



1    **Numerical modeling of physical and biochemical processes in the**  
2    **subsurface and their impacts on the self-potential signature**

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## 1 Abstract

2 Subsurface contamination is a significant problem due to excessive fertigation and industrial  
3 and domestic wastewater discharge. With numerical modeling and geophysical tool development,  
4 subsurface contaminant research has become easier to implement and study. However, there is still  
5 a gap in coupling the biochemical processes and geophysical signals. Such a coupling model is  
6 needed to facilitate understanding subsurface processes and provide further theoretical basis to  
7 practice and field monitoring. Thus, this research aims to simulate the self-potential (SP) signature  
8 in response to physical and biochemical dynamics in the subsurface. For the physico-bio-chemical  
9 model, the processes of water flow, solute transport, biochemical reactions, microbial dynamics,  
10 adsorption, and gas flow are considered. Specifically, the biochemical cycles related to C, N, Mn,  
11 Fe, and S are incorporated in the model. The physico-bio-chemical model is then coupled with the  
12 SP model. The SP model is addressed by Poisson's continuity equation, based on streaming and  
13 redox potential contribution. The streaming potential is calculated by the effective excess charge  
14 density and the water flow velocity, while the Butler-Volmer equation solves the redox potential.  
15 The results show that redox processes dominate the SP signals. Oxygen and nitrate concentrations  
16 present positive relationships with redox potential and dominate the redox potential in the oxic and  
17 anoxic environment, respectively. Nitrification and dissolved organic carbon (DOC) aerobic  
18 oxidation rates show positive relationships with redox potential. In contrast, the denitrification rate  
19 presents a negative relationship. The higher reaction rates for different redox processes also  
20 correspond to their optimal redox potential ranges. The streaming potential affected by water  
21 content and flux contributes little to SP, and the negative values along with soil depth become less  
22 remarkable. Generally, the SP and redox potential model can better reflect redox species  
23 concentrations and reaction rates, while the streaming potential model can reflect the water content  
24 and flux dynamics. Thus, the research can guide the detection of redox-sensitive contamination  
25 and water leakage in the subsurface.

## 26 1 Introduction

27 Subsurface contamination caused by anthropogenic activities is a widespread problem. It can  
28 be attributed to various reasons, such as pesticide, herbicide and fertilizer applications, industrial  
29 and domestic wastewater discharge, etc. To study the physical and biochemical dynamics in the  
30 subsurface, experimental measurement and numerical modeling on contaminant fate and transport



are usually implemented. Moreover, with the advantage of being minimally invasive and of relatively low cost, geophysical methods have also become an emerging set of tools for subsurface monitoring, such as direct current electrical conductivity, self-potential (SP), and induced polarization.

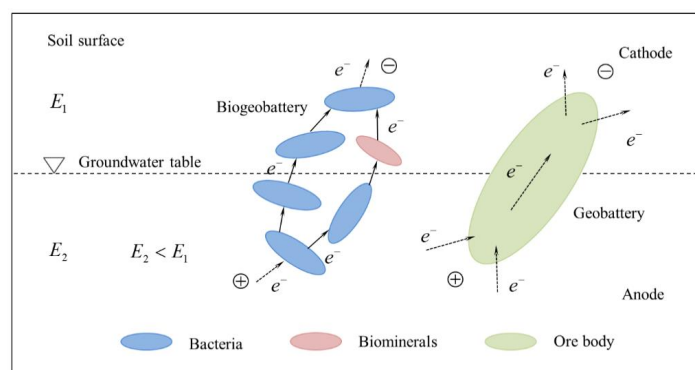
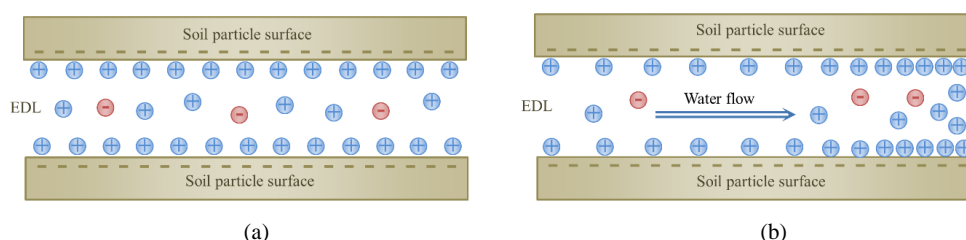
Self-potential (SP) is a passive geophysical method to measure the natural electrical potential distribution of the earth. The SP survey does not require the application of an external electric current. The potential differences are generated by the small, naturally induced currents that occur in the subsurface, recorded by a set of non-polarizing electrodes, typically at the earth's surface (Revil et al., 2009; Soueid Ahmed et al., 2020). Such naturally induced currents are attributed to electrical charge uneven distributions or electron transfer resulting from water flux or biochemical redox processes (Linde and Revil, 2007). Moreover, the signature is affected by the subsurface hydraulic and electrical properties (Hu et al., 2020). Thus, when combined with hydrological and biogeochemical processes, the SP technique can be specifically used to localize and quantify water flow (e.g., pumping test and water leakage) (Oliveti and Cardarelli, 2017) and redox-sensitive contaminant distribution (e.g., metallic and organic matter) (Revil et al., 2009; Forté and Bentley, 2013) and estimate soil hydraulic and electrical properties (e.g., hydraulic and electrical conductivity) (Jouniaux et al., 2020).

SP signature induced by water flow is defined as streaming potential. It occurs when a hydraulic potential gradient drives an electrolyte through a channel or pore with charged walls (Lyklema, 1995; Li, 2004) (Fig. 1 (b)). Along the water flux direction, electrical charges move downstream driven by the hydraulic potential gradient (i.e., carried by water flow), generating convection current in the porous media (Möckel et al., 1998). At narrow bottlenecks, where the electrical double layers of two grains overlap, charges may also accumulate. As the head gradient (pressure in horizontal systems) is the source driving force of streaming potential, streaming current density was previously formulated by the hydraulic potential gradient multiplying by a streaming current coupling coefficient (Sill, 1983). This traditional formulation has been proven to be useful and was applied to numerous hydrological investigations (Santos et al., 2002; Allègre et al., 2012; DesRoches and Butler, 2016). However, the direct process that derives the streaming currents is the water flow. Hence, the classic formulation, that does not consider the permeability, makes it difficult to extend to unsaturated conditions (Bolève et al., 2007). As a more intuitive phenomenon of streaming potential generation results from uneven charge distribution, driven by water flow,



1 later, a formulation that directly links streaming current density to water flow velocity and effective  
 2 excess charge density was developed (Bolève et al., 2007). This formulation is suitable to model  
 3 streaming potential in the field, as been widely applied to hydrological studies. Examples include  
 4 pumping tests (Soueid Ahmed et al., 2016), seepages from ditches or into sinkholes (Bolève et al.,  
 5 2007), water leakage in dams (Bolève et al., 2011; Soueid Ahmed et al., 2020), or contaminated  
 6 site affected by hydrological dynamics (Abbas et al., 2017). Moreover, the formulation was also  
 7 extended to the inertial laminar flow regime and unsaturated area with excellent agreement  
 8 between the theory and experimental data (Jardani et al., 2007; Mboh et al., 2012; Soueid Ahmed  
 9 et al., 2019), making it more applicable to vadose zone research (Soldi et al., 2020).

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14 Figure 1 Schematic of streaming and redox potential induced by physical and biochemical processes.  
 15 Panes (a) and (b) illustrate electrical charge distributions in a static state and induced by water flow, respectively. Pane  
 16 (c) illustrates electron transfer caused by redox processes (modified based on Jouniaux et al.(2009) and Revil et al.  
 17 (2010)). EDL is the abbreviation for electrical double layer.

18 SP associated with redox processes is induced by electron transfer, where electron donors  
 19 (e.g., organic carbon) deliver electrons to acceptors (e.g., oxygen or nitrate) driven by redox  
 20 potential (Jouniaux et al., 2009). In addition, the electrons can transfer through the bacterial



1 extracellular appendages (also known as nanowires) as well as biominerals formed by microbial  
 2 activities (Arora et al., 2007; Jouniaux et al., 2009). This process, where electrons transfer from  
 3 anode to cathode (Fig. 1(c)) is similar to that of a ‘geobattery’, in which electrons transfer through  
 4 the ore body (Revil et al., 2010; Fernandez et al., 2019). In such cases, the SP anomaly associated  
 5 with redox processes is induced in the surrounding conductive media (Castermant et al., 2008).  
 6 Thus, the redox potential and SP associated with redox processes can reflect biochemical  
 7 degradation. These are increasingly applied to experimental and field studies, such as estimating  
 8 carbon, nitrogen, and phosphorus removal by redox potential data (Yu et al., 1997) and detecting  
 9 ionic and organic contaminant plumes by SP technique (Revil et al., 2009; Forté and Bentley, 2013;  
 10 Cui et al., 2017; Fernandez et al., 2019). With software development, numerical modeling becomes  
 11 a more available and cost-effective method to study SP contributed by redox potential. The Nernst  
 12 equation is a widely used method to calculate the potential of redox couples (e.g.,  $\text{H}_2\text{O}/\text{O}_2$ ,  
 13  $\text{CH}_4/\text{CO}_2$ ,  $\text{NH}_4^+/\text{NO}_3^-$ ,  $\text{NO}_2^-/\text{NO}_3^-$ ,  $\text{Fe}^{2+}/\text{Fe}(\text{OH})_3$ ,  $\text{H}_2\text{S}/\text{SO}_4^{2-}$ ,  $\text{As}(\text{III})/\text{As}(\text{V})$ ) in  
 14 thermodynamic equilibrium reactions (Schüring et al., 2000) However, Meyer et al. (2014) showed  
 15 that the Nernst equation is unsuitable for all redox couples as numerous biochemical processes are  
 16 characterized by a kinetic state in a natural environment (rather than equilibrium). The Butler-  
 17 Volmer equation (Butler, 1924a, 1924b, 1932; Erdey-Grúz and Volmer, 1930) was adopted to  
 18 address this issue. The Butler-Volmer equation describes how the electrical currents pass through  
 19 an electrode in a redox kinetic state and generate a voltage difference in the bulk electrolyte  
 20 (Dickinson and Wain, 2020). This can better reflect the process of redox potential measurement  
 21 (Schüring et al., 2000) and is more suitable for redox potential modeling. Early research mainly  
 22 focused on parameter verification and sensitivity analyses (Tanaka and Tamamushi, 1964; Peiffer  
 23 et al., 1992). In recent studies, the Butler-Volmer equation was usually combined with a model of  
 24 the relevant biochemical process, which was used to analyze the relationship between substrate  
 25 concentration and redox potential (Picioreanu et al., 2007; Hamelers et al., 2011; Zhao et al., 2016;  
 26 Jin et al., 2020). However, there is a gap in using the Butler-Volmer concept to calculate redox  
 27 potential for complex kinetic reactions and analyze the signature in response to biochemical  
 28 dynamics on temporal and spatial scales.

29 Based on the very limited research of simulating the SP signature and its forming processes,  
 30 the objectives of this study are (1) to set up a physico-bio-chemical model that would couple the  
 31 processes of water flow, solute transport, gas flow, biochemical reactions, microbial dynamics,



1 and adsorption in the soil system; (2) to establish an SP model by considering the contribution of  
 2 streaming potential (related to water flux and effective excess charge density) and redox potential  
 3 (solved by Butler-Volmer Equation); (3) to couple the physico-bio-chemical and geophysical  
 4 models, and to calibrate and verify the models using experimental data; (4) to apply the models to  
 5 90cm vertical soil column in which the capillary fringe underneath a soil aquifer treatment (SAT)  
 6 site is modeled, and analyzed SP signals in response to subsurface water flow and biodegradation  
 7 processes ( mainly focusing on carbon and nitrogen transformation ).

