. The evaluation of the synergy of a water system is strongly related to multiple complex interactions, such as the interactions among different processes, users, and regions. However, these interactions have rarely been explicitly captured in prior evaluations of water allocation. One of the

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key network metrics used in network analysis, connectivity, is a promising measure of the degree of coordination among different objectives in complex systems (Weitz et al., 2018). Connectivity reflects the connectedness of a given link to all possible links in the network, and the strength of each link is weighted, reflecting the number and strength of correlations (Felipe-Lucia et al., 2020). In this study, connectivity is used to embody coordination in the context of synergy, as shown in Eq. (26). Due to the limited supply of water resources, competition among different objectives is unavoidable, and the objectives cannot be fully optimized to equal extents, i.e., , an increase in one target output may decrease another output. Therefore, equilibrium is integrated as another vital part of the synergy devoted to maintaining a balance among the satisfaction of each goal in a system. The equilibrium based on the principle of information entropy (Gao et al., 2013; Zivieri, 2022) is shown in Eq. (27). Information entropy is a measure of the uncertainty associated with a random variable and is used to quantify the information contained in a message, usually in bits or bits/symbols; furthermore, it has been widely used to represent the fairness or equilibrium of a system (Chen et al., 2022; Zhao et al., 2022). When H is low, the level of equilibrium in the system is high. By combining the quantification of coordination and equilibrium, the synergy degree is appropriately determined (Eq. (29)). Notably, the total synergy index (TSI) of a system is used for both generating candidate management alternatives in the generation phases of PTSOA and performing assessments of the associated level of synergy, as shown in Eqs. (41-44).

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parameter ob_i , respectively.





$$SSI_{ob_i} = \frac{\sum_{j=1}^{N} c_{ij} \times (ob_i + ob_j)}{\sum_{j=1}^{N} (ob_i + ob_j)}, i \neq j$$

$$(41)$$

610
$$H(S) = -\sum_{i=1}^{N} \frac{(1 - u_{ob_i})}{N} \log \frac{(1 - u_{ob_i})}{N}$$
 (42)

$$u_{ob_i} = \frac{ob_i - ob_{i,\min}}{ob_{i,\max} - ob_{i,\min}}$$
(43)

$$TSI = \frac{\sum_{i=1}^{N} SSI_{ob_i}}{H(S)}$$

$$(44)$$

where SSI_{ob_i} is the connectivity of the ith object; c_{ij} is the Pearson correlation between the ith object and jth object; ob_i and ob_j are the values of the ith and jth objective functions, respectively; TSI is the synergy index of the system; H(S) is the overall equilibrium of all objects based on the principle of information entropy; u_{ob_i} is the standardized value of the ith object; N is the total number of objects in the system; $ob_{i,min}$ and $ob_{i,max}$ are the minimum and maximum critical thresholds of the

2.4.2 Hierarchical optimal algorithm design for the PTSAO model

Based on the algorithm design with a hierarchical objective function proposed by Li et al. (2022), a new level is added to the original two levels of the algorithm, and the alternative generation phase is improved for better synergy. In this algorithm, the objective functions in the upper decision level is first satisfied, and then the lower-level objective function provides an optimal result based on the results of optimal allocation





626 in the upper level. To provide as comprehensive solutions as possible, the decision 627 alternatives need to be classified into different sets for further selection. In addition, the 628 synergy degree of the result of each layer is calculated to select optimal decisions 629 among all Pareto front solutions. The detailed steps of the hierarchical optimal algorithm are as follows: 630 631 I. In the first level, calculate the objective function (city level) values for the social, 632 economic and sustainable development components, and sort the results with 633 NSGA-III (Pourshahabi et al., 2020; Chen et al., 2017) to obtain each Pareto 634 front $F_1, F_2, ..., F_i$. 635 II. Classify the Pareto fronts into K (K is determined based on the diversity of 636 policies) elements with the K-means algorithm (Liu et al., 2022), which is used 637 to partition a data set into K distinct and nonoverlapping clusters. To perform 638 K-means clustering, we first specify the desired number of clusters K. Then, the 639 K-means algorithm is used to assign each observation to exactly one of the K 640 clusters. 641 III. Calculate the synergy degree of each individual in the front, and select the 642 solution that yields the greatest synergy in each cluster. K solutions are obtained 643 in the first layer. 644 IV. Use the selected K solutions in the first layer to establish constraints in the second layer. Solve the objective function of the second layer with NSGA-III. 645 646 V. Calculate the synergy degree of each individual in the front and select the





solution that yields the greatest synergy as well as the two solutions that maximize the conventional and unconventional water supply department objective functions in all Pareto fronts with K preconditions.

VI. The three selected solutions in the second layer are used to establish constraints in the third layer. Solve the objective function of the third layer with NSGA-III under the three preconditions.

VII. The synergy degree of each individual in the front is calculated, and the solution that yields the greatest synergy in the third layer is selected. Three solutions are obtained considering the synergy in the former two layers. Finally, the synergistic configurations optimal for all stages in the whole process are identified considering the synergy among decision levels, processes and time scales.

