# Response to Anonymous Referee #2

## Response to General Comments

1. Overall, I find the paper interesting. However, the introduced model has fundamental shortcomings: in particular, it does not include any stream flow routing, which implies that all the points along the stream behave independently of one another. This is obviously unrealistic and can certainly affect the results significantly. At the minimum, the authors should much better discuss and justify the stream storage approach they have adopted (section 2.4). This is critical if they want their work to be considered relevant – so far, I have doubts.

   Response: *Thank you for your feedback on our manuscript. We agree with your observation about the limitations of the model, specifically the absence of stream flow routing. In our revisions, we have added a clearer explanation and justification for our approach to address your concerns about stream flow routing and have listed the limitation in our updated set of simplifying assumptions. Our work is relevant to hydrogeologists who at present rely on fixed-stage analytical models or complex numerical methods as discussed further below.*

2. The main contribution of the paper is not very clear: is it about an analytical solution, or is it about the implications of a poorly recognized phenomenon? In particular, the authors refer to (Zlotnik, 2004) several times, but without explaining what the difference between (Zlotnik, 2004) work and this work is. Moreover, the authors should clarify that numerical models (at least MODFLOW) do allow for simulating stream stage drawdown induced by pumping. Currently, they almost insinuate the contrary (the introduction is quite unclear about it, up to the point of being misleading).

   Response: *In this response comment by the reviewer, we strive below and in the revised manuscript, to clarify the main contribution of our work, which was identified explicitly in lines 47 – 51 of the original manuscript. The contribution is about both a new analytical solution and "the implications of a poorly recognized phenomenon," namely transient stream drawdown and channel storage. Our work differs from that of Zlotnik (2004) in our treatment of the boundary condition at the stream-aquifer interface. Whereas Zlotnik (2004) uses a fixed-stream stage in a leaky aquifer, we allow stream stage to respond to groundwater pumping; whereas Zlotnik (2004), and all models that use the fixed-stage condition, cannot predict transient stream drawdown response to pumping, our model predicts this in addition to stream depletion. The model of Zlotnik (2004) suffers from the same limitation as the other models reviewed in our introduction, namely that they predict stream depletion but not associated stream drawdown response; to reiterate, these models are based on the fixed-stage assumption and as such, stream drawdown is by definition zero. We have developed an analytical solution that overcomes this limitation; we have then apply the solution to a field example. We develop the solution by first modifying the boundary condition at the stream-aquifer interface using a mass balance condition regulated by a finite stream channel storage coefficient. We demonstrate that fixed-stage models coincide with our model in the limit as the stream channel storage coefficient approaches infinity, $C_r \to \infty$. The numerical model is only used in our work to validate the analytical solution. We state however, that MODFLOW also relies on the same fixed-stage condition used by the earlier analytical models. To make our contribution clearer here in the response, we include Figure 1 where we plot the predicted behavior of the old models (a) separate from the new model (b). The model of Fox et al. (2002) is the general representation of all fixed-stage models for confined aquifer flow including those that account for aquifer leakage such as the model of Zlotnik (2004). To highlight the main contribution our work, the introduction is revised as follows:*

   ***"Given the limitations of the stream depletion models reviewed above, an alternative theory is proposed here where a new boundary condition is imposed at the stream-aquifer interface by invoking the mass-balance principle and introducing the concept of finite stream channel storage. Hence, in this study, two semi-analytical models are developed for the cases of non- or minimally-penetrating streams (NPS) and fully-penetrating streams (FPS) in a confined aquifer, taking into account the effect of finite stream channel storage and***