## 8 **2 Model development**

9 First, the model's soil physical and biochemical components (i.e., water flow, solute transport,  
 10 gas flow, biochemical reactions, microbial dynamics, and equilibrium adsorption) are set up. Then,  
 11 the SP model is established based on Poisson's continuity equation. Finally, the physico-bio-  
 12 chemical and geophysical models are coupled by relating the physical and biochemical properties  
 13 in the soil water system.

### 14 **2.1 Physical and biochemical processes**

15 In the physio-bio-chemical model, the Richards equation describes the water flow. The soil  
 16 hydraulic properties are solved by the soil-hydraulic functions of van Genuchten-Mualem (van  
 17 Genuchten, 1980). Multi-solute transport is simulated by the advection-dispersion equation,  
 18 involving biochemical reactions, adsorption, and gas transfer as the sink/source terms.  
 19 Biochemical reactions are described by multiple-Monod, second-order, and chemical equilibrium  
 20 reactions. The adsorption is solved by the equilibrium between the concentrations in the aqueous  
 21 phase and the soil surface. Finally, the advection-diffusion equation describes the gas flow,  
 22 considering Henry's equilibrium law to address the transfer between aqueous and gas phases.

#### 23 **2.1.1 Water flow**

24 The water flow is solved by the Richards equation, describing one-dimensional water  
 25 movement in partially saturated porous media.

$$26 \quad \frac{\partial \theta_w(h)}{\partial t} = \frac{\partial}{\partial z} \left[ K(h) \left( \frac{\partial h}{\partial z} + 1 \right) \right] \quad (1)$$



where  $\theta_w$  is the volumetric water content (-),  $h$  is the matric head (m),  $t$  is the time (s),  $z$  is the spatial coordinate (positive upward) (m), and  $K$  is the unsaturated hydraulic conductivity ( $\text{m s}^{-1}$ ).

The unsaturated soil hydraulic properties, i.e., soil water content and unsaturated hydraulic conductivity, are in nonlinear functions of the pressure head. The model implements the soil hydraulic functions of van Genuchten (1980), who used the statistical pore size distribution model of Mualem (1976) to describe the unsaturated hydraulic conductivity function in terms of soil water retention parameters,

$$\theta_w(h) = \begin{cases} \theta_r + \frac{\theta_s - \theta_r}{\left[1 + (\alpha h)^n\right]^m} & h < 0 \\ \theta_s & h \geq 0 \end{cases} \quad m = 1 - 1/n, n > 1 \quad (2)$$

$$K(h) = K_s S_e^l \left[1 - \left(1 - S_e^{1/m}\right)^m\right]^2 \quad S_e = \frac{\theta_w - \theta_r}{\theta_s - \theta_r} \quad (3)$$

where  $\theta_r$  is the residual water content (-),  $\theta_s$  is the saturated water content (-),  $\alpha, m, n$  are empirical parameters related to pore and grain size distribution, affecting the shape of hydraulic functions ( $\text{m}^{-1}$ ), (-), (-),  $K_s$  is the saturated hydraulic conductivity ( $\text{m s}^{-1}$ ),  $S_e$  is the effective saturation (-), and  $l$  is the pore connectivity parameter, often taken to be 0.5 (-).

## 2.1.2 Solute transport

The solute transport equation is derived from the advection-dispersion equation, considering the sink/source terms.

$$\frac{\partial \theta_w C_i}{\partial t} = \frac{\partial}{\partial z} \left( \theta_w D_i^w \frac{\partial C_i}{\partial z} \right) - \frac{\partial q_w C_i}{\partial z} + \theta_w s_{c,i} \quad (4)$$

where  $C_i$  is the concentration of species  $i$  in the aqueous phase ( $\text{mol L}^{-1}$ ),  $D_i^w$  is the dispersion coefficient of species  $i$  in the aqueous phase ( $\text{m}^2 \text{s}^{-1}$ ),  $q_w$  is the volumetric flux density ( $\text{m s}^{-1}$ ), and  $s_{c,i}$  is the sink/source term of species  $i$  in the aqueous phase ( $\text{mol L}^{-1} \text{s}^{-1}$ ).

The dispersion coefficient in the aqueous phase,  $D_i^w$  is given by (Bear, 1972), with the tortuosity factors  $\tau_w$  and  $\tau_g$  for aqueous and gas phases (in Section 2.1.6) are the functions of the water and air contents and are described by the relationship of Millington and Quirk (1961),



$$\theta_w D_i^w = D_L |q_w| + \theta_w D_{i,w} \tau_w \quad \tau_w = \frac{\theta_w^{7/3}}{\theta_s^2} \quad (5)$$

where  $D_L$  is the longitudinal dispersivity (m),  $D_{i,w}$  is the molecular diffusion coefficient of species  $i$  in free water ( $\text{m}^2 \text{s}^{-1}$ ), and  $\tau_w$  is the tortuosity factor in the aqueous phase (-).

The sink/source terms, involving biochemical reactions, adsorption (positive sign means desorption), and gas transfer, are presented as follows. The theory of each process will be further discussed in Sections 2.1.3, 2.1.5, and 2.1.6, respectively.

$$\theta_w s_{c,i} = \theta_w r_i + S_{ce,i} - S_i + \Gamma_i \quad (6)$$

where  $r_i$  is the total kinetic reaction rates that species  $i$  is involved ( $\text{mol L}^{-1} \text{s}^{-1}$ ),  $S_{ce,i}$  is the change of species  $i$  mass flux caused by chemical equilibrium reactions ( $\text{mol dm}^{-3} \text{s}^{-1}$ ), such as complexation/hydrolysis and precipitation/dissolution,  $S_i$  is the adsorption change of species  $i$  attributed to the equilibrium with soil surface site density ( $\text{mol dm}^{-3} \text{s}^{-1}$ ), and  $\Gamma_i$  is the gas transfer of species  $i$  addressed by Henry's equilibrium law ( $\text{mol dm}^{-3} \text{s}^{-1}$ ).

### 2.1.3 Biochemical reactions

#### (1) Multiple-Monod kinetics

In multiple-Monod kinetics, it is assumed that the concentrations of all species involved in the reactions affect the biochemical processes (Molz et al., 1986). The multiple-Monod kinetics for the biochemical process  $p$  can be expressed as (Monod, 1949)

$$r^p = \mu_{\max}^p f_{\theta_w} X_m I_b(X_m) I_{nc}^p(C_i) \prod_i \left[ \frac{C_i}{K_i^p + C_i} \right] \quad (7)$$

where  $r^p$  is the kinetic reaction rate for process  $p$  ( $\text{mol L}^{-1} \text{s}^{-1}$ ),  $\mu_{\max}^p$  is the maximum kinetic reaction rate for process  $p$  ( $\text{mol C or N / g cell s}^{-1}$ ),  $f_{\theta_w}$  is the reciprocal of  $\theta_w$  (-),  $X_m$  is the biomass of bacteria  $m$  responsible for the reaction ( $\text{g dm}^{-3}$ ), and  $K_i^p$  is the half-saturation constant of species  $i$  for reaction process  $p$  ( $\text{mol L}^{-1}$ ).

In this research, the microbes are assumed to be retained in the soil, bacterial concentration (in the aqueous phase) is acquired by dividing the biomass by water content.  $I_b(X_m)$  and  $I_{nc}^p(C_i)$  are referred to as the biomass and non-competitive inhibition terms, expressed by follows (Segel, 1975; Kindred and Celia, 1989).





$$I_b(X_m) = \left[ \frac{k_{b,m}}{k_{b,m} + f_{\theta_w} X_m} \right] \quad (8)$$

$$I_{nc}^p(C_i) = \left[ \frac{k_{I,C_i}^p}{k_{I,C_i}^p + C_i} \right] \quad (9)$$

where  $k_{b,m}$  is the biomass inhibition constant ( $\text{g L}^{-1}$ ), and  $k_{I,C_i}^p$  is the substrate inhibition constant for reaction process  $p$  ( $\text{mol L}^{-1}$ ).

## (2) Second-order reactions

In the natural environment, some chemical reactions related to soil substrate concentrations may occur spontaneously and are contributed less by microbial communities. In such a case, the second-order reaction is a more reasonable and easier method to solve the reaction instead of Monod kinetics. The second-order reactions in the model are considered to be proportional to the product of two reactant concentrations. The reaction rate for process  $p$  can be expressed as

$$r^p = k^p [C_1][C_2] \quad (10)$$

where  $k^p$  is the second-order reaction rate constant for process  $p$  ( $\text{mol}^{-1} \text{L s}^{-1}$ ).

The total kinetic reaction rate for species  $i$  is equal to the sum of the rates for all processes in which the species  $i$  is involved, hence

$$r_i = \sum_p y_i^p r^p \quad (11)$$

where  $y_i^p$  is the specific coefficient of species  $i$  for reaction process  $p$  (-). When the species is the primary substrate,  $y_i^p$  is equal to 1. Otherwise, it is the ratio of species  $i$  relative to the primary substrate for reaction process  $p$ , as determined from stoichiometry.

## (3) Chemical equilibrium reactions

Chemical equilibrium reactions, such as aqueous complexation/hydrolysis and precipitation/dissolution, widely exist and affect biochemical processes in soil water systems. The equilibrium constants can be shown as

$$K_i^h = \frac{\prod_j C_j^{\gamma_j}}{C_i} \quad (12)$$



$$K_i^{SP} = \prod_j C_j^{\gamma_j} \quad (13)$$

where  $K_i^h$  and  $K_i^{SP}$  are the equilibrium constants for species  $i$  complexation/hydrolysis and precipitation/dissolution, respectively, also known as hydrolysis equilibrium and solubility product constants,  $C_j$  is the concentration of specific  $j$  related to the chemical equilibrium reaction ( $\text{mol L}^{-1}$ ), and  $\gamma_j$  is the stoichiometric coefficient of species  $j$  (-), a positive value means species  $j$  is on the opposite side to species  $i$  of the chemical equilibrium equation, while a negative value refers to on the same side.

#### 2.1.4 Microbial dynamics

Microbial growth is assumed to be limited by Monod kinetics which the corresponding microbial community is involved in the reaction. The microbial dynamics is controlled by the microbial growth and death or maintenance rates, which is shown as

$$\frac{dX_m}{dt} = Y_m \theta_w r^p - X_m d_m \quad (14)$$

where  $Y_m$  is the microbial yield coefficient for bacteria  $m$  when mediating reaction process  $p$  (g cells / mol C or N), and  $d_m$  is the microbial death or maintenance rate constant for bacteria  $m$  ( $\text{s}^{-1}$ ).

#### 2.1.5 Adsorption

Equilibrium is considered between the concentrations in the aqueous phase and soil surface. The equilibrium constant is shown as

$$K_i^S = \frac{C_i X_\gamma}{C_i \cdot X^\gamma} \quad (15)$$

where  $K_i^S$  is the surface equilibrium constant for species  $i$  adsorption,  $X^-$  is the soil surface site density ( $\text{mol L}^{-1}$ ),  $\gamma$  is the stoichiometric coefficient of soil surface site (-),  $C_i X_\gamma$  is the surface complex concentration of species  $i$  ( $\text{mol L}^{-1}$ ).

#### 2.1.6 Gas flow

The advection-diffusion equation describes the gas flow, considering gas transfer (sink/source term of solute transport) between gas and aqueous phases.



$$\frac{\partial \theta_g G_i}{\partial t} = \frac{\partial}{\partial z} \left( \theta_g D_i^g \frac{\partial G_i}{\partial z} \right) - \frac{\partial q_g G_i}{\partial z} - \Gamma_i \quad (16)$$

$$\theta_g D_i^g = \theta_g D_{i,g} \tau_g \quad \tau_g = \frac{\theta_g^{7/3}}{\theta_s^2} \quad (17)$$

where  $G_i$  is the concentration of species  $i$  in the gas phase ( $\text{mol cm}^{-3}$ ),  $\theta_g$  is the volumetric gas content (-),  $D_i^g$  is the diffusion coefficient of species  $i$  in the gas phase ( $\text{m}^2 \text{s}^{-1}$ ),  $q_g$  is the volumetric gas flux density ( $\text{m s}^{-1}$ ),  $D_{i,g}$  is the molecular diffusion coefficient of species  $i$  in the gas phase ( $\text{m}^2 \text{s}^{-1}$ ), and  $\tau_g$  is the tortuosity factor in the gas phase (-).