3. Application

660 3.1 Study area

Yiwu city is selected as a case study to validate the applicability of the PTSOA model.

Yiwu city is in Southeast China, located from 119°49 'E-120°17' E and 29°02 '13 "N
29°33' 40" N. The city covers an area of 1105 km². The area is characterized by a

scarcity of water resources, and the conventional water supply is under severe stress.

The regional water consumption depends heavily on transported water and external





water transfer. The per capita water resources total 622 m³, only 22.6% of the provincial average and 19.1% of the national average. Moreover, the problem of water pollution has become a bottleneck constraint for the development of Yiwu city. Therefore, it represents a typical water-scarce city with limited conventional water. Notably, water quality in Yiwu has been subjected to significant environmental stress because of the negative effects of wastewater discharge with the rapid development of industry. The current water quality is poor, with Class V water, and the main pollutant concentrations exceed the corresponding standards (Zhejiang Natural Resources and Statistical Yearbook on Environment, 2020). As shown in Fig. 3, the Yiwu River crosses the city from northeast to southeast. Additionally, there are six ecological water compensation outlets in six main tributaries in the Yiwu River.



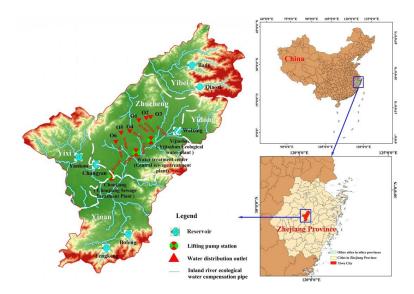


Fig. 3. Map of the study area

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3.2 Generalization of the water system

An initial multisource, complementary and mutually regulated system has been developed for Yiwu, and this system spans the entire urban water cycle (water sourcewater supply-water use-drainage-drainage collection-recycling and reuse). To apply the optimal water allocation model to the complex real-world water system, all stakeholders in the water system should be schematized into a topological system, as shown in Fig. 4. The diagram comprises five modules: water sources, water supply, water use, water drainage and external discharge for all stakeholders. The first module includes seven main reservoirs, two water diversion projects, the Central Sewage Treatment Plant and the Yiwu River. The seven reservoirs and two water diversion projects (as shown in Table 1) supply high-quality water. There are complex connections between the first and second modules. For example, two reservoirs supply water to one waterworks or one reservoir feeds two or three waterworks simultaneously. The reservoirs also supply some of the agricultural and ecological waters to subareas of the city. The Yiwu River, with a total length of 38.39 km and 21 first-class tributaries in the city, and the Central Sewage Treatment Plant, as shown in Table 2, are low-quality water sources. Additionally, excluding water from reservoirs, most agricultural irrigation water is supplied from surface water stored in hundreds of small reservoirs and mountain ponds. Since there are no data available for agricultural irrigation water, which accounts for only a small portion of the total water

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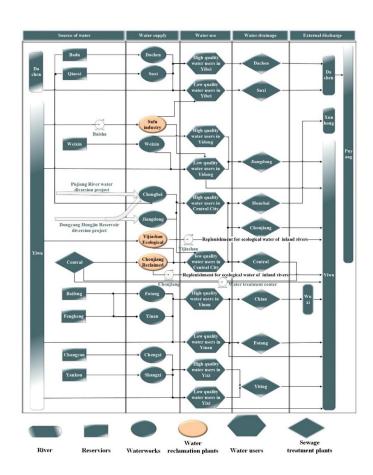




demand in the area, this water volume is ignored in the model. For the second module, high-quality water piped from reservoirs is transported to nine urban and rural centralized waterworks (as shown in Table 2). The Yiwu River distributes low-quality water to the Yijishan Ecological Water Plant and Sufu Industrial Water Plant through the Yijishan and Baisha Water Pump Stations, respectively. The water discharged at the Central Sewage Treatment Plant is transferred to the Choujiang Industrial Water Plant. Based on the water supply project distribution and the economic as well as social development levels, Yiwu is divided into five districts, as shown in Table 3: the Central District, Yidong District, Yibei District, Yinan District and Yixi District. The third module comprises both high-quality water users (high-quality water users consist of urban and rural domestic water users and industrial water users in the water supply network of urban and rural public water plants) and low-quality water users (lowquality water users include industrial water users, municipal water users and ecological water replenishment for inland rivers) in each district. There are nine sewage treatment plants in the fourth module (which focuses on the drainage stage), as shown in Table 2. The unreused water from sewage treatment plants is discharged to the external environment. Reuse processes are also considered in the system.