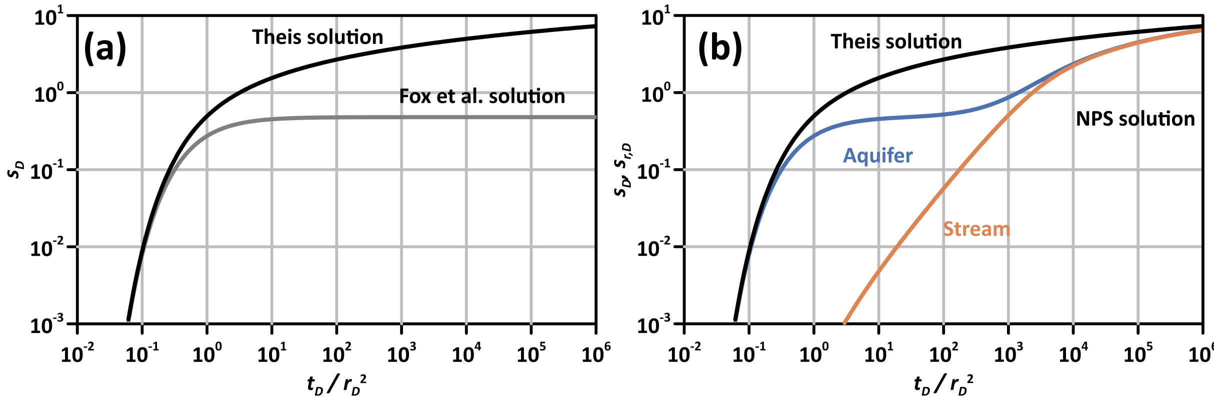


Figure 1: Graphs highlighting key differences between the fixed-stage model Fox et al. (2002) and the developed in our work. Note that the model of Fox et al. (2002), as the case is with all fixed-stage models, predicts steady-state aquifer drawdown at late-time and predicts a stream drawdown response of zero ($s_r = 0$ by definition). Our model predictions differ from Fox et al. (2002) in (a) the nature of aquifer drawdown and (b) transient stream drawdown response.

***the resulting drawdown of the stream. It is reiterated again here that stream drawdown is distinguished from stream depletion because it defines a decline stream stage whereas the latter only refers to a decrease in stream discharge rate. The model developed herein are the first semi-analytical models in the hydrogeology literature to accomplish this, overcoming the limitation of existing analytical models that assume streams to have fixed-stage. The solutions are validated by comparing them with a numerical model based on the finite-element method (FEM) and with field observations of aquifer and stream drawdown. Finally, the newly developed models are applied to field observations of stream and aquifer drawdown in a parameter estimation exercise by fitting the models to both aquifer and stream drawdown data, which demonstrates their practical application."***

3. Regarding the presentation, I find the paper well written and illustrated in general, although several grammar mistakes and typos need to be corrected. On the other hand, many things need to be clarified, calling for a significant amount of work. Please refer to the detailed comments below.

   Response: *Thank you for your detailed comments. We strive to address them all below.*

## Response to Detailed Comments

1. L19-20: Suggest citing the nice, comprehensive USGS report by Barlow and Leake (2012), and perhaps remove some of the less relevant references given here.

   Response: *We appreciate your suggestion and have incorporated it by including the reference in our sentence and reducing the overall number of references in the list. Note that our list of references include more recent papers published after 2012. The revised sentence now reads:*

   ***"Groundwater pumping in basins with or bounded by streams can lead to reduced stream flows, with undesirable impacts on both human use and ecosystem function (Barlow and Leake, 2012; Foglia et al., 2013; Zipper et al., 2018; Tolley et al., 2019; Kwon et al., 2020) due to drying up of streambeds and disconnected stream-groundwater systems."***

2. L36-37: This part is not clear at all. First, I do not think Harbaugh (2005) talks about empirical hydrographs. Second, you should explicitly mention the MODFLOW packages STR1, SFR1 and SFR2, which do allow for simulating stream stage drawdown induced by pumping.