Henry's equilibrium law solves the gas transfer between the gas and aqueous phases. The equilibrium constant is shown as

$$K_i^H = \frac{C_i}{P_i} \quad (18)$$

where  $K_i^H$  are Henry's law constants of species  $i$  ( $\text{mol L}^{-1} \text{atm}^{-1}$ ), and  $P_i$  is the partial pressure of species  $i$  in the gas phase (atm).

## 2.2 Self-potential

The SP distribution is attained by the solution to the following Poisson's equation.

$$\nabla \cdot (\sigma \nabla \varphi) = \nabla \cdot j_s \quad (19)$$

where  $\sigma$  is the electrical conductivity of porous media ( $\text{S m}^{-1}$ ),  $\varphi$  is the self-potential (V), and  $j_s$  is the source current density ( $\text{A m}^{-2}$ ).

In sandy soil, the surface conductivity can be ignored as it is much lower than the electrolyte conductivity (Duy Thanh et al., 2019). Based on Archie's law (Archie, 1942), the formulation of porous media conductivity is linked to the pore water conductivity, presented as

$$\sigma = \sigma_w \phi^m \quad (20)$$

$$\sigma_w = C_f N_A e \sum_{i=1}^N |z_i| \beta_i C_i \quad (21)$$

where  $\sigma$  is the pore water conductivity ( $\text{S m}^{-1}$ ),  $\phi$  is the connected porosity (-),  $m$  is the cementation exponent ( $m = 1.645$ ) (Revil et al., 2012),  $e$  is the elementary charge ( $e = 1.6 \times 10^{-19} \text{C}$ ),  $z_i$  is the valence of species  $i$ ,  $\beta_i$  is the mobility of species  $i$  ( $\text{m}^2 \text{s}^{-1} \text{V}^{-1}$ ),  $N_A$  is



1 Avogadro's number ( $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ ),  $C_f$  is the constant ( $C_f = 10^3$ ), and  $C_f N_A$  is the unit  
 2 conversion factor from  $\text{mol L}^{-1}$  to number  $\text{m}^{-3}$ .

3 When SP signals are induced by water flow and redox processes, occurring in a variably  
 4 saturated environment, Eq. 19 can be written as

$$5 \quad \nabla \cdot (\sigma s_w^n \nabla \phi) = \nabla \cdot (j_{sc} + j_{rc}) \quad (22)$$

6 where  $s_w$  is the water saturation (-),  $n$  is the saturation exponent ( $n = 1.98$ ) (Revil, 2013),  $j_{sc}$  is  
 7 the streaming current density ( $\text{A m}^{-2}$ ), and  $j_{rc}$  is the current density associated with redox  
 8 processes ( $\text{A m}^{-2}$ ).

9 The SP contributed by streaming and redox potential separately can be described as Eqs. 23  
 10 and 24, induced by water flow and redox processes, respectively.

$$11 \quad \nabla \cdot (\sigma s_w^n \nabla \phi_s) = \nabla \cdot j_{sc} \quad (23)$$

$$12 \quad \nabla \cdot (\sigma s_w^n \nabla \phi_r) = \nabla \cdot j_{rc} \quad (24)$$

13 where  $\phi_s$  is the streaming potential (V), and  $\phi_r$  is the SP associated with redox processes (V).

14 In the following, the formulations of streaming current density and current density associated  
 15 with redox processes will be introduced explicitly in Sections 2.2.1 and 2.2.2. Then, SP, streaming  
 16 potential, and SP associated with redox processes can be calculated based on Eqs. 22, 23, and 24,  
 17 respectively.

## 18 **2.2.1 Streaming potential**

19 The electrolyte flow through porous media induces the streaming potential. The streaming  
 20 current density equals the effective excess charge density per unit pore volume multiplying the  
 21 water flow velocity. Considering saturation, the equation can be expressed as

$$22 \quad j_{sc} = \frac{Q_v v}{s_w} \quad (25)$$

23 where  $Q_v$  is the effective excess charge density per unit pore volume dragged by water flow ( $\text{C m}^{-3}$ ), and  $v$  is the water flow velocity ( $\text{m s}^{-1}$ ).

25 Based on the observation by Jardani et al. (2007), the effective excess charge density is mainly  
 26 related to the porous material permeability and the formulation is written as

$$27 \quad Q_v = -9.2 - 0.82 \log_{10} \kappa \quad (26)$$



1 where  $\kappa$  is the permeability at saturation ( $\text{m}^2$ ), and the relationship between permeability and  
 2 saturated hydraulic conductivity is expressed as

$$3 \quad \kappa = \frac{\eta_w K_s}{\rho_w g} \quad (27)$$

4 where  $\eta_w$  is the dynamic viscosity (  $\eta_w = 9 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$  ),  $\rho_w$  is the water density  
 5 (  $\rho_w = 10^3 \text{ kg m}^{-3}$  ), and  $g$  is the standard gravity (  $g = 9.8 \text{ m s}^{-2}$  )

6 The water velocity is related to the volumetric flux through the water content,  $v = \frac{q_w}{\theta_w}$ , which  
 7 can be simulated by coupling with the water flow model (Eqs. 1-3).

## 8 **2.2.2 Redox potential**

9 The current density contributed by redox processes is associated with the redox potential  
 10 gradient, formulated as

$$11 \quad j_{rc} = -\sigma s^n \nabla E \quad (28)$$

12 where  $E$  is the redox potential (V).

13 To calculate the redox potential, we assume the redox reaction for a single redox couple  $j$  is  
 14 presented as



16 where  $n_j$  is the number of electron transfer per molecule oxidized or reduced,  $\text{Ox}_j$  is the oxidized  
 17 species (electron acceptors), and  $\text{Red}_j$  is the reduced species (electron donors).

18 For the non-equilibrium multi-ion electrolyte, the net current flows through the measuring  
 19 circuit equal to the sum of individual current induced by various redox couples. Considering Ohm's  
 20 law, the measured potential at the electrode interface can be calculated as

$$21 \quad E = R_l \sum_{j=1}^m i_{\text{net}_j} \quad (30)$$

22 where  $R_l$  is the potentiometer input resistance ( $\Omega$ ), and  $i_{\text{net}_j}$  is the individual current induced by  
 23 redox couple  $j$  (A).

24 The individual current of each redox couple can be calculated by the Butler-Volmer equation,  
 25 which is expressed as the currents from both oxidation (positive sign) and reduction (negative sign)  
 26 reactions.



$$i_{\text{net}_j} = i_{0_j} \left\{ \exp \left[ -\alpha_j \frac{n_j F}{RT} (E - E_{\text{eq}_j}) \right] - \exp \left[ (1 - \alpha_j) \frac{n_j F}{RT} (E - E_{\text{eq}_j}) \right] \right\} \quad (31)$$

In some theoretical researches,  $n_j$  is ignored (Shinagawa et al., 2015), and Eq. 31 is written as

$$i_{\text{net}_j} = i_{0_j} \left\{ \exp \left[ -\frac{\alpha_j F}{RT} (E - E_{\text{eq}_j}) \right] - \exp \left[ \frac{(1 - \alpha_j) F}{RT} (E - E_{\text{eq}_j}) \right] \right\} \quad (32)$$

where  $i_{0_j}$  is the exchange current of redox couple  $j$  (A),  $\alpha_j$  is the transfer coefficient of redox couple  $j$  (-), also known as cathodic transfer coefficient,  $F$  is the Faraday constant ( $F = 96,484 \text{ C mol}^{-1}$ ),  $R$  is the gas constant ( $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$ ),  $T$  is the temperature ( $T = 298.15 \text{ K}$ ), and  $E_{\text{eq}_j}$  is the equilibrium potential of redox couple  $j$  (V). It can be calculated based on the Nernst equation (Eq. S.6) in Section S.1 (Supplement).

The exchange current represents the balanced Faradaic activity at equilibrium condition. The specific derivation and explanation for exchange current are presented in Section S.1 (Supplement).

$$i_{0_j} = k_{0_j}^{\text{el}} C_{\text{Red}_j}^{\alpha} C_{\text{Ox}_j}^{1-\alpha} n_j F A \quad (33)$$

where  $k_{0_j}^{\text{el}}$  is the standard rate constant of redox couple  $j$  ( $\text{m s}^{-1}$ ),  $C_{\text{Red}_j}$  and  $C_{\text{Ox}_j}$  are concentrations of reduced and oxidized species ( $\text{mol L}^{-1}$ ), and  $A$  is the electrode surface area ( $\text{m}^2$ ).

### 3 Case study

The case study presented here is inspired by an SAT facility, the Shafdan site, Israel, where dynamic water fluxes reach the capillary fringe. The Shafdan's SAT includes six infiltration basins, where each basin is around  $2 \times 10^5 \text{ m}^2$  and divided into 9-14 sub-basins (ponds) for alternating flooding (Goren et al., 2014). The SAT basin for treated wastewater (TWW) degradation mainly consists of sandy soil (Goren et al., 2014). The TWW is intermittently discharged into the SAT infiltration basins, creating an alternating oxic and anoxic environment to facilitate contaminant degradation in the subsurface. We assume the capillary fringe suffers an intermittent discharge cycle of 2 flooding days and 4 drying days with a water flux of  $9 \text{ cm d}^{-2}$  (Fig. S1(a), Supplement). The chemical species types and concentrations in the TWW are presented in Table S.1 (Supplement).



The simulation is conducted on a 90cm vertical capillary fringe with 1cm numerical elements, where the groundwater table fluctuates across the vertical soil resulting from TWW discharge. The TWW discharge scenario, including water flux, species concentrations, and gas flow, are set as the upper boundary conditions, while the vertical drainage across the lower boundary of the soil profile depends on the position of groundwater level (Hopmans and Stricker, 1989). The biochemical simulation keeps running with intermittent TWW discharge (i.e., 2 flooding days and 4 drying days) until the subsurface environment becomes pseudo-stable. Then, the physio-bio-chemical dynamics are coupled with the geophysical model and the lower boundary is selected as the reference point for geophysical simulation.

The coupled model (i.e., physico-bio-chemical and geophysical models) runs for two alternating flooding and drying cycles (i.e. 12 days). The study aims to analyze redox potential and SP associated with redox processes in response to redox contaminant concentrations and reaction rates, and streaming potential in response to water content and flux. On the spatial scale, SP, redox potential, and streaming potential along with soil profile on the 1<sup>st</sup>, 2<sup>nd</sup>, 4<sup>th</sup>, and 6<sup>th</sup> days are analyzed. The 1<sup>st</sup> and 4<sup>th</sup> days represent the flooding and drying days, while the 2<sup>nd</sup> and 6<sup>th</sup> days represent the change days (i.e., from flooding to drying days and from drying to flooding days, respectively). Moreover, soil depths of 15cm, 45cm, and 75cm represent the unsaturated (0-30cm), alternating (30-60cm), and saturated (60-90cm) zones that are chosen for temporal scale analyses.

### 3.1 Biochemical processes

For biochemical cycling, C, N, Mn, Fe, S are chosen as the typical redox-sensitive species, and the main biochemical reactions are shown in Fig. 2.

#### 3.1.1 Primary redox reactions

Biochemical kinetic reactions are generally classified as primary and secondary redox reactions. The organic matter degradation pathways are the primary redox reactions (Wang and van Cappellen, 1996; Thullner et al., 2005). These include DON mineralization, DOC aerobic oxidation, denitrification, Mn(IV), Fe(III) and  $\text{SO}_4^{2-}$  reduction.

