

Dear Gerrit de Rooij,

thank you very much for reviewing our manuscript. We have considered all of your statements carefully. Please find below our detailed answers to all comments. We are convinced that your comments have led to considerable improvement of our work and thank for this constructive input. In those cases, in which we disagree with your assessment, we provide a detailed justification for not following your recommendations.

In the first part, we answer to all major comments, in the second part we list all the annotations you provided in the pdf together with the specific replies. In cases, where the annotations are similar to the major comments, we refer to them. For convenience, we numbered the comments.

Kind regards,

Andre Peters, Tobias Hohenbrink, Sascha C. Iden, Martinus Th. van Genuchten, and Wolfgang Durner

## Major comments

1. The paper is generally well-written and clear, and the contribution to soil physics is relevant and suitable for HESS. Below are a few (somewhat) major comments. These, in addition to minor comments, also appear in the annotated manuscript.

We thank you for this quick and detailed review and your general positive judgement.

2. The Introduction is well-written and convincingly argued. I think the paper can be embedded in the literature a bit better. I provide two additional references that themselves have additional references that may be worthy of inclusion. I have been following the work of some of the authors, so I know they are well aware of developments in the literature. Perhaps they can use the depth of that awareness to add a few relevant papers. There is no need for a full-blown review though.

Thank you for the suggestions. We will consider them carefully. We agree with the statement that there is no need for a full-blown review, and have tried to find the right balance between recalling model features that have already been published and the need to introduce readers that are not that familiar with the issue to some model basics. In general, we want to keep the presentation concise and want to focus on the innovative feature of our paper, i.e., the absolute prediction of capillary conductivity. We will, however, incorporate the two suggested references into the introduction.

3. L. 201-202

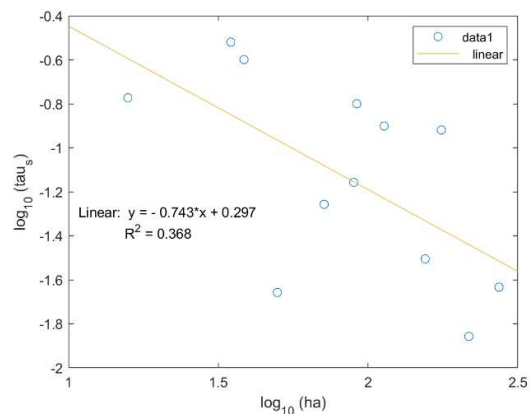
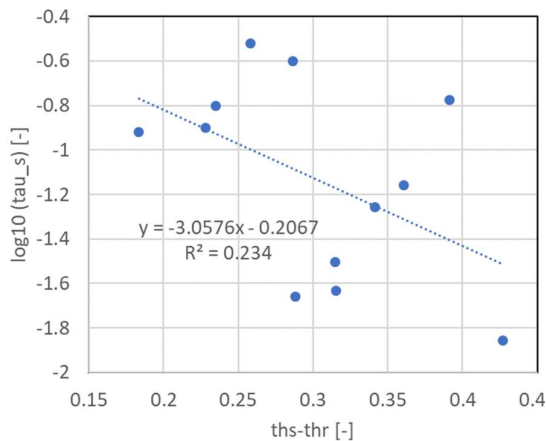
Does the hypothesis of a mildly varying  $\tau_{sub_s}$  not implicitly require that the conductivity of a soil that is so wet that all but the largest pores (whose size and shape are determined almost entirely by the soil macrostructure) are filled does not vary much for different textures? I am not convinced that is the case, but have to admit that my reservations are based more on intuition than hard data. I understand the proof will come later in the paper.

The texture, and thus the pore size distribution, can be completely different within the framework of the developed theory. Thus, the predicted  $K_{sc}$  does not vary much within a texture class, but of course it varies greatly between textures. This, of course, also leads to very different conductivities throughout the moisture range. In contrast, the parameter  $\tau_{sub_s}$

would theoretically vary only in a very small range, even between soils of different textures (see lines 208ff). Note also that our interpretation of  $\tau_s$  takes into account not only path lengthening due to tortuosity, but also other effects in general, such as different fluid properties near particle surfaces and pore linkages deviating from the assumption of the Mualem integral (see lines 214ff). Therefore, we determine it empirically and expect a somewhat larger variation than for a theoretically derived  $\tau_s$  (considering only the orbital strain), but still a variation that is orders of magnitude smaller than for the classical  $K_s$  parameter.

4. Still, I would be interested in a more elaborate treatment of the implications of the range of  $\tau_{sub_s}$  for the range of  $K_{sc}$  in Eq. (20). It seems to me that the additional variability in  $K_s$  must stem from the other terms in Eq. (20) except the constant  $\beta$ . Perhaps dot plots of those terms for the soils for which you predicted  $K_{sc}$  could help. I am not sure if that is the best way to explore this, but the interaction between the three non-constant terms in Eq. (20) is of interest but largely neglected.