   Response: *In order to improve the clarity of the sentence, we have restructured the related phrases as:*

   ***"Numerical models, such as MODFLOW (Harbaugh, 2005) and MIKE SHE (Refsgaard***

***et al., 2010), through their respective stream packages (e.g., STR1, SFR1, and SFR2), treat stream boundary conditions and source/drainage terms in a similar manner to the analytical models previously discussed. These models also allow for the specification of spatially variable stream stages using observed hydrographs (Harbaugh, 2005) or formulas such as the Manning equation (Prudic et al., 2004), allowing for the effect of stream flow to be considered. However, the approach is highly nonlinear and requires iterative methods at every time step, which can be computationally demanding."***

3. L40-41: Of what source terms are you talking about here?

   Response: *The source term for the pump-induced stream infiltration. To clarify, this sentence has been revised as:*

   ***"To reiterate, when considering source terms for stream infiltration in partially-penetrating scenarios, they are treated as linear functions of the difference in head across the streambed, with a fixed stage."***

4. L42-44: Not true with the STR1, SFR1 and SFR2 packages, as already mentioned.

   Response: *We have revised this paragraphs as:*

   ***"As mentioned already, the models utilize either a constant-head or Robin boundary condition (or source term), or disregard stream storage at the stream-aquifer interface. These models rely on a fixed (or more generally, prescribed) stream stage or rapid infiltration from the stream, with the stream acting as an infinite source of storage that can continuously provide recharge during periods of groundwater pumping."***

5. L45-46: Please explain the maximum SDR concept.

   Response: *We have added the texts "* ***defined as a fraction of the pumping rate supplied by stream depletion" after the "maximum SDR".***

6. L47: Rather than "because the stream flow rate is two orders or more higher than the pumping rate", I think it should be "when the stream flow rate is two orders or more higher than the pumping rate".

   Response: *Thank for the suggestion. In the revised manuscript, the related sentence has been rewritten as:*

   ***"According to Kollet and Zlotnik (2003), stream stage decline due to aquifer pumping can only be ignored in cases when the stream flow rate is two orders of magnitude or more higher than the pumping rate."***

7. L56: This sentence appears unfinished at "Theis (1941)".

   Response: *The associated sentence has been deleted as suggested by another reviewer.*

8. L55-63: This is diverting from the introduction; I would suggest putting it in the Discussion section.

   Response: *As per the suggestion of another reviewer, sentences 55-63 in the Preprint have been removed.*

9. L68: Rather than "test the hypothesis", I guess you mean "make the assumption".

   Response: *We appreciate the suggestion. In our revision, we have substituted "make the assumption" for the original language.*

10. Figure 1b: the piezometric level should be lower than the stream level to have inflow from the stream to the aquifer!

    Response: *We assume in our work that the background flow before pumping initiation is from the aquifer to the stream, contributing to the groundwater recession period of the stream hydrograph. Flow into the aquifer from the stream occurs only when the cone of depression around the well has propagated all the way to the stream's edge. This explains the delayed drawdown response of the stream predicted by the model developed in our work. Note that the classical definition of stream depletion includes groundwater intercepted by the pump preventing its flow to the stream in addition to water captured directly from the stream. This conception of depletion holds even in our case.*

11. L87-88: Where is the origin of the $y$ axis?

    Response: *We appreciate your feedback regarding the missing explanation of the origin of the $y$ axis. We have now included the following statement in our description: "The $y$ axis origin is located at the point where the line perpendicular to the pumping well and the stream intersects the closest stream bank." Additionally, we have updated the diagram (see the figure below) to clearly indicate the location of the axis origins.*