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Fig. 4. Schematic diagram of Yiwu city

719 3.3 Parameter determination

According to the flow duration curve of the annual natural inflow data for 51 years (1963-2014), three years with exceedance probabilities of 50%, 75% and 90% are selected to represent normal (1984.1–1985.1, annual mean inflow: $1.33 \times 10^8 \text{ m}^3$), dry (2008.1–2009.1, annual mean inflow: $1.11 \times 10^8 \text{ m}^3$), and extremely dry (1971.1–1972.1, annual mean inflow: $0.63 \times 10^8 \text{ m}^3$) scenarios, respectively. In addition to





inflow, the data used in the PTSOA model mainly include the data for the parameters in each layer. Water demand values were calculated using the Yiwu City Water Resources Comprehensive Plan 2020, as shown in Table 1.

Table 1 Water demands of various regions in Yiwu in 2020 (10⁴ m³)

Subregion	Yibei	Yidong	Zhucheng	Yixi	Yinan
water demand	1695	572	11813	2198	2045

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The water resources fees paid to the government total 0.3 yuan/m³. The parameters of the reservoirs and external water division projects in Yiwu city are listed in Table 2.

Table 2 Parameters of the reservoirs and external water division projects

				Flood limit	
	Water	Initial	Dead	storage	Absolute storage
Reservoirs &	Fee	storage	storage	capacity	capacity
External sources	yuan/m³	10^4 m^3	10^4m^3	10^4 m^3	10^4 m^3
Badu	0.99	1359	49	2688	2639
Qiaoxi	1.30	1505	77	2933	2856
Weixin	0.37	500	17	483	466
Baifeng	1.05	1013	15	2010	1995
Fengkeng	1.15	778	55	1501	1446
Yankou	1.49	1820	499	3140	2641
Changyan	0.70	491	41	940	899
Pujiang Project	1.00	0	0	3000	3000
Dongyang Project	1.00	0	0	5000	5000

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The Tennant method is applied to calculate the ecological water demand. In this method, the relationship between the annual average discharge and habitat quality is considered, and the percentage of the annual average natural runoff is used as the recommended value of the ecological water demand for a given river channel.



- According to the recommended values, the percentage of runoff required for the fish
- 739 spawning period from April to September is 30% and the percentage runoff in the
- 740 general water consumption period (October to March) is 10%.
- Based on observations obtained with the F601 evaporator (a standard evaporation
- 742 instrument widely used in China), evaporation is calculated as:

$$EP = E \times k \tag{45}$$

- 744 where EP (mm) is the evaporation of a reservoir; E (mm) is the observed evaporation;
- 745 and k is a reduction coefficient. According to observations, this coefficient is the same
- 746 for every reservoir and varies throughout the year (Zhao, 2014). The prices of
- conventional water and reclaimed water are 1.7 and 2.6 yuan/m³, respectively.
- The monthly mean monitoring data for effluent pollutant concentrations and the
- 749 daily maximum processing capacities of sewage treatment plants were obtained from
- 750 the monitoring systems of the sewage treatment plants. For example, the concentrations
- 751 of COD, NH3-N, TN, and TP in the sewage of the Jiangdong Sewage Treatment Plant
- 752 are 13.80 (mg/L), 0.22 (mg/L), 6.02 (mg/L), and 0.13 (mg/L), respectively. The daily
- 753 maximum processing capacity of Jiangdong Sewage Treatment Work is 12 (10⁴ t/d).
- 754 The effluent quality of sewage treatment works satisfies the Class A Standard used in
- 755 China. The maximum capacities of the Baisha pump station, Yijiashan pump station,
- 756 Choujiang pump station and water treatment centre pump station are 13 t/d, 13.5 t/d, 10
- 757 t/d, and 4.5 t/d, respectively.
- 758 Additionally, the environmental capacities of the six tributaries that are replenished





with ecological water are calculated according to Eqs. (33)-(35), and the results are listed in Table 3. COD, TP and TN are selected as representative pollutants in the tributaries to guarantee the water environmental quality of inland rivers. The water quality goals for the tributaries must conform to the Class III standard according to GB 5749-2006 in China. The unit electricity price of pump stations in Zhejiang Province is 0.41 yuan/kW·h. GB50014-2006 (2014 edition) stipulates that the comprehensive urban domestic sewage quota should be 80~90%, and the urban comprehensive domestic sewage quota should be 90% in areas with extensive drainage facilities. According to the "Yiwu Water Resources Bulletin 2020", the urban comprehensive domestic sewage quota is set to 90%, and the sewage treatment rate is set to 100%. The benefits per unit water supply for different users in different subregions are determined from the Yiwu Water Price Adjustment Plan 2020.

Table 3 Area and environmental capacity of tributaries

Name of this buttom	Area (km²)	Class III			
Name of tributary	Alea (kiii-)	COD (t/a)	TN (t/a)	TP (t/a)	
Chengdong	3.4	188.1	4.7	0.4	
Chengzhong	8.7	432.7	31.5	3.8	
Chengxi	6.3	302.5	9.5	2.3	
Chenganan	7.1	318.8	0	3.6	
Hongxi	12.5	778.8	138.8	7.9	
Dongqingxi	38	1271.4	221.5	12.7	

772 4. Results and discussion

773 By solving the PTSOA model for Yiwu city, synergistic optimal water allocation results

for different layers (across different decision levels, water use sectors, and subregions)





are obtained under normal, dry and extremely dry conditions. Pareto sets are obtained across 500 runs of the PTSOA model with the proposed hierarchical optimization algorithm.