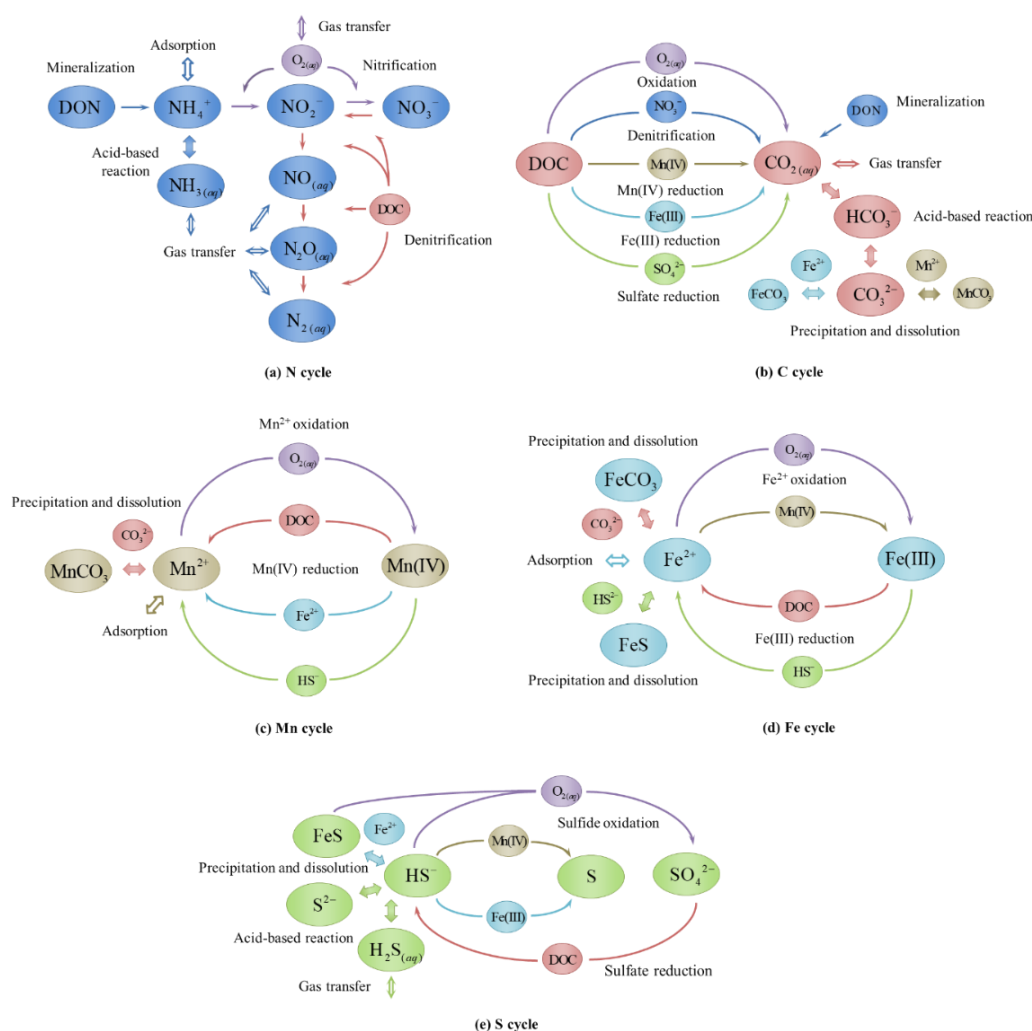


Figure 2 Biochemical cycles of different redox species in the capillary fringe. Panes (a), (b), (c), (d), and (e) represent N, C, Mn, Fe, and S cycles, respectively. DOC and DON are the abbreviations for dissolved organic carbon and nitrogen.

DON mineralization is mainly performed by carbon-oxidizing and denitrifying bacteria in oxic and anoxic environments, respectively (Coelho et al., 2000). For DOC oxidation, the reduced species can be generally listed as  $\text{O}_2$ ,  $\text{NO}_3^-$ ,  $\text{Mn(IV)}$ ,  $\text{Fe(III)}$  and  $\text{SO}_4^{2-}$  according to the redox sequence. In the oxic environment, DOC tends to be consumed by carbon-oxidizing bacteria through aerobic respiration. When oxygen content becomes less, DOC oxidized by other redox-





1 sensitive species occurs. DOC oxidized by  $\text{NO}_3^-$  can also be defined as denitrification.  $\text{NO}_3^-$  is  
 2 first reduced to  $\text{NO}_2^-$ , and then  $\text{NO}_2^-$  is sequentially converted to NO and  $\text{N}_2\text{O}$ . Finally,  $\text{N}_2\text{O}$   
 3 is transformed to non-harmful  $\text{N}_2$ . For DOC oxidized by Mn(IV), Fe(III) and  $\text{SO}_4^{2-}$ , we define  
 4 the reaction processes as Mn(IV), Fe(III) and  $\text{SO}_4^{2-}$  reductions. In DOC oxidation, the existence  
 5 of higher redox potential species tends to restrict the reaction of the lower one. (Roden, 2008).

6 The primary redox reactions are solved by multiple-Monod kinetics (Eqs. 7-9), which  
 7 consider the biomass, biomass and non-competitive inhibition, and substrate concentrations that  
 8 affect the reaction rates. The kinetic equations and rates for the primary redox reactions are  
 9 presented in Table 1.

### 10 **3.1.2 Secondary redox reactions**

11 The reduced species (i.e.,  $\text{NH}_4^+$ ,  $\text{NO}_2^-$ ,  $\text{Mn}^{2+}$ ,  $\text{Fe}^{2+}$  and  $\text{HS}^-$ ) result from primary  
 12 reactions or other pathways that can be oxidized in an oxidizing environment. These processes are  
 13 usually referred to as secondary redox reactions. The processes of  $\text{NH}_4^+$  and  $\text{NO}_2^-$  oxidized by  
 14  $\text{O}_2$  are defined as nitrification and are simulated by multiple-Monod kinetics (Eqs. 7-9). However,  
 15 for the secondary redox reactions related to  $\text{Mn}^{2+}$ ,  $\text{Fe}^{2+}$  and  $\text{HS}^-$ , the kinetic rates are solved by  
 16 second-order reactions (Eq. 10). This is because these chemical reactions can occur spontaneously  
 17 and are contributed less by microbial communities (Jaffe et al., 2002; Thullner et al., 2005). The  
 18 kinetic equations and rates for secondary reactions are described in Table 1.



Table 1 Primary and secondary redox reactions in the biochemical model.

Primary redox reaction	Reaction rate
$RD_x \cdot NH_3 \xrightarrow[\mu_{max}^{miner}]{\mu_{max}^{miner} \frac{O_2}{O_2 + K_{O_2}}} xCH_2O + NH_4^+ + OH^-$	$\mu_{aer}^{miner} = \mu_{max, aer}^{miner} f_{\theta_e} X_{oxid} \left[ \frac{k_{p, oxid}}{k_{p, oxid} + f_{\theta_e} X_{oxid}} \right] \left[ \frac{DON}{K_{DON, aer} + DON} \right] \left[ \frac{O_2}{K_{O_2}} \right]$
$CH_2O + O_2 \xrightarrow{\mu_{max}^{oxid}} CO_2 (aq) + H_2O$	$\mu_{aer}^{miner} = \mu_{max, aer}^{miner} f_{\theta_e} X_{denit} \left[ \frac{k_{p, denit}}{k_{p, denit} + f_{\theta_e} X_{denit}} \right] \left[ \frac{k_{LO_2}}{K_{LO_2} + O_2} \right] \left[ \frac{DON}{K_{DON, aer} + DON} \right] \left[ \frac{NO_3^-}{K_{NO_3^-} + NO_3^-} \right]$
$CH_2O + 2NO_3^- \xrightarrow{\mu_{max}^{denit1}} 2NO_2^- + CO_2 (aq) + H_2O$	$\mu_{oxid} = \mu_{max}^{oxid} f_{\theta_e} X_{oxid} \left[ \frac{k_{p, oxid}}{k_{p, oxid} + f_{\theta_e} X_{oxid}} \right] \left[ \frac{CH_2O}{K_{CH_2O} + CH_2O} \right] \left[ \frac{O_2}{K_{O_2} + O_2} \right]$
$CH_2O + 4NO_3^- + 4H^+ \xrightarrow{\mu_{max}^{denit2}} 4NO + CO_2 (aq) + 3H_2O$	$\mu_{denit1} = \mu_{max}^{denit1} f_{\theta_e} X_{denit} \left[ \frac{k_{p, denit}}{k_{p, denit} + f_{\theta_e} X_{denit}} \right] \left[ \frac{k_{LO_2}}{K_{LO_2} + O_2} \right] \left[ \frac{CH_2O}{K_{CH_2O} + CH_2O} \right] \left[ \frac{NO_3^-}{K_{NO_3^-} + NO_3^-} \right]$
$CH_2O + 4NO \xrightarrow{\mu_{max}^{denit3}} 2N_2O + CO_2 (aq) + H_2O$	$\mu_{denit2} = \mu_{max}^{denit2} f_{\theta_e} X_{denit} \left[ \frac{k_{p, denit}}{k_{p, denit} + f_{\theta_e} X_{denit}} \right] \left[ \frac{k_{LO_2}}{K_{LO_2} + O_2} \right] \left[ \frac{CH_2O}{K_{CH_2O} + CH_2O} \right] \left[ \frac{NO_3^-}{K_{NO_3^-} + NO_3^-} \right]$
$CH_2O + 2N_2O \xrightarrow{\mu_{max}^{denit4}} 2N_2 + CO_2 (aq) + H_2O$	$\mu_{denit3} = \mu_{max}^{denit3} f_{\theta_e} X_{denit} \left[ \frac{k_{p, denit}}{k_{p, denit} + f_{\theta_e} X_{denit}} \right] \left[ \frac{k_{LO_2}}{K_{LO_2} + O_2} \right] \left[ \frac{CH_2O}{K_{CH_2O} + CH_2O} \right] \left[ \frac{NO}{K_{NO} + NO} \right]$
$CH_2O + 2MnO_2 (s) + 4H^+ \xrightarrow{\mu_{max}^{Mn(IV)}} 2Mn^{2+} + CO_2 (aq) + 3H_2O$	$\mu_{denit4} = \mu_{max}^{denit4} f_{\theta_e} X_{denit} \left[ \frac{k_{p, denit}}{k_{p, denit} + f_{\theta_e} X_{denit}} \right] \left[ \frac{k_{LO_2}}{K_{LO_2} + O_2} \right] \left[ \frac{CH_2O}{K_{CH_2O} + CH_2O} \right] \left[ \frac{N_2O}{K_{N_2O} + N_2O} \right]$
$CH_2O + 4Fe(OH)_3 (s) + 8H^+ \xrightarrow{\mu_{max}^{Fe(III)}} 4Fe^{2+} + CO_2 (aq) + 11H_2O$	$\mu_{Mn(IV)} = \mu_{max}^{Mn(IV)} f_{\theta_e} X_{Mn(IV)} \left[ \frac{k_{p, Mn(IV)}}{k_{p, Mn(IV)} + f_{\theta_e} X_{Mn(IV)}} \right] \left[ \frac{k_{LO_2}}{K_{LO_2} + O_2} \right] \left[ \frac{CH_2O}{K_{CH_2O} + CH_2O} \right] \left[ \frac{f_{\theta} MnO_2}{K_{MnO_2} + f_{\theta} MnO_2} \right]$
$CH_2O + \frac{1}{2} SO_4^{2-} + \frac{1}{2} H^+ \xrightarrow{\mu_{max}^{SO_4}} \frac{1}{2} HS^- + CO_2 (aq) + H_2O$	$\mu_{Fe(III)} = \mu_{max}^{Fe(III)} f_{\theta_e} X_{Fe(III)} \left[ \frac{k_{p, Fe(III)}}{k_{p, Fe(III)} + f_{\theta_e} X_{Fe(III)}} \right] \left[ \frac{k_{LO_2}}{K_{LO_2} + O_2} \right] \left[ \frac{CH_2O}{K_{CH_2O} + CH_2O} \right] \left[ \frac{f_{\theta} Fe(OH)_3}{K_{Fe(OH)_3} + f_{\theta} Fe(OH)_3} \right]$
	$\mu_{sulf} = \mu_{max}^{sulf} f_{\theta_e} X_{sulf} \left[ \frac{k_{p, sulf}}{k_{p, sulf} + f_{\theta_e} X_{sulf}} \right] \left[ \frac{k_{LO_2}}{K_{LO_2} + O_2} \right] \left[ \frac{CH_2O}{K_{CH_2O} + CH_2O} \right] \left[ \frac{SO_4^{2-}}{K_{SO_4^{2-}} + SO_4^{2-}} \right]$