This is basically right and we are quite thankful for this comment. We even thought about introducing such a correlation. However, since this correlation can only be done for the 12 calibration data sets (which contain the sufficient information (see line 259)) we decided not to do so. Since  $\beta$  is constant and  $(ths - thr)$  varies only moderately, the main correlation stems between  $\tau_s$  and the shape of the capillary saturation function. Below you can see on the left the correlation between  $\tau_s$  and  $(ths - thr)$  and on the right the correlation between  $\tau_s$  and  $h_a$ . Note that the parameter  $h_a$  (air entry for the non-capillary part) is here given as the suction at which capillary saturation is 0.75 (outlined in appendix 2, Lines 477ff). We used this value because it lumps the effect of the shape parameters  $\alpha$ ,  $n$ , and  $m$  together. We see a weak correlation for both. Especially on the right-hand side there is a tendency for lower values for  $\tau_s$  with higher values for  $h_a$  (i.e. for fine textured soils).



However, in considering whether it is advisable to include this information in the paper, we feel that it would distract from the central content, the development of the theory. Therefore, we believe that this topic deserves its own discussion once more data is available to validate our model.

5. Eq. (20)

The presence of a residual water content confuses me a little given the tendency in the past 10 - 15 years or so to get rid of it. As a case in point, you quote Schneider and Goss, who provided a finite matric potential for oven-dryness. Does this not contradict the existence of a non-zero residual water content?

We are sorry that there is a misunderstanding here. The meaning of parameter  $\theta_{hr}$  in the PDI model is defined in Eq (2) and in the text (line 120) as the "maximum adsorbed water content". The water content of the non-capillary part decreases towards zero at  $h = 10^4.8$  m according to Schneider and Goss. The PDI model thus covers the entire water content range from a maximum water content at saturation to zero at oven dryness. There is no residual water content in the conventional sense.

6. L.228

Please add some explanation for your choices for the capillary saturation functions. There are many alternatives, some of which are part of a set of curves that distinguish between adsorbed water and capillary-bound water. There are also versions without asymptote at zero or a non-zero residual water content. Given the argumentation in the Introduction, I did not expect equations with an asymptote to be included. Perhaps you indeed used the non-asymptotic version of Fredlund and Xing? If I recall correctly, they propose several functions, some of which have an asymptote at zero water content.

With the previous explanation, it is hopefully now clear that the capillary saturation function as part of the PDI model system is only that part of the overall retention curve that reflects the "capillary" pores, and that the asymptote marks the transition from the capillary-dominated to the "film flow"-dominated water regime. This is described in the manuscript (Section 2.1). Any sigmoidal curve that covers the range of 1 to zero can be chosen as the basic capillary saturation function. The fact that this function asymptotically approaches zero at a certain suction does not mean that the total water content converges towards this asymptote. For calculation of the capillary fraction of the conductivity function, we consider this capillary saturation function to be the correct function to be used in the Mualem integral.

We use in this paper the van Genuchten, the Kosugi and the Fredlund-Xing basic saturation functions for the capillary saturation part. These are the most commonly used functions in the field of soil hydrology (VG cited ~30'000 times in Scholar, Kosugi ~850 times) and geotechnics (Fredlund-Xing ~4300). Of course, other models, which account for decreased water contents towards oven-dryness exist and may be used. But we do not think that they add any further gain in knowledge to the aim of the paper, which is the prediction of total conductivity in the wet range. We refer to Table 2 for the mathematical expression of these (also the basic Fredlund and Xing) models and will give a short explanation for the choice of the four capillary saturation functions we used in the revised manuscript.

7. L. 263

Basically, you fit  $\tau$ -s instead of  $K_s$  to fit the data everywhere except in the range near saturation. In effect you are still scaling a conductivity curve, you just call the scaling parameter something else.

The new element is that you use the fitted value as a predictor for soils where you did not fit it to. These soils each have their own values of the residual and saturated water contents,  $F(1)$ , and  $F(\Gamma=0)$ , from which emerges an individual value of  $K_{sc}$  using Eq. (20).

Is this a correct description of the procedure? If so, it makes any relationships/correlations between the three non-constant terms of Eq. (20) even more interesting (see my earlier comment).

It is true that we „fit  $\tau_s$  instead of  $K_s$  to fit the set of calibration data everywhere except in the range near saturation.“ As described earlier in the manuscript, the essence of our reasoning is that this  $\tau_s$  parameter - unlike  $K_s$  - is independent or only weakly dependent on the pore-size distribution and we seek therefore a more or less "universal value" for it that is independent of texture and other parameters.