4.1 Results of the first layer of the PTSOA model for synergistic

optimal water allocation

To demonstrate the relationship among conflicting objectives, sets of Pareto solutions for the first layer under normal, dry and extremely dry conditions are shown in Fig. 5.

The optimization using the Pareto concept allows the operator to choose an appropriate solution depending on the prevailing circumstances and analyse the trade-off among the conflicting objectives. In each of the figures, the total water supply shortage, total water supply benefit and total amount of water retained in reservoirs in Yiwu city are plotted. The colour of the markers indicates the classification of the solutions of the K-means method, as described in Sect. 2.4.2. All of the decision alternatives are classified into six groups marked in different colours for broad-scale decision-making. The names of the classes are marked in the figure in red (for example, K1-1 represents the first class of solutions in the normal scenario, and K3-2 represents the second class of solutions in the extremely dry scenario). The red arrows indicate optimization directions. The ideal solution is located at the top-right corner (low total water supply shortage, high total water supply benefit, and relatively high total amount of reserved water in reservoirs) of the plot. The geometries of the tradeoffs vary significantly across





the applications, as is expected given the different hydrological conditions. Generally, the total water supply shortage and the total amount of water retained in reservoirs show an inverse relationship. In contrast, the total water supply benefit shows a direct and positive influence on the total water supply shortage. The water supply reliability of the selected decision alternatives is greater than 95% under normal, dry and extremely dry conditions. The total amount of reserved water in reservoirs under normal scenarios varies in the range of 2.91×10^7 m³ to 6.14×10^7 m³, which is much higher than that under the extremely dry scenario, with a value of 1.44×10^7 m³ to 2.93×107 m³. This finding demonstrates that the optimal allocation is able to reconcile the present demand and future needs, even in extremely dry scenarios. The total water supply shortage in all scenarios is less than 5% of the water demand, which indicates that the guaranteed water supply is greater than 95%.

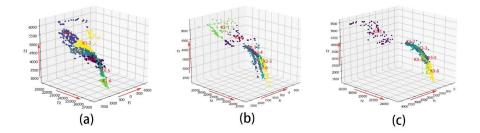


Fig. 5. Sets of Pareto solutions after 500 model simulations with the hierarchical optimal algorithm under (a) normal, (b) dry and (c) extremely dry scenarios. The red arrow indicates the direction of optimization.

We further present the TSI (total synergy index), SSI (total connectivity) and H

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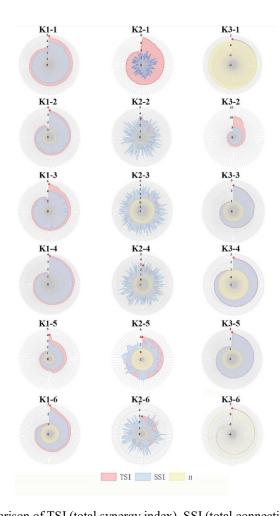
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(overall equilibrium) values for different classes characterized based on the optimal PTSOA solutions under three scenarios, as shown in Fig. 6. In the PTSOA model, the Pareto solutions with the best TSI values are input to the second layer for further optimization. Thus, the red points in Fig. 6 represent the selected schemes for all classes. We observe that the variation in the TSI is consistent with that in the SSI in some, but not all cases. In some cases, difference are mainly caused by the influence of H, which influences the optimal hydrological equilibrium, especially in dry conditions. Although normal conditions are most conducive to achieving equilibrium, the better H value in extremely dry conditions than in dry conditions seems nonintuitive. However, these results suggest that when water is very limited, equally limited water is supplied to all users, thus enhancing the overall equilibrium. We note that the SSI is higher in the normal scenario than in the other two scenarios. We attribute this to relatively abundant water being useful for stakeholders to achieve synergy due to the reduced competition compared to other cases. The TSI values reach maximums of 5.36, 7.37 and 10.82 under normal, dry and extremely dry conditions, respectively. Since the TSI is used to illustrate the synergy of allocation plans under certain conditions, the three kinds of TSI values are not comparable.





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Fig. 6. Comparison of TSI (total synergy index), SSI (total connectivity) and H (overall equilibrium) values among various Pareto solutions in different classes for

the (K1) normal, (K2) dry, and (K3) extremely dry scenarios.

As an example, Fig. 7 provides the specific water supply decision alternatives for the first layer that maximize synergy in each cluster under normal conditions. The water allocation plans for the seven main reservoirs and two external water diversion projects in every month of the configuration period are displayed. All reservoirs and water works





are represented by abbreviations based on their full names in Fig. 7. For example, QX-CB is the label for the water supplied from Qiaoxi Reservoir to Chengbei Water Works. The water volumes supplied by Qiaoxi Reservoir to Chengbei Water Works (ranging from 1.78×10^7 m³ to 3950×10^4 m³) and from the Pujiang External Water Division Project to Chengbei Water Works (ranging from 2.57×10^7 m³ to 3×10^7 m³) are relatively high in all clusters. This result is consistent with the fact that Chengbei Water Works is one of the main conventional water sources for the central city area, a region that accounts for more than 50% of the total water demand of Yiwu city. The water supplied by the two external water diversion projects from August to December is higher than that in other months. The mean monthly precipitation in these months is only 58-74% of the mean annual precipitation in Yiwu, so more external water is supplied for replenishment. Baifeng and Fengkeng Reservoirs supply similar volumes of water to their two connected waterworks.





