Response to Anonymous Referee #1

Major comments

1. I have the impression the paper does not give enough credit to the body of work in which earlier analytical solutions to the flow problem discussed here were derived. The Laplace-Fourier transformation used here was adopted from Butler Jr. et al. (2001), who in turn adopted it from earlier work done in the former Soviet Union. I had to look this up to find out.

   Response: Thank you for your comment. The Laplace and Fourier transform techniques used in this manuscript are not adopted from the works of Butler Jr. et al. (2001) but from the textbooks of Engineering Mathematics. They are standard methods of applied and engineering mathematics for solution of partial differential equations. We have added a clarifying statement **in the beginning of Subsection 2.2** in the revised manuscript citing an example standard text from which the transforms used in our work to solve flow equations may be found, viz., "To solve the flow problem described above, the governing equation is first transformed into a dimensionless form. Details of the nondimensionalization of the governing equations and their solutions may be found in the Appendix. Laplace and Fourier-cosine transforms are applied to the dimensionless governing equations, which are then solved by standard methods for ordinary differential equations. The respective inversion formulae of the transforms are finally used to numerically obtain the applicable flow solutions in space-time. The transform and inversion formulae can be found in standard textbooks of Engineering Mathematics, and interested readers may refer to the reference text of Haberman (2012). Similar solution approaches have been used in the hydrogeology literature by Butler Jr et al. (2001) and others."

2. The authors should also clarify in the Introduction what this paper contributes to this existing body of work: what is the added value of their solutions? I am not implying this contribution does not exist, but I found it hard to figure out what it is, as I do not have the time to study the older papers in detail myself. It appears the answer is given in the sentence starting in line 463. It would help to move this forward.

   Response: Thank you for this comment. In this response we strive to clarify the knowledge gap addressed in the revised manuscript. We note that the advancement of the work is not in developing a new procedure or approach to solving the same flow problem as in the cited works. We use standard methods of applied mathematics to solve a new flow model. The knowledge gap addressed in our work was identified explicitly in lines 47 – 51 of the original manuscript. To highlight the main contribution our work, the knowledge gap statement is revised as follows: "Given the limitations of the stream depletion models reviewed above, an alternative theory is proposed here where a new boundary condition is imposed at the stream-aquifer interface by invoking the mass-balance principle and introducing the concept of finite stream channel storage. Hence, in this study, two semi-analytical models are developed for the cases of non- or minimally-

penetrating streams (NPS) and fully-penetrating streams (FPS) in a confined aquifer, taking into account the effect of finite stream channel storage and the resulting drawdown of the stream. It is reiterated again here that stream drawdown is distinguished from stream depletion because it defines a decline stream stage whereas the latter only refers to a decrease in stream discharge rate. The model developed herein are the first semi-analytical models in the hydrogeology literature to accomplish this, overcoming the limitation of existing analytical models that assume streams to have fixed-stage. The solutions are validated by comparing them with a numerical model based on the finite-element method (FEM) and with field observations of aquifer and stream drawdown. Finally, the newly developed models are applied to field observations of stream and aquifer drawdown in a parameter estimation exercise by fitting the models to both aquifer and stream drawdown data, which demonstrates their practical application."

3. The paper is awkwardly structured, with section 6 introducing elements that one would expect in a Materials and Methods section that is not there. On at least one occasion this leads to a reference to a flow system that was not yet discussed, which is an indicator of a poorly structured paper.
   Response: We have extensively revised the full content and restructured the manuscript to follow the standard format as follows.

   - Introduction
   - Methods
     - Mathematical formulation
     - Solution
     - Field Experiment
     - Analysis of Stream & Aquifer Drawdown
   - Results
     - Model Predicted Behavior
     - Parameter Estimation & Model Fit to Data
   - Discussion
   - Conclusion

4. The section on the existing aquifer is too large. It is OK to use field data to test the practical use of the solutions, but the detailed description of the geological setting and the legal context distracts from the main message of the paper. So does the parameter fitting exercise, which seems more suitable for an engineer's report for local use.
   Response: We agree that the section may be too large for the aquifer information and that some of the details may distract from the main message of the manuscript. In the revision, we have reduced the section to focus on the key information that is relevant to the main message of the manuscript.