Secondary redox reaction	Reaction rate
$\text{NH}_4^+ + \frac{3}{2} \text{O}_{2(\text{aq})} \xrightarrow{r^{\text{mi}}} \text{NO}_2^- + \text{H}_2\text{O} + 2\text{H}^+$	$r^{\text{mi}_1} = \mu^{\text{mi}_1} f_{\theta} X_{\text{mi}_1} \left[ \frac{k_{\theta, \text{mi}_1}}{k_{\theta, \text{mi}_1} + f_{\theta} X_{\text{mi}_1}} \right] \left[ \frac{\text{NH}_4^+}{K_{\text{NH}_4^+}^{\text{mi}_1} + \text{NH}_4^+} \right] \left[ \frac{\text{O}_2}{K_{\text{O}_2}^{\text{mi}_1} + \text{O}_2} \right]$
$\text{NO}_2^- + \frac{1}{2} \text{O}_{2(\text{aq})} \xrightarrow{r^{\text{mi}_2}} \text{NO}_3^-$	$r^{\text{mi}_2} = \mu^{\text{mi}_2} f_{\theta} X_{\text{mi}_2} \left[ \frac{k_{\theta, \text{mi}_2}}{k_{\theta, \text{mi}_2} + f_{\theta} X_{\text{mi}_2}} \right] \left[ \frac{\text{NO}_2^-}{K_{\text{NO}_2^-}^{\text{mi}_2} + \text{NO}_2^-} \right] \left[ \frac{\text{O}_2}{K_{\text{O}_2}^{\text{mi}_2} + \text{O}_2} \right]$
$\text{Mn}^{2+} + \frac{1}{2} \text{O}_{2(\text{aq})} + \text{H}_2\text{O} \xrightarrow{r^{\text{Mn}, \text{O}}} \text{MnO}_{2(\text{s})} + 2\text{H}^+$	$r^{\text{Mn}, \text{O}} = k^{\text{Mn}, \text{O}} [\text{Mn}^{2+}] [\text{O}_2]$
$\text{Fe}^{2+} + \frac{1}{4} \text{O}_{2(\text{aq})} + \frac{5}{2} \text{H}_2\text{O} \xrightarrow{r^{\text{Fe}, \text{O}}} \text{Fe}(\text{OH})_3(\text{s}) + 2\text{H}^+$	$r^{\text{Fe}, \text{O}} = k^{\text{Fe}, \text{O}} [\text{Fe}^{2+}] [\text{O}_2]$
$\text{Fe}^{2+} + \frac{1}{2} \text{MnO}_{2(\text{s})} + 2\text{H}_2\text{O} \xrightarrow{r^{\text{Fe}, \text{Mn}}} \text{Fe}(\text{OH})_3(\text{s}) + \frac{1}{2} \text{Mn}^{2+} + \text{H}^+$	$r^{\text{Fe}, \text{Mn}} = k^{\text{Fe}, \text{Mn}} [\text{Fe}^{2+}] \left[ \frac{f_{\theta} \text{MnO}_2}{f_{\theta} \text{MnO}_2 + 1} \right]$
$\text{HS}^- + 2\text{O}_{2(\text{aq})} \xrightarrow{r^{\text{S}, \text{O}}} \text{SO}_4^{2-} + \text{H}^+$	$r^{\text{S}, \text{O}} = k^{\text{S}, \text{O}} [\text{HS}^-] [\text{O}_2]$
$\text{HS}^- + \text{MnO}_{2(\text{s})} + 3\text{H}^+ \xrightarrow{r^{\text{S}, \text{Mn}}} \text{Mn}^{2+} + \text{S}(\text{s}) + 2\text{H}_2\text{O}$	$r^{\text{S}, \text{Mn}} = k^{\text{S}, \text{Mn}} [\text{HS}^-] \left[ \frac{f_{\theta} \text{MnO}_2}{f_{\theta} \text{MnO}_2 + 1} \right]$
$\text{HS}^- + 2\text{Fe}(\text{OH})_3(\text{s}) + 5\text{H}^+ \xrightarrow{r^{\text{S}, \text{Fe}}} 2\text{Fe}^{2+} + \text{S}(\text{s}) + 6\text{H}_2\text{O}$	$r^{\text{S}, \text{Fe}} = k^{\text{S}, \text{Fe}} [\text{HS}^-] \left[ \frac{f_{\theta} \text{Fe}(\text{OH})_3}{f_{\theta} \text{Fe}(\text{OH})_3 + 1} \right]$
$\text{FeS}(\text{s}) + 2\text{O}_{2(\text{aq})} \xrightarrow{r^{\text{FeS}, \text{O}}} \text{Fe}^{2+} + \text{SO}_4^{2-}$	$r^{\text{FeS}, \text{O}} = k^{\text{FeS}, \text{O}} \left[ \frac{f_{\theta} \text{FeS}}{f_{\theta} \text{FeS} + 1} \right] [\text{O}_2]$



### 3.1.3 Equilibrium reactions

In addition to primary and secondary redox reactions, there are also equilibrium reactions in the aqueous phase (i.e., complexation and hydrolysis), between aqueous and solid phases (i.e., precipitation and dissolution), between the aqueous phase and the solid surface (i.e., adsorption and desorption), and between aqueous and gas phases (i.e., dissolution and volatilization). The equilibrium constants for aqueous, solid, surface, and gas phases are presented as Eqs. 12, 13, 15, and 18, respectively. The constants for the equilibrium reactions are shown in Table 2.

Table 2 Equilibrium reactions in the biochemical model.

Complexation and hydrolysis	Hydrolysis equilibrium constant
$\text{HCO}_3^- \xrightleftharpoons{K_{\text{HCO}_3^-}^h} \text{CO}_3^{2-} + \text{H}^+$	$K_{\text{HCO}_3^-}^h = \frac{[\text{CO}_3^{2-}][\text{H}^+]}{[\text{HCO}_3^-]}$
$\text{CO}_2(\text{aq}) + \text{H}_2\text{O} \xrightleftharpoons{K_{\text{H}_2\text{CO}_3}^h} \text{CO}_3^{2-} + 2\text{H}^+$	$K_{\text{H}_2\text{CO}_3}^h = \frac{[\text{CO}_3^{2-}][\text{H}^+]^2}{[\text{CO}_2]}$
$\text{NH}_4^+ \xrightleftharpoons{K_{\text{NH}_4^+}^h} \text{NH}_3(\text{aq}) + \text{H}^+$	$K_{\text{NH}_4^+}^h = \frac{[\text{NH}_3][\text{H}^+]}{[\text{NH}_4^+]}$
$\text{HS}^- \xrightleftharpoons{K_{\text{HS}^-}^h} \text{S}^{2-} + \text{H}^+$	$K_{\text{HS}^-}^h = \frac{[\text{S}^{2-}][\text{H}^+]}{[\text{HS}^-]}$
$\text{H}_2\text{S}(\text{aq}) \xrightleftharpoons{K_{\text{H}_2\text{S}}^h} \text{HS}^- + \text{H}^+$	$K_{\text{H}_2\text{S}}^h = \frac{[\text{HS}^-][\text{H}^+]}{[\text{H}_2\text{S}]}$
Precipitation and dissolution	Solubility product constant
$\text{MnCO}_3(\text{s}) \xrightleftharpoons{K_{\text{MnCO}_3}^{\text{SP}}} \text{Mn}^{2+} + \text{CO}_3^{2-}$	$K_{\text{MnCO}_3}^{\text{SP}} = [\text{Mn}^{2+}][\text{CO}_3^{2-}]$
$\text{FeCO}_3(\text{s}) \xrightleftharpoons{K_{\text{FeCO}_3}^{\text{SP}}} \text{Fe}^{2+} + \text{CO}_3^{2-}$	$K_{\text{FeCO}_3}^{\text{SP}} = [\text{Fe}^{2+}][\text{CO}_3^{2-}]$
$\text{FeS}(\text{s}) + \text{H}^+ \xrightleftharpoons{K_{\text{FeS}}^{\text{SP}}} \text{Fe}^{2+} + \text{HS}^-$	$K_{\text{FeS}}^{\text{SP}} = \frac{[\text{Fe}^{2+}][\text{HS}^-]}{[\text{H}^+]}$
$\text{CaCO}_3(\text{s}) \xrightleftharpoons{K_{\text{CaCO}_3}^{\text{SP}}} \text{Ca}^{2+} + \text{CO}_3^{2-}$	$K_{\text{CaCO}_3}^{\text{SP}} = [\text{Ca}^{2+}][\text{CO}_3^{2-}]$
$\text{CaSO}_4(\text{s}) \xrightleftharpoons{K_{\text{CaSO}_4}^{\text{SP}}} \text{Ca}^{2+} + \text{SO}_4^{2-}$	$K_{\text{CaSO}_4}^{\text{SP}} = [\text{Ca}^{2+}][\text{SO}_4^{2-}]$
$\text{MgCO}_3(\text{s}) \xrightleftharpoons{K_{\text{MgCO}_3}^{\text{SP}}} \text{Mg}^{2+} + \text{CO}_3^{2-}$	$K_{\text{MgCO}_3}^{\text{SP}} = [\text{Mg}^{2+}][\text{CO}_3^{2-}]$