Eq. (20) was introduced to give a simple link between the classic scheme and our proposed scheme. And you are right, although  $\tau_s$  is constant,  $K_{sc}$  is individual for the different soils, which is reflected in the integration of  $1/h$  with respect to capillary saturation in Eqs. (18) to (20). For an explanation of the nature of  $\tau_s$ , please refer to our response to major comment 3.

8. Figs. 5 and 6

Due to 'space limitations' you present only 6 of the predicted curves. Particularly in the box plots of Figs. 5 and 6, there seems to be plenty of space for more red dots. I would not mind seeing a few more, perhaps even all of them. But what is the meaning of the red cross in the  $v_{Gc}$  column of the left panel in Fig. 5?

Please note that all of the 23 curves are shown in the supplemental material, while the 6 examples are only randomly selected cases, shown for illustration in Fig 5. We are of course happy to present more or even all of the illustrations, but felt that they did not add any value to the paper if they were shown directly in this manuscript.

## Minor comments

9. Lines 42 - 44: "Recently, Peters et al. (2021) combined the mechanistic models of Lebeau and Konrad (2010) and Tokunaga (2009) with the conceptual model of Peters (2013) to obtain a simple prediction scheme for the absolute non-capillary conductivity function  $K_{nc}(h)$ ."

Perhaps, the work of Weber et al. should be mentioned here as well:

Weber, T.K.D., W. Durner, T. Streck, and E. Diamantopoulos (2019): A modular framework for modeling unsaturated soil hydraulic properties over the full moisture range. WRR 55, 4994-5011. doi 10.1029/2018WR024584.

Addendum: I see you mention this paper in l. 130.

Since Weber et al. did not develop an absolute prediction scheme, mentioning it here would not be in the right context.

10. Lines 47- 48: "Today, the capillary bundle model of Mualem (1976a), who refined the assumptions of the CCG model, is most frequently used." Assouline and Or (2013) offer an insightful critique of this and similar models. (Assouline, S., and D. Or (2013): Conceptual and parametric representation of soil hydraulic properties: a review, Vadose Zone J. doi:10.2136/vzj2013.07.0121=

Thanks for this useful hint, we will add the following to the manuscript:

"See Assouline and Or (2013) for an insightful critique of this and similar models."

11. Line 78: Fig. 1 (add a period). Also I think HESS prefers labeling of panels in figures, so you can refer to the panel by its label here.

[Will be done](#)

12. Line 185: "Applying his model to a variety of data, Mualem found empirically that  $\lambda \approx 0.5$ ."

This is one of the issues discussed by Assouline and Or.

[We are aware of this issue \(see Peters et al., 2011\). However, for a complete prediction, we need one certain value. For our calibration data set, the value of 0.5 worked quite well. For the test data set, the full prediction with  \$\lambda = 0.5\$  worked also very well. This might be due to the fact that we use the Mualem model to describe only part of the data, which are dominated by capillary effects. In our view, including a discussion of  \$\lambda\$  would distract from the key point of the paper and thus rather complicate than add value to the paper.](#)

13. Line 196: "...Mualem's integral (occurring in Equation (16) in Equation (12)),..."  
Correct this, please.

[Agreed. We apologize for this negligence and will correct it.](#)

14. Line 202 "We hypothesize that  $\tau_s$  varies only moderately among different textures,..."

Does this not implicitly require that the conductivity of a soil that is so wet that all but the largest pores (whose size and shape are determined almost entirely by the soil macrostructure) are filled does not vary much for different textures? I am not convinced that is the case, but have to admit that my reservations are based more on intuition than hard data. I understand the proof will come later in the paper.

Nevertheless, I would be interested in a more elaborate treatment of the implications of the range of  $\tau_{sub_s}$  for the range of  $K_{sc}$  in Eq. (20).

[See our reply to the major comments 3 and 4.](#)

15. Eq. (20):  $K_{sc} = \beta \tau_s (\theta_s - \theta_r)^2 [F(1) - F(\Gamma_0)]^2$

The presence of a residual water content confuses me a little given the tendency in the past 10 - 15 years or so to get rid of it. As a case in point, you quote Schneider and Goss, who provided a finite matric potential for oven-dryness. Does this not contradict the existence of a non-zero residual water content?

[Please see our reply to the major comment 5.](#)

16. Line 220: "Therefore, we seek in this contribution empirically a value of."

...an empirical value...

[Will be changed.](#)

17. Line 228: "The basic capillary saturation functions are given by the function of Kosugi (1996), the constrained and unconstrained van Genuchten functions (van Genuchten, 1980), and the Fredlund and Xing (1994) saturation function."