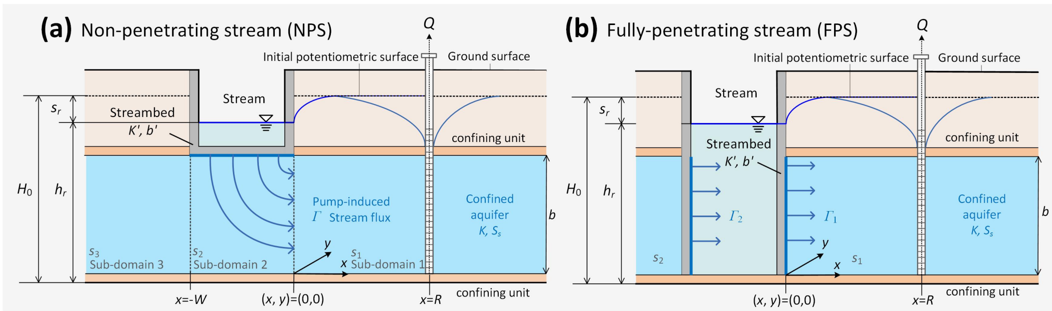


Figure 2: The schematic diagram of the conceptual model of the stream-aquifer system used for the (a) non-penetrating stream (NPS) and (b) fully-penetrating stream (FPS) models derived herein.

12. L95: Are you sure of that division by $\pi$??

    Response: *Yes. Positive.*

13. L99: Stream drawdown was previously noted $H_r(t)$, why is it noted $h_r(t)$ now?

    Response: *This is a typo: $H_r$ should be $h_r$. Note that $h_r(t)$ in the original manuscript is not stream drawdown; it is stream stage. These are two different things. Stream drawdown is $s_r(t)$ and was defined as $s_r(t) = H_0 - h_r(t)$ in the original manuscript. In the revised manuscript, stream stage and stream drawdown are expressed as functions on $y$ and $t$.*

14. L99-100: This would imply that the initial aquifer head is equal to the initial stream stage; isn't it too restrictive?? This is even contradictory with the conceptual model in Figure 1b. PS: I think what you are doing is correct; but the way you write it makes it appear as if it would only work for "flat" initial conditions, whereas the superposition principle allows more than that.

    Response: *Exactly. We rely on the superposition principle here. We subtract out the background trend in aquifer head driving toward the stream so as to work with a flat surface (to pose a homogeneous problem in drawdown, not head) that is mathematically tractable. The resulting solution may then be added to the background aquifer head.*

15. Note: the same goes with the temporal variations of the boundary conditions. When in L79 you write that $s_r(t) = H_0 - H_r(t)$, you are implying that the stream stage cannot have natural variations. But the superposition principle allows for natural variations. What you would need to do is to define drawdown as the change in stream stage due to pumping; for example, $s_r(t) = H_{rn}(t) - H_{rp}(t)$, where $H_{rn}(t)$ would represent the stream stage under natural (i.e., non-pumping) conditions, and $H_{rp}(t)$ the stream stage under pumping conditions.

    Response: *Again, this is exactly correct. Given that the superposition principle allows for natural variations and that such variations may take on non general forms, it is important to reduce the problem to a form that may then be used with the superposition principle for specific applied problems encountered in practice. We cannot conceive of a single general solution that accounts for all such natural variations. Hence, it is important to remove such specific natural fluctuations, and to pose the problem only as a drawdown or deviation from background, non-pumping induced behavior. The response attributable to pumping may then be used with the superposition principle to recover site-specific behaviors. Our focus is simply pumping induced drawdown and depletion.*

16. L101-102: These variables were already defined in L76-77; avoid repetitions.

    Response: *Noted and corrected in revised manuscript.*

17. L103: "is" should be "in".

    Response: *We have corrected it in the revised manuscript.*

18. L113-114: Do you really need those two conditions? In other words, aren't the continuity conditions for drawdown (L110-111) sufficient for the problem to be well-posed?