Adsorption	Surface equilibrium constant
$\text{NH}_4^+ + \text{X}^- \xrightleftharpoons{K_{\text{NH}_4^+}^S} \text{NH}_4\text{X}$	$K_{\text{NH}_4^+}^S = \frac{[\text{NH}_4\text{X}]}{[\text{NH}_4^+][\text{X}^-]}$
$\text{Fe}^{2+} + 2\text{X}^- \xrightleftharpoons{K_{\text{Fe}^{2+}}^S} \text{FeX}_2$	$K_{\text{Fe}^{2+}}^S = \frac{[\text{FeX}_2]}{[\text{Fe}^{2+}][\text{X}^-]^2}$
$\text{Mn}^{2+} + 2\text{X}^- \xrightleftharpoons{K_{\text{Mn}^{2+}}^S} \text{MnX}_2$	$K_{\text{Mn}^{2+}}^S = \frac{[\text{MnX}_2]}{[\text{Mn}^{2+}][\text{X}^-]^2}$
$\text{Na}^+ + \text{X}^- \xrightleftharpoons{K_{\text{Na}^+}^S} \text{NaX}$	$K_{\text{Na}^+}^S = \frac{[\text{NaX}]}{[\text{Na}^+][\text{X}^-]}$
$\text{K}^+ + \text{X}^- \xrightleftharpoons{K_{\text{K}^+}^S} \text{KX}$	$K_{\text{K}^+}^S = \frac{[\text{KX}]}{[\text{K}^+][\text{X}^-]}$
$\text{Ca}^{2+} + 2\text{X}^- \xrightleftharpoons{K_{\text{Ca}^{2+}}^S} \text{CaX}_2$	$K_{\text{Ca}^{2+}}^S = \frac{[\text{CaX}_2]}{[\text{Ca}^{2+}][\text{X}^-]^2}$
$\text{Mg}^{2+} + 2\text{X}^- \xrightleftharpoons{K_{\text{Mg}^{2+}}^S} \text{MgX}_2$	$K_{\text{Mg}^{2+}}^S = \frac{[\text{MgX}_2]}{[\text{Mg}^{2+}][\text{X}^-]^2}$
Gas dissolution and volatilization	Henry's law constant
$\text{CO}_2(\text{g}) \xrightleftharpoons{K_{\text{CO}_2}^H} \text{CO}_2(\text{aq})$	$K_{\text{CO}_2}^H = \frac{C_{\text{CO}_2}}{P_{\text{CO}_2}}$
$\text{NH}_3(\text{g}) \xrightleftharpoons{K_{\text{NH}_3}^H} \text{NH}_3(\text{aq})$	$K_{\text{NH}_3}^H = \frac{C_{\text{NH}_3}}{P_{\text{NH}_3}}$
$\text{NO}(\text{g}) \xrightleftharpoons{K_{\text{NO}}^H} \text{NO}(\text{aq})$	$K_{\text{NO}}^H = \frac{C_{\text{NO}}}{P_{\text{NO}}}$
$\text{N}_2\text{O}(\text{g}) \xrightleftharpoons{K_{\text{N}_2\text{O}}^H} \text{N}_2\text{O}(\text{aq})$	$K_{\text{N}_2\text{O}}^H = \frac{C_{\text{N}_2\text{O}}}{P_{\text{N}_2\text{O}}}$
$\text{N}_2(\text{g}) \xrightleftharpoons{K_{\text{N}_2}^H} \text{N}_2(\text{aq})$	$K_{\text{N}_2}^H = \frac{C_{\text{N}_2}}{P_{\text{N}_2}}$
$\text{H}_2\text{S}(\text{g}) \xrightleftharpoons{K_{\text{H}_2\text{S}}^H} \text{H}_2\text{S}(\text{aq})$	$K_{\text{H}_2\text{S}}^H = \frac{C_{\text{H}_2\text{S}}}{P_{\text{H}_2\text{S}}}$
$\text{O}_2(\text{g}) \xrightleftharpoons{K_{\text{O}_2}^H} \text{O}_2(\text{aq})$	$K_{\text{O}_2}^H = \frac{C_{\text{O}_2}}{P_{\text{O}_2}}$



### 1 3.1.4 Microbial dynamics

2 In this component of the model, the microbial communities are conceptualized to have 7 types,  
 3 namely, (i) heterotrophic carbon-oxidizing bacteria; (ii) heterotrophic denitrifying bacteria; (iii)  
 4 heterotrophic Mn(IV)-reducing bacteria; (iv) heterotrophic Fe(III)-reducing bacteria; (v)  
 5 heterotrophic sulfate-reducing bacteria (Lensing et al., 1994); (vi) autotrophic ammonia-oxidizing  
 6 bacteria; and (vii) autotrophic nitrite-oxidizing bacteria. In order to simplify the model, we ignore  
 7 the direct competition between different types of bacteria. However, carbon-oxidizing bacteria and  
 8 autotrophic bacteria may indirectly compete for oxygen, while heterotrophic bacteria compete for  
 9 organic matter for metabolism. The general equation for microbial dynamics is presented as Eq.  
 10 14. The character, biochemical process involvement, and microbial dynamics for different bacteria  
 11 are shown in Table 3.

### 12 3.2 Self-potential response

13 For streaming potential, the physical output parameters (e.g., water pressure head, saturation,  
 14 and flux) from the physio-bio-chemical model are used as the input data to simulate streaming  
 15 potential and current density. The simulation can follow the processes in Eq. 23 and Section 2.2.1.

16 Redox species concentrations and water saturation are used to couple biochemical and redox  
 17 potential models for redox potential calculation. The redox reaction rates for Fe, Mn, and S are  
 18 weak because the concentrations of Fe and Mn species are quite low, and the environment is also  
 19 suitable for sulfate stable existence. Thus, we ignore these species and only consider redox species  
 20 of O, N, and C for redox potential simulation. Based on the reactions, 7 pairs of redox couples (i.e.  
 21  $\text{H}_2\text{O}/\text{O}_2$ ,  $\text{NH}_4^+/\text{NO}_2^-$ ,  $\text{NO}_2^-/\text{NO}_3^-$ ,  $\text{NO}/\text{NO}_2^-$ ,  $\text{N}_2\text{O}/\text{NO}$ ,  $\text{N}_2/\text{N}_2\text{O}$ ,  $\text{CH}_2\text{O}/\text{HCO}_3^-$ )  
 22 are considered in the model. Then, the redox potential can be simulated by the processes in  
 23 Sections 2.2.2 and S.1 (Supplement). After that, the SP and current density associated with redox  
 24 processes can also be calculated based on Eqs. 24 and 28.

25 Based on the simulated processes of source current densities described in the above two  
 26 paragraphs, the SP signals can be finally calculated as Eq. 22. After that, we can analyze SP signals  
 27 (including streaming potential and SP associated with redox processes) in response to water  
 28 content and flux, as well as redox species concentrations and reaction rates, and finally, apply the  
 29 SP model to practice.

30



Table 3 Microbial communities in the biochemical model.

Microbial community	Character	Biochemical process	Microbial dynamics
Heterotrophic carbon-oxidizing bacteria	Aerobic heterotrophic bacteria	DOC aerobic oxidation DON mineralization	$\frac{dX_{\text{oxid}}}{dt} = Y_{\text{oxid}} \theta_w r^{\text{oxid}} - X_{\text{oxid}} d_{\text{oxid}}$
Heterotrophic denitrifying bacteria	Anaerobic heterotrophic bacteria	Denitrification DON mineralization	$\frac{dX_{\text{denit}}}{dt} = Y_{\text{denit}} \theta_w (r^{\text{denit}_1} + r^{\text{denit}_2} + r^{\text{denit}_3} + r^{\text{denit}_4}) - X_{\text{denit}} d_{\text{denit}}$
Heterotrophic Mn(IV)-reducing bacteria	Anaerobic heterotrophic bacteria	Mn(IV) reduction	$\frac{dX_{\text{Mn(IV)}}}{dt} = Y_{\text{Mn(IV)}} \theta_w r^{\text{Mn(IV)}} - X_{\text{Mn(IV)}} d_{\text{Mn(IV)}}$
Heterotrophic Fe(III)-reducing bacteria	Anaerobic heterotrophic bacteria	Fe(III) reduction	$\frac{dX_{\text{Fe(III)}}}{dt} = Y_{\text{Fe(III)}} \theta_w r^{\text{Fe(III)}} - X_{\text{Fe(III)}} d_{\text{Fe(III)}}$
Heterotrophic sulfate-reducing bacteria	Anaerobic heterotrophic bacteria	Sulfate reduction	$\frac{dX_{\text{sulf}}}{dt} = Y_{\text{sulf}} \theta_w r^{\text{sulf}} - X_{\text{sulf}} d_{\text{sulf}}$
Autotrophic ammonia-oxidizing bacteria	Aerobic autotrophic bacteria	Ammonium transformation to nitrite	$\frac{dX_{\text{nh}_2}}{dt} = Y_{\text{nh}_2} \theta_w r^{\text{nh}_2} - X_{\text{nh}_2} d_{\text{nh}_2}$
Autotrophic nitrite-oxidizing bacteria	Aerobic autotrophic bacteria	Nitrite transformation to nitrate	$\frac{dX_{\text{nh}_3}}{dt} = Y_{\text{nh}_3} \theta_w r^{\text{nh}_3} - X_{\text{nh}_3} d_{\text{nh}_3}$



### 1    **3.3 Data calibration and verification**

2            The experimental data is first used for model calibration and verification.

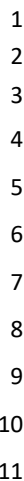
#### 3    **3.3.1 Experiment setup**

4            The experimental data was courtesy of Zhang and Furman (2021), and the experiment was  
 5    conducted in a soil column of 91cm operating for 29 days. The studied soil (98.47% sand, 0.66%  
 6    clay, and 0.87% silt) was sampled from the SAT site, Yavne 2, Shafdan, Israel (Sopilniak et al.,  
 7    2017). The schematic of soil biochemical and geophysical experiments are shown in Fig. 3. The  
 8    water level was kept at around 49cm with solution injected from the top every 3 or 4 days. The  
 9    water flux was 1.2-2.0cm/h, and the duration lasted for 3-5h in each flooding event. The water  
 10   input flux and chemical species concentrations are shown in Fig. S.1(b) and Table S.1  
 11   (Supplement), respectively. To control the water table was stable at around 49cm, water discharge  
 12   was conducted simultaneously at the bottom of the soil column.

13           Soil water matric potential was measured every 15 min by self-made tensiometers (porous  
 14   ceramic cup (#230, SDEC<sup>®</sup>), PVC tubes (ACF0007, Tygon<sup>®</sup>), and a transducer (MPX2100DP,  
 15   Freescale<sup>®</sup>). A sensor measured the oxygen volume percentage at the headspace (KE-25, Figaro<sup>®</sup>).  
 16   The chemical species concentrations (i.e., DOC, DON,  $\text{NH}_4^+$ , and  $\text{NO}_3^-$ ) used for model  
 17   calibration were measured around 6 hours before water flooding and drainage events. The  
 18   tensiometers were inserted at the soil depths of 1cm, 11cm, ... 81cm. However, the chemical  
 19   species were measured only at the depths of 41cm, 51cm, ... 91cm, as the soil water was hard to  
 20   extract from the unsaturated soil above 41cm. The extracted pore water (55ml) was sampled with  
 21   Rhizon CSS samplers (19.21.24F, Rhizosphere<sup>®</sup>) and filtered through a 0.22 $\mu\text{m}$  filter unit (PVDF  
 22   membrane, Millex<sup>®</sup>GV) before the measurement. In the pore water, dissolved inorganic nitrogen  
 23   species ( $\text{NH}_4^+$ ,  $\text{NO}_2^-$  and  $\text{NO}_3^-$ ) were measured by the spectrophotometer (GENESYS 10S UV-  
 24   Vis, Thermo Fisher Scientific<sup>®</sup>). DOC and total dissolved nitrogen were measured by the TOC-V  
 25   (Shimadzu<sup>®</sup>). DON was attained by the concentrations of total dissolved nitrogen subtracting  
 26   dissolved inorganic nitrogen (the sum of  $\text{NH}_4^+$ ,  $\text{NO}_2^-$  and  $\text{NO}_3^-$ ).

27





Redox electrodes (#461, ecoTech®) were inserted at the soil depths of 1cm, 11cm, ...81cm, and 86cm to measure the redox potential every 15 min with the reference electrode (#4622, ecoTech®) inserted at 16cm depth. The SP differences were measured every 5cm along the soil column by a portable spectral induce polarization (PSIP) equipment (Ontash & Ermac®) twice a day. The stimulus electrodes were disconnected from the soil column. Two sets of measurements were implemented. The first set was to measure the SP differences of 6-11cm, 16-21cm, ...76-81cm soil layers with 8 couples of potential electrodes, while the second set was to measure the data of 11-16cm, 21-26cm, ...71-76cm soil layers with 7 pairs of potential electrodes.



### 3.3.2 Parameter collection

As such, a model contains several 10s of parameters, part of the parameters were calibrated, while the majority of parameters were collected from the literature. In cases, the parameters recommended by the HP1 software (Šimůnek et al., 2009; Parkhurst and Appelo, 2013) were adopted. The main parameters for soil physical properties, multiple-Monod kinetics and microbial dynamics, second-order reactions, and various equilibrium reactions are shown in Tables S.2-S.5 (Supplement), while parameters for SP modeling are listed in Tables S.6 and S.7 (Supplement) and described in Section 2.2.