Please add some explanation for your choices here. There are many alternatives, some of which distinguish between adsorbed water and capillary-bound water, and did away with the asymptote at the residual water content. Given the argumentation in the Introduction, I expected at least some of those to be included. Perhaps you indeed used the non-asymptotic version of Fredlund and Xing? If I recall correctly, they propose several functions, some of which have an asymptote at zero water content.

[Please see our reply to the major comment 6.](#)

18. Line 229: “the constrained and unconstrained van Genuchten functions (van Genuchten, 1980)”

What does that mean?  $m = 1-1/n$  for the constrained version, and  $n$  and  $m$  independent for the unconstrained version?

[We refer to Tab 2, where it should become clear. Furthermore, we will change the text to “...the van Genuchten functions \(van Genuchten, 1980\) with the usual constraint \( \$m=1-1/n\$ \) and also in unconstrained form \( \$m\$  independent from  \$n\$ \), ...”](#)

19. Line 229: “... and the Fredlund and Xing (1994) saturation function.”

The version with the correction factor that ensures there is no asymptote at zero water content?

[Please see our reply to the major comment 6. Tab 2 shows the functions we use. We use the basic function without the correction, because this is not required in the PDI model framework. We will add here a sentence to clarify this issue:](#)

*“... and the Fredlund and Xing (1994) saturation function. The latter is used as basic function without the correction that forces the function to be zero at  $h_0$ , because this is not required in the PDI model framework.”*

20. Lines 234 – 235: ---"and that closed-form expressions can be derived easily also for “classical” models that use a residual water content and neglect the non-capillary components.” This suggest that none of the functions you chose have a residual water content, but they have, don't they?

[Please see our reply to the major comment 5](#)

21. Line 252: “Mualem’s model (as all capillary bundle models) in combination with water retention models that gradually approach saturation can produce a non-physical sharp decrease in the hydraulic conductivity near saturation (...)”

Madi et al. (2018) developed a mathematical criterion that can be used to test individual WRC models.

Madi, R., G.H. de Rooij, H. Mielenz, and J. Mai (2018): Parametric soil water retention models: a critical evaluation of expressions for the full moisture range. HESS 22, 1193-1219, doi 10.5194/hess-22-1193-2018.

[Thank you. We will add this information in the revised manuscript.](#)

22. Line 262: “Details about the soils are given in the original literature and are summarized in Table 3. For each of the 4 models in Table 2, we determined a value for  $\tau_s$  by fitting them to the 12 data sets and estimating the WRC parameters and  $\tau_s$ .”

Basically, you fit tau-s instead of Ks to fit the data except in the range near saturation. In effect you are still scaling a conductivity curve, you just call the scaling parameter something else. The new element is that you use the fitted value as a predictor for soils where you did not fit it to. These soils each have their own values of the residual and saturated water contents,  $F(1)$ , and  $F(\Gamma-0)$ , from which emerges an individual value of  $K_{sc}$  using Eq. (20).  
Is this a correct description of the procedure?

Please see our reply to the major comments 3 and 7.

23. Table 3

Note that Mualem did not use the original lab-measured data points but instead used smooth curves through these points to derive the points presented in his catalogue.

Thanks for the hint. Not many know this but we are aware of this.

I am unable to find the source paper (Pachepsky et al., 1984). The only link Google dug up leads to another paper.

We are sorry about that and admit that the early original paper is hard to find on the internet. We have a pdf of that publication and upon request are happy to share it.

24. Fig. 5: Is there a red cross here? What does it signify?

Point well taken - the red cross indicates an outlier. Following the default setting in Matlab® (<https://de.mathworks.com/help/matlab/ref/boxchart.html>), data are treated as outlier if they are 1.5 inter quartile range away from the top or bottom of the box. We will explain this in the figure caption in the revised manuscript:

*“The red cross indicates an outlier, defined by the Matlab® default settings as 1.5 times the inter quartile range away from the top or bottom of the box (<https://de.mathworks.com/help/matlab/ref/boxchart.html>).”*

25. Line 353: In such cases,  $\lambda$  might be estimated and only  $\tau_s$  might be fixed.

Assouline (2010) even set it to zero. See Assouline and Or (2013) for a discussion of this and the full reference to Assouline (2010).

Setting it to zero means that the tortuosity is independent of saturation. We think that this is not a preferable choice.

26. Line 370: “Following Jarvis (2007), we may choose for this a suction of about 0.06 m (pore diameter approximately 0.5 mm) up to which the macropore conductivity can be neglected.”

Repeats earlier text (except the publication year).

The repetition here is intentional to make the manuscript more readable. The publication of the paper year is indeed 2007. We apologize for this negligence and will correct it on line 256.