5. The figure captions mostly are inadequate. Figures should be intelligible without having to refer to the main text, but the captions are too short to allow this. In some cases, not even all curves are properly

explained.

Response: Thank for the suggestion. The figure captions are recast to include all information conveyed in the graphs in the revision.

6. I am not convinced the figures always focus on the most interesting aspects of the solutions, and are perhaps more driven by mathematical rather than hydrological considerations.

   Response: The proposed model is newly developed by considering the stream storage reflected by C_r. This is never been proposed in the past study. Therefore, the model verification and the study on this new parameter are required for further analysis like field application. That is the reason these cover most parts of the figures.

7. The sections discussing the figures should be gathered in a section 'Results and Discussion'. The section that is called 'Results' in the current version repeats the discussion in those preceding sections and can then be deleted.

   Response: Thank you. As responded in the previous comment, we reconstructed our manuscript following the format:

   - Introduction
   - Methods
     - Mathematical formulation
     - Solution
     - Field Experiment
     - Data Analysis Methods
   - Results
     - Model Predicted Behavior
     - Parameter Estimation & Model Fit to Data
   - Discussion
   - Conclusion

8. There are minor and major grammar errors that sometimes hamper understanding (see detailed comments for an incomplete listing). I gave up on section 5 because I could not comprehend large parts of the text.

   Response: Thank you for your feedback on our manuscript. We apologize for any grammar errors that may have affected your understanding of our work. We have carefully reviewed your comments and made several changes to improve the readability of the paper. In particular, we have checked and resolved the grammar issue in the manuscript with the help of the primary author, who is a native speaker and highly skilled in scientific writing. We hope that these changes will improve your understanding of our work.

9. The paper does not always adhere to SI units.

   Response: They are now consistent with the SI system. We have carefully checked the units throughout the manuscript to ensure that they are correct and we appreciate your help in pointing out any inconsistencies.

10. If a revision will be submitted, the authors should place the figures in the text, not at the end. In addition, the figures not always adhere to HESS standards.
    Response: In the revision, we placed the figures in the text, as you have suggested. We apologize if the figures do not comply with HESS standards and we will carefully review and update them to meet journal standards.

11. I am not sure about the level of novelty of the paper. If the improvement of earlier analytical solutions of the flow problem of interest is considered marginal, I may have overrated the scientific value of the paper.
    Response: We would like to mention that our solution improves the model by considering pumping-induced transient stream drawdown, which has not been considered in existing models of stream depletion. It is important to note that stream depletion and stream drawdown are distinct phenomena, which can be confusing. This is why we refer to depletion without drawdown as a *depletion model paradox* (note that we have removed the use of paradox in the revision). Our solution addresses this issue by accurately reflecting the stream drawdown behavior observed in the study field. This improvement significantly helps us to better understand the stream depletion problem, improve parameter estimation, and develop sound strategies for subsurface water resources management when the stream capacity is finite.

12. Considerable rewriting is necessary in my opinion, which is why I recommend major revisions.
    Response: We agree that considerable rewriting is necessary. We have revised the paper to address the issues that you have raised. We will make every effort to thoroughly and accurately address all of your concerns in the revision.

13. Minor comments are in the annotated manuscript.
    Response: We thoroughly reviewed the minor comments you provided in the annotated manuscript and made the necessary changes in response to them.

References

Butler Jr, J. J., Zlotnik, V. A., and Tsou, M.-S. (2001). Drawdown and stream depletion produced by pumping in the vicinity of a partially penetrating stream. *Groundwater*, 39(5):651–659.

Haberman, R. (2012). *Applied partial differential equations with Fourier series and boundary value problems*. Pearson Higher Ed.