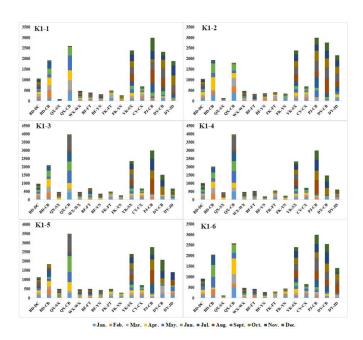


Fig. 7. Water supply from each reservoir to connected water works in each month in

the normal scenario (10⁴ m³)

4.2 Results of the second layer of the PTSOA model for synergistic optimal water allocation

The 6×3 decision alternatives selected in the six clusters of the optimal first-layer results in the normal, dry and extremely dry scenarios are input into the second layer for further optimization. As shown in Fig. 8, the total amount of water retained in water works and the amount of unconventional water supplied show a negative correlation. In the alternative generation phase of game bargaining between the two objectives, the greater the total amount of water retained in water works is, the greater the amount of unconventional water supplied will be, which indicates that more conventional water





863 will be saved when more unconventional water is supplied. Conversely, the amount of unconventional water supplied is affected by the total amount of water retained in water 864 865 works. 866 In the second layer, three alternatives in each scenario are selected as prior 867 conditions for further optimization. In addition to the two individual extrema of the two 868 objectives, the alternative that yields the best synergy is also identified, and it is similar to that in the first layer. In the normal scenario, the TSI values are -0.90, -1.02 and -0.88 869 in the cases with the optimal conventional water supply, unconventional water supply 870 and synergy, respectively. The most synergistic approach includes only 7.08×10⁴ m³ 871 872 more conventional water retained than that in the conventional water supply cases and only 9.72×10⁴ m³ more than that in the optimal unconventional water supply case. 873 874 Therefore, not only is the best TSI value obtained, but the requirements of both 875 conventional and unconventional water supply departments are met. The TSI of the 876 most synergistic solution is the highest under dry conditions, with a value of -0.79. 877 Overall, the total amount of water retained in the water works ranges from 3.95×10^7 m^3 to $5.75 \times 10^7 m^3$, $3.12 \times 10^7 m^3$ to $5.31 \times 10^7 m^3$, and $2.43 \times 10^7 m^3$ to $3.96 \times 10^7 m^3$ for the 878 879 three types of conditions. The total amount of unconventional water supplied ranges from 5.95×10^7 m³ to 7.48×10^7 m³, 6.34×10^7 m³ to 7.56×10^7 m³, and 6.28×10^7 m³ to 880 7.37×10⁷ m³ in the normal, dry and extremely dry scenarios, respectively. It is notable 881 882 that the drier the conditions are, the lower the amount of water retained in water works 883 and the greater the amount of unconventional water supplied. This approach is useful

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for cities to mitigate the risk of drought. Additionally, based on the constraints regarding the contaminants allowed to be discharged, more than 1272.21 t and 48.81 t of COD and ammonia nitrogen emissions are avoided per year. In other words, the balancing of the two objectives is beneficial for managers to determine an equilibrium solution that satisfies the relevant demand and successfully avoids surplus conventional or unconventional water supply in terms of sustainable development.

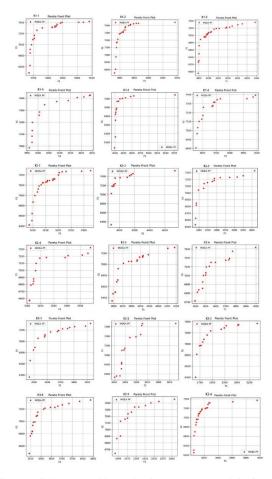
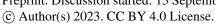


Fig. 8. Pareto fronts of the second layer in the PTSOA model after 500 simulations

with the hierarchical optimal algorithm in the normal, dry and extremely dry

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scenarios. F1 represents the total amount of water retained in water works (10⁴ m³), and F2 represents the amount of unconventional water supplied (10⁴ m³). The direction of optimization is from the top-right corner to the bottom-left corner.