The soil physical and biochemical state variables (e.g., solute concentrations, water, and gas pressure) first measured in the 4th day were used as initial conditions. The simulation ran for 25 days. The upper boundary conditions were set as variable water flux, gas flow, and solute concentrations from the top. The lower boundary conditions were set as variable water flux that equals the surface flux and the solute-free drainage.

### 3.3.3 Calibrated and verified results

In physico-bio-chemical processes, water pressure, DOC, DON, ammonium, and nitrate concentration data at 41 cm and 61 cm depth was used to calibrate the model, while the 51cm and 71cm data was used for verification. In geophysical response, the redox potential of 41cm and SP difference of 26-31cm were used for model calibration, while those of 81cm and 51-56cm were used for model verification. The Nash-Sutcliffe efficiency coefficient (NSE) was used to evaluate the simulation results of contaminant concentrations, and the formulation is presented as follows.

$$E_{ns} = 1 - \frac{\sum (C_s - C_m)^2}{\sum (C_m - \bar{C}_m)^2} \quad (34)$$

where  $C_s$ ,  $C_m$  and  $\bar{C}_m$  are the simulated, measured, and average measured values, respectively ( $\text{mmol L}^{-1}$ ), and  $E_{ns}$  is the Nash-Sutcliffe efficiency coefficient ( $0 \leq E_{ns} \leq 1$ ). Thus, when  $E_{ns}$  is close to 1, the simulated values are close to the measured values, and vice versa.

#### (1) Physico-bio-chemical model

The physico-bio-chemical model calibration and verification results are shown in Fig. S.2 (Supplement). The measured and simulated water pressure values and different contaminant



concentrations can generally match well at different soil depths. NSEs are also relatively higher for different species.

The abnormal increasing trend for measured water pressure at 41cm is due to the gas bubble mixed in the tensiometer, which causes measurement errors. Additionally, an accident of water leakage occurred on the 22nd day, the water pressure of measured data presents abnormal trends at different depths after that day.

For chemical species concentrations, the NSE values of DOC and DON at 51cm and 71cm are relatively lower. Because the insoluble SOM decomposition is ignored in the model, some simulated concentrations of DON and DOC are a little lower than the measured data. Moreover, there are no initial concentrations above 41cm, affecting the simulated results at the adjacent soil depth. Thus, NSEs of ammonium and nitrate concentrations are also a little lower at 41cm depth.

## (2) Self-potential model

The geophysical calibrated and verified results are shown in Fig. 4. We can see that the changing trends of measured and simulated values for redox potential are matched well.

As for the SP difference, the simulated values are relatively stable in the unsaturated area (i.e. 26-31cm), while the measured values slightly fluctuate around the simulated results. In the experiment, the soil is heterogeneous even at the same layer, but the electrodes are usually in contact with limited soil. In the unsaturated area, the water flow can easily affect soil water character and their contact with electrodes. Thus, the SP differences show slight fluctuation. In numerical modeling, the soil is considered to be homogeneous at the same layer. Oxygen concentration is stable and dominates the SP signature in unsaturated soil, so SP differences of simulated results are relatively stable on the temporal scale.

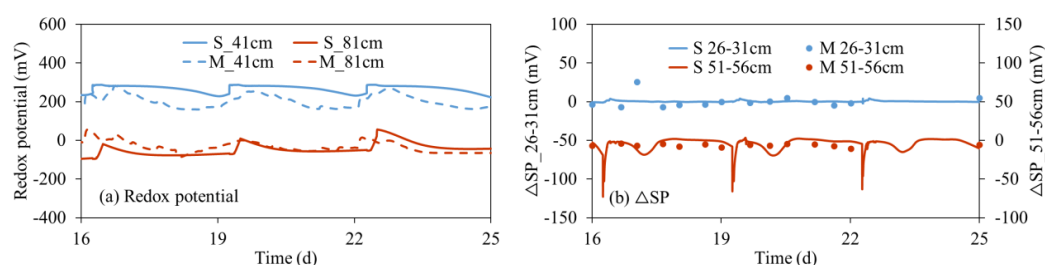


Figure 4 Measured and simulated values of redox potential and SP differences for model calibration and verification. Panes (a) and (b) represent calibrated and verified results for redox potential and SP differences, respectively. Capitals M and S represent measured and simulated values.



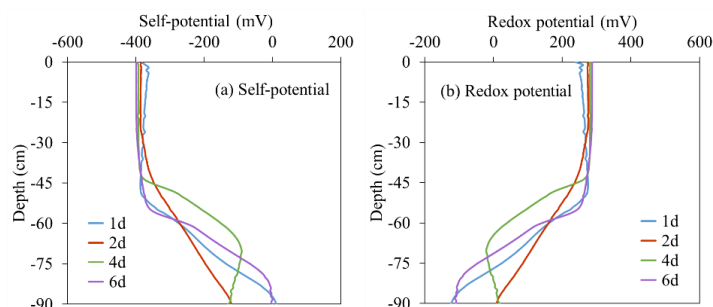
In the saturated zone (i.e., 51-56cm), the measured values are relatively stable, but simulated values tend to fluctuate periodically. In the experiment, the electrodes are fully in contact with the aqueous phase in the saturated soil, so the measurement errors caused by the heterogeneous soil environment are not as obvious as those in unsaturated soil. Moreover, the SP data was not measured during water infiltrates when the chemical species, reaction rates, and water saturation can be further affected by water flow. However, the periodical water recharge and discharge will cause the transient oxic environment and dynamic biochemical changes in the saturated area, so the SP signals show transient fluctuation trends in the simulated values.

## 4 Results and discussion

### 4.1 Spatial analyses

#### 4.1.1 Redox potential

SP and Redox potential in response to redox species concentrations and reaction rates along soil depth are shown in Fig. 5. SP associated with redox processes is not shown in the figure because it is visually hard to distinguish from SP, caused by the little contribution from streaming potential. In the following, we mainly focus on redox potential analyses as SP presents almost opposite trends to redox potential. Redox potential is higher and stable above 40cm due to the oxic environment, but declines below 40cm because of the gradual decrease in oxygen concentration (Fig. 5(b)).





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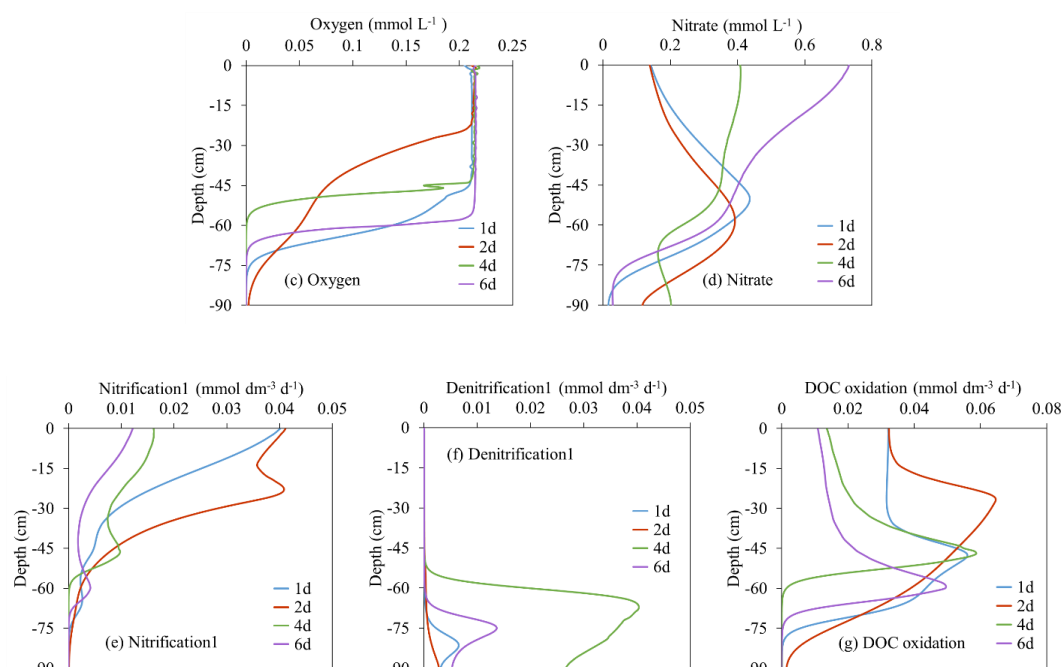


Figure 5 Self-potential and redox potential in response to redox species concentrations and reaction rates along with the soil profile. Panes (a) and (b) indicate SP and redox potential. Panes (c) and (d) refer to oxygen and nitrate concentrations. Panes (c), (d), and (e) represent the reaction rates for ammonium transformation to nitrate (i.e. nitrification 1), nitrate conversion to nitrite (i.e. denitrification 1), and DOC aerobic oxidation, respectively.

Redox potential is mostly affected by oxygen and nitrate concentrations (Fig. 5(c) and (d)). In the unsaturated area (upper layer), the changing trends of redox potential are similar to those of oxygen concentrations for the 4 typical days. However, the trends are mainly affected by nitrate concentrations in the saturated zone (lower layer), where oxygen concentrations are low. In the alternating area (middle layer), the redox potential is dominated by both oxygen and nitrate concentrations, and the trends are relatively complex.

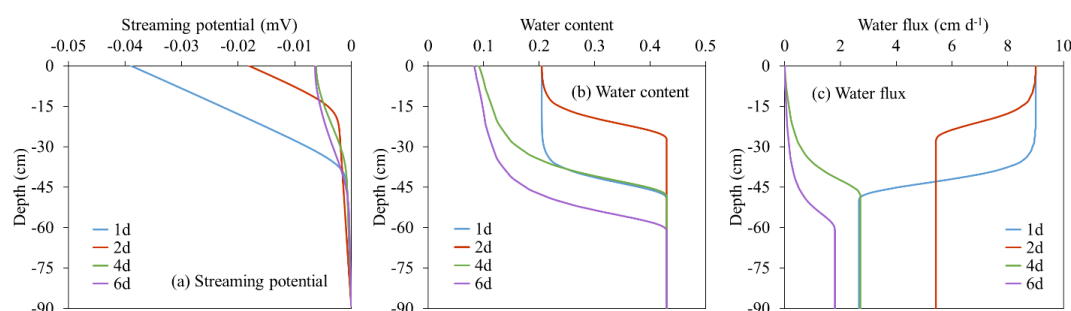
Generally, nitrification and DOC aerobic oxidation are stronger in a higher redox environment (unsaturated and alternating soil) (Fig. 5(e) and (f)), while denitrification is higher in a lower redox environment (saturated soil) (Fig. 5(g)). The redox potential values of 100-350mV, -50-50mV, and 50-250mV are generally considered as the optimal ranges for nitrification, denitrification, and DOC aerobic oxidation in wastewater treatment (Gerardi, 2016; Marin et al., 2016). The most suitable redox environment for nitrification is above 50cm based on the optimal



1 ranges. From 50cm to 75cm, redox potential for different days tends to be gradually lower than  
 2 100mV in sequence. Below 75cm, the redox potential values for the 4 days are all less than 100mV.  
 3 Meanwhile, the nitrification rates show gradual downward trends along with soil depth. The  
 4 reaction rates above 50cm layer are relatively higher, but the rates from 50cm to 75cm tend to be  
 5 lower, and the rates are close to 0 below 75cm. Moreover, the higher denitrification rates in deeper  
 6 soil on different days also correspond to the optimal redox potential range (-50-50mV). As for  
 7 DOC aerobic oxidation, the optimal redox potential range is slightly lower than that of nitrification.  
 8 This causes DOC oxidation rates not always to be the highest in the upper soil, and the relatively  
 9 higher reaction rates can also extend to relatively deeper soil. Thus, the stronger reaction rates tend  
 10 to shift from unsaturated to upper saturated soil for the 4 typical days, corresponding to the suitable  
 11 redox potential range (50-250mV).