    Response: *Yes, one does really need both conditions! Otherwise the problem is ill-posed and insoluble. Head continuity conditions alone are not sufficient to achieve well-posedness. The reviewer is welcome to attempt solving the flow problem with only the head continuity conditions. We have outlined all our solution steps transparently in the appendix. Specifically, the general solutions for the three cases, relation to $x_D$, each have six unidentified coefficients. To determine these six coefficients (unknowns), six conditions are required, namely the two infinite-far boundary conditions and four continuity conditions, as outlined in L113-114 of the Preprint.*

19. L133: I think it should be Hantush (1965), not Hantush and Jacob (1955).

    Response: *Yes, thank you. Noted and corrected.*

20. L136: Same comment.

    Response: *Revised and corrected.*

21. L163-164: So, in the end you are assuming a no-flow boundary at $x = -W$ in all cases?? This would not be in line with all the rest of the paper, so is this sentence just a mistake??

    Response: *We consider both cases where there is no flow at $x = -W$ and where the far-side half-space contributes to flow. The former case in realizable in practical cases where the stream flows along a fault line where vertical displacement places an impermeable formation in contact with the stream-aquifer system. The latter occurs where a stream simply erodes the overlying layers above an aquifer as well as the aquifer itself.*

22. L144-146: These equations would imply that $s_r$ is a function of $y$ (and $x$ in the NPS case). Is this correct (previously, you wrote $s_r$ only as a function of $t$, as in $s_r(t)$?

    Response: *Yes, $s_r$ is a function of both $y$ and $t$, and also of $x$ in the NPS case as correctly noted by the reviewer. We have clarified this in the revised manuscript.*

23. Furthermore, this approach implies that the stream stage can vary in each point independently of what happens upstream and downstream of that point. How reasonable is this assumption? If the stream stage decreases in one point, it should also decrease downstream of that point, even if no water if withdrawn further downstream.

    Response: *Thank you for pointing out this. The stream drawdown model suggests that the equation for stream flow is valid in instances where the flow rate is low or the variation in flow rate between two points is insignificant. For cases where such flow is appreciable, one may, in principle, invoke the superposition principle.*

24. L175: "ration" should be "ratio".

    Response: *Thank you. This has been corrected in our revised manuscript.*

25. L198: Why talking in terms of dimensionless time but not in dimensionless space? Be consistent.

    Response: *The variable should be represented as $y_D$, rather than simply $y$.*

26. L221-226: There is a mistake here: $sD = -\gamma/2 - \ln(u)$ should be $sD = -\gamma/2 - ln(u)/2$; and then you should get $A = 1.5\sqrt{\alpha}$. Furthermore, rather than deriving this well-known formula for the radius of influence, you should simply refer to Bear (1979). You could also refer to Bresciani et al. (2020) for justifying your choice of radius of influence formula over other existing formula.

    Response: *Thank you for your thorough review. We have revised the calculation of $A$ from $A = 1.73\sqrt{\alpha}$ to $A = 2\exp(-\gamma/2)\sqrt{\alpha}$, which is approximately equal to $1.5\sqrt{\alpha}$. As a result, the related statements have also been updated:*

    *“ **The late-time drawdown approximation of Cooper Jr and Jacob (1946) can be used to***

***determine the radius of influence, $R_{D,\infty}$, of a pumping well. This is done by considering a cone of depression centered around the well and defining $R_{D,\infty}$ as the point where dimensionless drawdown, $s_D$, is equal to zero. The formula for $s_D$ is $s_D = -(\gamma + \ln(u))/2$, where $u = r_D^2/4\alpha t_D$ and $r_D^2 = (x_D - 1)^2 + y_D^2$. In this equation, $\alpha$ represents the hydraulic diffusivity $K/S_s$, and $\gamma \simeq 0.577216$ is Euler's constant. This results in $R_{D,\infty} = 2\exp(-\gamma/2)\sqrt{\alpha} \approx 1.5\sqrt{\alpha}$ (Bear, 1979). Additional information on well radius of influence for different aquifer and well configurations can be found in Bresciani et al. (2020). In evaluating stream depletion solution, $L_D \approx R_{D,\infty}$ can be set."***

27. L230-250: How is the stream modelled in the numerical model? Does it also neglect downstream transmission of stream stage drawdown, as in the analytical model?