4.3 Results of the third layer of the PTSOA model for synergistic

optimal water allocation

After selecting the three scenarios that yield the best synergy and the two best objective functions for characterizing all Pareto fronts of the second layer in each scenario, these 3×3 solutions are input to the third layer for further optimization. Fig. 9 shows the tradeoffs among the five objectives in the third layer of the PTSOA model for the (S1) normal, (S2) dry, and (S3) extremely dry scenarios (these abbreviations are used to distinguish these results from those of the above two layers). The number following the '-' represents the selected solution from the second layer. For example, S1-1 represents the normal scenario with the minimum total amount of water retained in water works, S1-2 represents the normal scenario with the maximum unconventional water supply and S1-3 represents the normal scenario with the maximum synergy degree in the second layer. In each of these plots, the abscissa denotes the identifier for the objective functions, which ranges from 1 to 5, and the ordinate gives the objective values in the Pareto fronts (10⁴ yuan). The five dimensions include the comprehensive benefits of the Yibei (1.0 dimension), Yidong (2.0 dimension), Yixi (3.0 dimension), Yinan (4.0 dimension) and central city (5.0 dimension) subregions. As shown in the figure, the





central city achieves the most comprehensive benefit among the five cities. This is primarily attributed to the large population and intensive industry in this area. However, the benefits in the other four subregions are also high compared to recent levels and those achieved with traditional allocation methods, as shown in Table 9. Interestingly, the comprehensive benefits in the subregions are greater in the scenario with the maximum synergy degree under normal conditions than in the other two scenarios. Technically, the total comprehensive benefits in the five subregions in this scenario are approximately 2.3×10^8 - 5.1×10^8 yuan higher than those in other cases, which indicates that the solution with the highest synergy degree in the second layer is the best choice for managers in normal years. However, the various subregions obtain the greatest benefits when maximizing the unconventional water supply in dry and extreme scenarios. This result indicates that increasing the use of unconventional water in dry and extremely dry years would significantly increase the potential benefits.





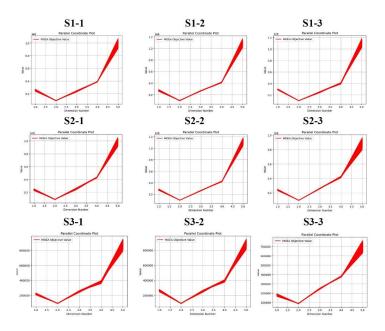


Fig. 9. Illustration of parallel-reference Pareto sets from the third layer in the PTSPOA model attained across all runs for the (S1) normal, (S2) dry, and (S3)

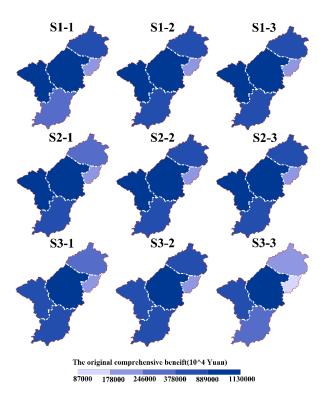
extremely dry scenarios

Fig. 10 presents the optimal comprehensive benefit in each subregion. In all scenarios, the central city is associated with the highest comprehensive benefit, followed by Yixi and Yinan, and the comprehensive benefit in Yidong is relatively low. This result may be related to this subregion having the smallest area (72.2 km²) and the smallest population (7.7×10⁴ people). The comprehensive benefits vary among different solutions and scenarios. Among the three normal decision alternatives, F1, F2 and F5 are highest in S1-3, with values of 3.03×10⁹ yuan, 9.90×10⁸ yuan and 1.12×10¹⁰ yuan, respectively. This indicates that considering the synergy degree could increase the





comprehensive benefit in most subregions in the normal scenario. Among the alternatives in the dry and extremely dry scenarios (excluding F4 and F5), other objectives are highest in S2-2, with values of 2.84×10^9 yuan, 9.63×10^8 yuan and 2.67×10^8 yuan, respectively. It suggests that maximizing the unconventional water supply is beneficial for the system in dry conditions. Additionally, F4 is highest, with a value of 2.29×10^9 yuan, in S2-3 among the three solutions in the dry scenario, and F5 is highest, with a value of 9.17×10^9 yuan, in S3-1 in the extremely dry scenario.



F10. Comprehensive benefit in each area after the regional collaborative allocation of

948 water resources





4.4 Discussion

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To assist policymakers in understanding the complex and systemic nature of water systems and reveal the dynamic interactions among objectives, network analysis and optimization was applied. By revealing the interactions among different objectives, we determine the level of synergy in complicated water systems, identify the challenges and opportunities for sustainable development of water systems in cities with various subregions, and provide valuable insights and specific action priorities for these regions. In the networks shown in Fig. 11, each node represents an individual objective (F1, F2, F3, F4, and F5 represent the comprehensive benefits in Yibei, Yidong, Yixi, Yinan and the central city, respectively), and pairwise objectives that are significantly (P < 0.05) correlated are connected by a link, where the strength of each link is related to the Pearson correlation coefficient. The obtained networks with 5 nodes were weighted and undirected (directionality can be estimated only if the direction of causality is known). The size of the circles in the figure indicates the connectivity of each objective. We considered trade-offs (i.e., negative correlations wherein one objective improves while the other worsens) among the objectives. In most scenarios, F5 was the relatively dominant objective, signifying that other objectives disproportionately deteriorated as progress was made towards the benefit of the central city, as shown in Fig. 11. It is evident that the trade-offs are more balanced in the scenarios with the highest degrees of synergy (S1-3, S2-3, and S3-3), which indicates that the tradeoffs and competitions