    Response: *Yes, to ensure consistency, the effect of downstream transmission is not considered during verification for testing the accuracy of the drawdown evaluated by our solutions. The stream is modeled in the numerical model exactly as it is modeled in the analytical solution. The former is included only for the purpose of validating the latter.*

28. L254: Space missing before the parenthesis.

    Response: *Thank you. The issue has been resolved.*

29. L253-254: Where are you evaluating the solution?

    Response: *The solution is evaluated using the Mathematica and MATLAB codes mentioned earlier. Links to the codes are supplied in the revised manuscript under the Code and data availability section as required by the journal. We include the links here, viz: Mathematica and MATLAB codes developed for the study are available at the hyperlinks: HydroShare and MATLAB File Exchange. The raw data analyzed in this work are available from the corresponding author upon request.*

30. L255-256: What is the conceptual difference between the models of Hantush (1965) and Fox et al. (2002)?

    Response: *Please see Figure 2 above, which is also included in the manuscript. The model of Hantush (1965) assumes a stream that has fully incised across the entire thickness of the aquifer (fully-penetrating stream, Figure 1b), whereas Fox et al. (2002) considers a stream that has only partially incised the aquifer formation (partially- and non-penetrating stream, Figure 1a).*

31. L281: Indeed, the stream stage response to pumping should also depend on the stream discharge rate!! This is a fundamental aspect of the problem that has been completely ignored in the approach taken in this study.

    Response: *In the Conclusion section, we acknowledged the limitation of not incorporating stream discharge (or velocity) in the model, and we add the sentence to mention it:*

    ***"Additional work is needed to incorporate stream discharge (or velocity) in the model and to conduct longer pumping tests than reported herein in order to better constrain parameter estimates.".***

32. L283: "unpmped" should be "unpumped".

    Response: *The typo has been corrected.*

33. L284: I disagree: does not imply an infinite reserve of the stream.

    Response: *It does, and we demonstrate this mathematically. Our solution reduces to the solutions of Hantush (1965) and Fox et al. (2002) in the limit as $C_{D,r} \to \infty$. Fixed-stage or zero stream drawdown implies the stream has infinite storage. This is a well-known and established fact in hydrogeology that a constant-head boundary condition implies an infinite supply of water, which is why this condition is sometimes called a recharge boundary. The reviewer is of course at liberty to present cogent argument in support of the position they take here.*

34. L318: I think "the case of $C_{D,r}$" should read "the case of $C_{D,r} \to \infty$".

    Response: *Noted and corrected.*

35. L339: Delete "that".

    Response: *Done as suggested.*

36. Figure 11: Show the direction of stream flow.

    Response: *Stream flow is neglected in our model; it is outside the scope of our current work. As noted previously, the principle of superposition may be invoked to account for stream drawdown impacts of stream flow. Here, we are concerned only with stream depletion and drawdown. The reviewer is reminded that the models of Hantush (1965); Fox et al. (2002); Butler Jr et al. (2001, 2007); Zlotnik (2004); Hunt (1999), etc., all neglect stream flow.*

37. Figure 12: What does the 0 represent (on both axes)?

    Response: *The values of 0 on both axes represent the datum used as a reference point for the data plot.*

38. Figure 12: Why do we still see significant trends if the data have been detrended?

    Response: *Figure 12 of the original manuscript shows the raw data relative to the datum; it does not show the detrended and denoised data. Only drawdown data shown later in the manuscript are detrended and denoised. Detrending here means remove of the long-term background recession trend from the drawdown data.*

39. Figure 12: The data show that the stream stage that is furthest to the pumping well (Stenner-P1) is the most affected, which is quite weird. Give possible explanations of this outcome.

    Response: *In the original manuscript, we gave a possible explanation as aquifer heterogeneity (spatial K variability) and anisotropy (directional K variability). Yes, the subsurface is definitely a weird place.*

40. L376: Where are the recovery data

    Response: *In the revised manuscript, the stream drawdown plots have included data from the recovery period (data collected after pump shutdown.), ensuring completeness. (see the figure below). Only drawdown data (pumping-phase) are analyzed with model fitting and parameter estimation because the recovery-phases were disrupted by follow-up pumping at the study site. The figure showing stream drawdown data with recovery has been updated in the revised manuscript as shown here in Figure 3.*

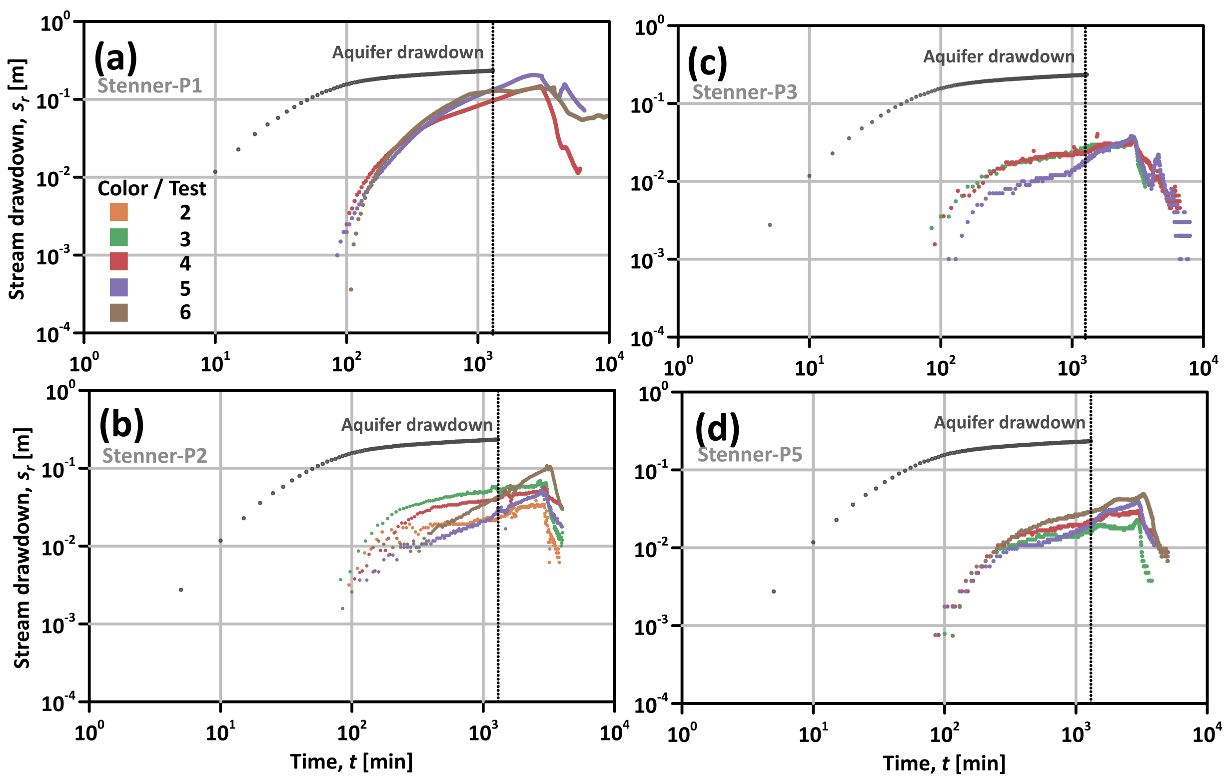


Figure 3: Log-log plots of transient stream stage drawdown response to pumping observed in stream channel stilling wells (a) Stenner-P1, (b) Stenner-P2, (c) Stenner-P3, and (d) Stenner-P5. The data also show stage recovery after pumping (dashed lines for pumping after 48 hrs). Aquifer response (gray dots) is included for comparison.

41. L402: One “is” must be deleted.

    Response: *Done as suggested.*

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