among the objectives are alleviated when synergy is considered. The links show that the conflicts of interest between F4 and F5 in scenarios S1-1 and S2-2 are extremely notable, suggesting that the comprehensive benefits in Yinan and the central city correspond to strong negative interactions in these cases. The connectivity of most objectives was relatively low in the tradeoff network in the extremely dry scenario, but F5 played a dominant role in terms of negative interactions among objectives, although the connectivity of F5 was lower than other connectivities in most normal and dry scenarios. Moreover, as the scenario varied from normal to extremely dry, the impact of individual regional targets on the whole system diminished.

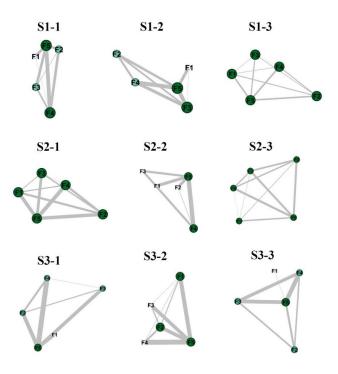
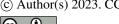


Fig. 11. Network analysis of the results of layer 3





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For comparison, we applied five widely used MOEAs, namely, NSGA-II, SPEA-II, ε-MOEA, IBEA, and MOEA/D, to solve cases with 3+2+5 mathematical objectives (3 objectives in the first layer, 2 objectives in the second layer and 5 objectives in the third layer) with the same constraints given previously for Yiwu city under normal, dry and extremely dry conditions. The constraints and common parameters, such as the maximum number of model simulations and the simulated binary crossover (SBX) rate, are set to those used in the PTSOA model. However, it is difficult to determine feasible decision alternatives with MOEAs, even though the number of iterations is increased to 20000 (which is far beyond that considered in the previous modelling) because the complexity of the system overshadows the optimization capabilities of these traditional models. These results reconfirm the superiority, efficiency and decoupling capability of the proposed model for optimal allocation cases involving complex water systems with multiple stakeholders, multiple sources, multiple decision-makers and embodied reused systems. By embedding the targets into hierarchical layers, the excessive abandonment of some promising alternatives is avoided, and optimal allocation is progressively achieved. In general, the hierarchical structure of the PTSOA model can simulate complicated systems with multiple complex objectives and constraints. In addition, the five MOEAs were used to solve the equations in the third layer of the PTSOA model, and the overall targets in the first layer were determined based on



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those used in the third layer of the PTSOA model. Additionally, the benefits in the current case with no optimization calculated based on the actual water supply are given for comparison. The current situation was categorized as a normal scenario, and other models were established with the same conditions to facilitate further comparison and analysis. There were distinct decision alternatives generated by each model, and the relevant results are listed based on their value ranges. As shown in Table 4, although NSGA-II and ε-MOEA yield slightly higher F2 values than PTSOA and F3 generated by IBEA (4.8×10⁸ -7.2×10⁸ yuan) is higher than obtained with PTSOA, PTSOA performs better than other models in most cases. The PTSOA is shown to be the best model for obtaining comprehensive benefits for the subregions in Yiwu in the normal scenario, demonstrating that the PTSOA model offers advantages including identifying the best alternatives and achieving greater subregional benefits than the other models. The proposed model yields a 1.76×10⁹-15.67×10⁹ yuan total comprehensive benefit improvement and can save approximately 3.2×10^7 - 4.7×10^7 (m³) of conventional water compared to the current values. It is also evident that the proposed model yields the highest TSI values, reflecting the improvement achieved by considering the synergy of the system. In terms of the targets in the first layer, except MOEA/D, other traditional models fail to retain enough water (water requirements for living under extreme drought conditions of the next configuration period) in the reservoirs to meet future basic needs. For MOEA/D, although it generates a slightly higher total water supply benefit, with a value of 2.81×10⁸-3.12×10⁸, the total water supply shortage and the total amount of





reserved water in the reservoirs are worse than the amounts obtained with the proposed model. PTSOA trades some economic benefits for enhanced water supply reliability and sustainable development, resulting in a decrease in the water supply from conventional water plants.

However, the consideration of reclaimed water in the proposed model effectively reduces the use of traditional water and improves the quality of the water environment by reducing sewage discharge, and other benefits are also achieved (such as meeting the quality standards for river water and guaranteeing that the ecological water demand of inland rivers is met). The results obtained by the PTSOA may help guide both the government and general public. Our proposed model is superior to traditional models. It can not only optimize water resource utilization and secure water supplies but also enhance the synergy and environmental quality of water systems. Considering synergy across various time scales, the proposed model ensures the synergistic allocation of water resources at yearly, monthly and daily scales while securing both present and future water supplies.

Table 4 Comparison of the comprehensive benefits in the five regions (F1, F2, F3, F4, and F5) and the TSI values in the current situation and obtained using NSGA-II,

SPEA-II, ε-MOEA, IBEA, MOEA/D, and PTSOA in the normal scenario

Comparison		TO				
	F1	F2	F3	F4	F5	- TSI
NSGA-II	2.72~2.86	0.91~1.03	2.57~2.60	3.21~3.37	7.38~9.95	-3.13~-2.82
SPEA-II	2.84~2.97	0.93~0.99	2.58~3.15	3.02~3.68	8.22~9.99	-2.39~-2.46





ε-ΜΟΕΑ	2.47~2.33	0.85~1.12	2.21~2.32	3.05~3.18	9.23~9.91	-3.41~-3.06
IBEA	2.57~2.88	0.87~0.92	3.05~3.11	3.20~3.32	5.27~8.28	-3.28~-3.11
MOEA/D	2.55~2.90	0.99~1.02	3.15~3.20	3.34~3.36	9.82~10.11	-2.37~-1.54
Current situation	2.05	0.83	2.49	3.11	9.87	-3.20
PTSOA	2.63~3.03	0.95~0.99	2.39~2.67	3.84~4.11	10.30~11.22	-1.66~-0.89

1040 **5. Conclusions**

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Applying optimal water allocation models to simultaneously enable economic benefits, water preferences and environmental demands at different decision levels, time scales and regions is a challenge. In this study, a new process-based three-layer synergistic optimal allocation model (PTSOA) is developed and applied to a real and complex water allocation system. The objective functions were divided into three layers to coordinate conflicts of interest among decision makers at different levels and time scales. Furthermore, the allocation of reclaimed water was embedded in the proposed model for synergistic optimal allocation of both conventional and unconventional water. A synergistic index based on network analysis was introduced to reduce competition among different stakeholders and facilitate the positive effect of stakeholder interactions. A hierarchical optimal algorithm was designed to solve the PTSAO model. The proposed model was applied to a representative city in Southeast China with scarce water resources and a developed industry. Achieving the optimal allocation of water resources in this kind of highly developed area offers a valuable reference for other counties in China. Key findings can be concluded from these results, as follows. First, the results demonstrated that the PTSOA model achieved synergistic allocation



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among hierarchical decision-makers across various time scales and in different regions, yielding the highest TSI (-1.66 to -0.89) among the models evaluated. Second, with a synergistic approach, a reasonable amount of conventional water is retained for future use in cases with potentially high risk, with volumes of 3.95×10⁷ m³, 3.12×10⁷ m³, and 2.43×10⁷ m³ retained in normal, dry and extremely dry scenarios, respectively. Moreover, 7.35×10^7 m³, 7.56×10^7 m³, and 7.37×10^7 m³ of conventional water is saved in the three scenarios. Third, considering both reclaimed water and conventional water in the optimization process efficiently improves the quality of municipal water, and more than 1272.21 t/year and 48.81 t/year of COD and ammonia nitrogen emissions are mitigated compared to those in the current situation. Distinct from previous models, the proposed optimal model was implemented with the consideration of spatial dimensions, which are important but often neglected. The results show that spatial allocation yields an improvement of 4-95% for the comprehensive benefits in different subregions compared to the benefits achieved with traditional models, and the total comprehensive benefit increases by 1.76×10⁹-15.67×10⁹ yuan compared to that in the current situation. The synergy index established based on network analysis is used to alleviate the competition among regions and facilitate water supply improvements. These results and conclusions provide valuable references for the evaluations of other complicated water allocation systems. The optimal allocation scheme is determined for a complex water system upon consideration stakeholder synergy and various hierarchical decision levels, time scales and regions. More in-depth studies of





1078 synergistic optimal water allocation are needed in the future. 1079 1080 Data availability. The data used to support the findings of this study are available 1081 from the corresponding author upon request. 1082 1083 Author contribution. JL and YPX designed all the experiments. JL and SWW collected 1084 and preprocessed the data. JL and WZ conducted all the experiments and analysed the 1085 results. JL wrote the first draft of the manuscript with contributions from SWC. YPX 1086 supervised the study and edited the manuscript. 1087 1088 Competing interests. At least one of the (co-)authors is a member of the editorial 1089 board of Hydrology and Earth System Sciences. 1090 1091 Disclaimer. Publisher's note: Copernicus Publications remains neutral with regard to 1092 jurisdictional claims in published maps and institutional affiliations. 1093 1094 Acknowledgements. The editors and two reviewers are greatly acknowledged for their 1095 constructive comments to improve the quality of this paper. The Water Resources 1096 Department of Zhejiang Province and the Yiwu City Water Construction Group Co., 1097 Ltd., are greatly acknowledged for providing the data regarding the water system of 1098 Yiwu city used in this study. 1099 1100 Financial support. This research is funded by the Major Project of Zhejiang Natural 1101 Science Foundation (LZ20E090001) and the Zhejiang Key Research and Development 1102 Plan (2021C03017).





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