#### 12 4.1.2 Streaming potential

13 The streaming potential in response to water content and flux along with soil depth is shown  
 14 in Fig. 6. The magnitude can be around 100 mV for redox potential, while the magnitude for  
 15 streaming potential is only around 0.01 mV. Thus, the streaming potential can be neglected in the  
 16 high biochemical dynamic and low water flux environment. However, to analyze the relationships  
 17 between steaming potential with water content and flux, we present the simulated results and  
 18 briefly describe them. Along with soil depth, the negative values of streaming potential become  
 19 less remarkable, and the changing trends are mainly affected by water content and flux. At the  
 20 same depth, the intensive water flux and low water content tend to result in more remarkable  
 21 negative values.



22  
 23 Figure 6 Streaming potential in response to water content and flux along with the soil profile. Panes (a), (b), and (c)  
 24 represents streaming potential, water content, and water flux, respectively.  
 25



## 4.2 Temporal analyses

### 4.2.1 Redox potential

SP and redox potential in response to redox species concentrations and reaction rates in time series are shown in Fig. 7. In the unsaturated zone (15cm), the fluctuating nitrate concentration shows little impact on redox potential. Oxygen concentration and redox potential are relatively high and stable and result in high nitrification and DOC oxidation rates. However, the reaction rates increase in flooding days (i.e. the 0-2<sup>nd</sup> day and 6-8<sup>th</sup> day) but decrease in drying days (i.e. the 2-6<sup>th</sup> day and 8-12<sup>th</sup> day), affected by water content changes resulting from dynamic water flow.

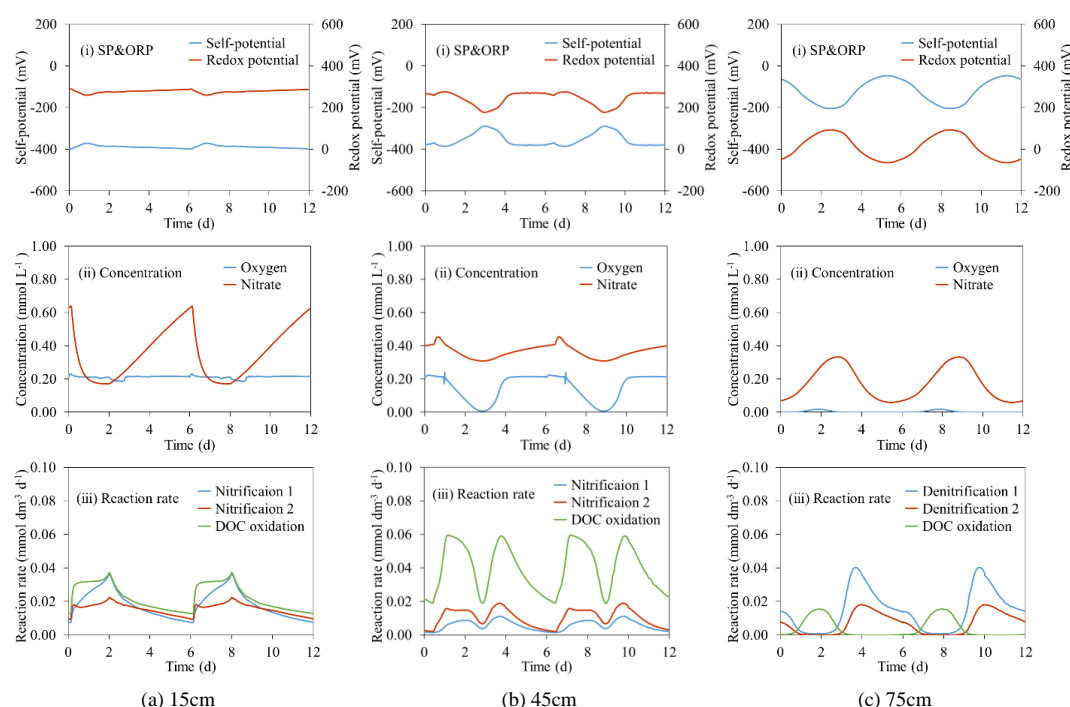


Figure 7 Self-potential and redox potential in response to redox species concentrations and reaction rates in time series. Panes (a), (b), and (c) represent 15cm, 45cm, and 75cm depths respectively. Sub-panes (i), (ii), and (iii) represent SP&ORP, redox species concentrations, and reaction rates at the corresponding depths. ORP means redox potential. Nitrification 2 indicates nitrite transformation to nitrate, and denitrification 2 refers to nitrite conversion to nitric oxide. Denitrification in 15cm and 45cm and nitrification in 75cm are not shown as the reaction rates are too low and can be ignored compared to other reactions.



In the alternating zone (45cm), redox potential fluctuates with water table changes (caused by the alternating flooding and drying days) and shows positive relationships with oxygen and nitrate concentrations. However, redox reaction rates (i.e., nitrification and DOC aerobic oxidation) present abnormal negative relationships with redox potential on the 0-1<sup>st</sup> day, 4-7<sup>th</sup> days, and 10-12<sup>th</sup> days. This is because the reaction rates are affected by the water content changes in partially saturated soil on these days (Fig. 8(b)). Denitrification rates are quite low in both unsaturated and alternating zones, so they are not shown in the figure and are not specifically discussed in the text.

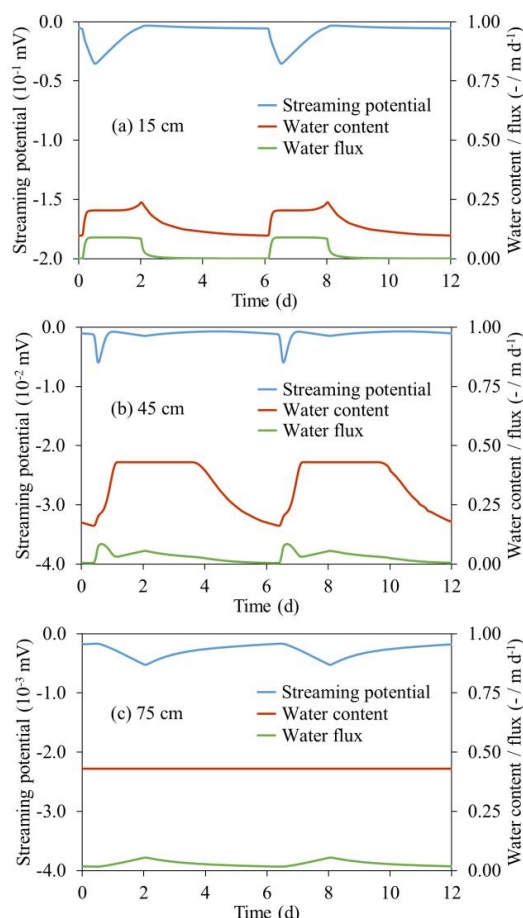
In the saturated zone (75cm), the oxygen content is low. Generally, redox potential presents positive relationships with nitrate concentration and DOC aerobic oxidation rate but a negative relationship with denitrification rate. As the infiltrating water from flooding facilitates oxygen transport to deeper soil, DOC oxidation shows periodical higher rates (around the 2<sup>nd</sup> and 8<sup>th</sup> days) in the saturated area. The redox potential is also within the optimal value for DOC oxidation. However, a similar situation does not occur to nitrification as the redox potential is below the optimal values for nitrification. On the other hand, the denitrification rate tends to be higher around the 4<sup>th</sup> and 10<sup>th</sup> days corresponding to the optimal redox potential range (-50-50mV), but the rate is close to 0 when the DOC oxidation rate periodical increases.

Based on the spatial and temporal analyses, the simulated SP and redox potential can better reflect redox species concentrations (i.e., oxygen and nitrate in the oxic and anoxic environment, respectively) and reaction rates (nitrification, denitrification, and DOC aerobic oxidation). Thus, the model can be used to assess redox-sensitive contaminant distribution and degradation spatially and temporally.

#### 4.2.2 Streaming potential

Streaming potential in response to water content and flux in time series are presented in Fig. 8. The streaming potential is mainly affected by water flux. At the same depth, the more intensive water flow tends to cause more obvious negative values in streaming potential. Moreover, the negative values in the upper soil are more remarkable than those in the lower soil. However, the water content effect on streaming potential is not obvious in temporal representation as the water flux impact is more significant and the temporal changes of water content are less remarkable than those on the spatial scale. Thus, the streaming potential signature can be used to estimate the effectiveness of soil moisture and water leakage detection (also see the spatial analyses section).





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 2 Figure 8 Streaming potential in response to water content and flux in time series. Panes (a), (b), and (c) represent 15cm,  
 3 45cm, and 75cm depths respectively.

## 4 5 Summary and conclusions

5 In this research, we couple the physio-bio-chemical and geophysical models and simulate the  
 6 SP signature in response to carbon and nitrogen transformation and transport under dynamic TWW  
 7 discharge in an SAT basin.

8 The physio-bio-chemical model can better simulate the physical and biochemical dynamics  
 9 in the subsurface, such as water pressure and carbon and nitrogen contaminants. The coupled  
 10 models show that SP signature is mostly contributed by redox processes in the case study. The  
 11 redox potential and SP signature can reflect the redox species concentrations, including oxygen  
 12 and nitrate in the oxic and anoxic environment, respectively. Moreover, the signals are also



1 sensitive to redox reaction rates, i.e., nitrification, denitrification, and DOC aerobic oxidation. The  
 2 higher reaction rates for different redox processes correspond to their optimal redox potential  
 3 ranges. The streaming potential contributes little to SP signature and is affected by water flux and  
 4 content. Thus, the model can be used to instruct or assess redox-sensitive contaminant monitoring  
 5 and soil moisture and water leakage detection in case studies.

6 However, studies related to redox potential solved by the Butler-Volmer equation are rare,  
 7 and we cannot compare the results with other studies for further model verification. Additionally,  
 8 the values for model parameters are usually broad, and in cases are based on very different models,  
 9 so the parameters must be repeatedly verified when applying the model to other cases. Confronting  
 10 this model with additional data, at the laboratory and field scales, is needed. However, such data  
 11 is relatively rare.

## 12 Appendices

13 Table A.1 Superscript symbols.

Superscript	Description
miner	Mineralization
oxid	DOC aerobic oxidation
denit <sub>1</sub>	Nitrate reduction
denit <sub>2</sub>	Nitrite reduction
denit <sub>3</sub>	Nitric oxide reduction
denit <sub>4</sub>	Nitrous oxide reduction
Mn(IV)	Mn(IV) reduction
Fe(III)	Fe(III) reduction
sulf	Sulfate reduction
nit <sub>1</sub>	Ammonium oxidation
nit <sub>2</sub>	Nitrite oxidation
Mn_O	Ferrous oxidized by oxygen
Fe_O	Bivalent manganese oxidized by oxygen



Fe_Mn	Bivalent manganese oxidized by Mn(IV)
S_O	Sulfide oxidized by oxygen
S_Mn	Sulfide oxidized by Mn(IV)
S_Fe	Sulfide oxidized by Fe(III)
FeS_O	Ferrous sulfide oxidized by oxygen

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Table A.2 Subscript symbols.

Subscript	Description
aer	Aerobic mineralization
anaer	Anaerobic mineralization
oxid	Heterotrophic carbon-oxidizing bacteria
denit	Heterotrophic denitrifying bacteria
Mn(IV)	Mn(IV)-reducing bacteria
Fe(III)	Fe(III)-reducing bacteria
sulf	Sulfate-reducing bacteria
nit <sub>1</sub>	Autotrophic ammonia-oxidizing bacteria
nit <sub>2</sub>	Autotrophic nitrite-oxidizing bacteria

3

#### 4 **Code availability**

5 The code for soil physical model is the built-in code in Hydrus-1D. The code for soil  
 6 biochemical and geophysical models is the programming code conducted in HP1 and Matlab. Any  
 7 requests for access to the code can be addressed to the corresponding authors.

#### 8 **Data availability**

9 The data for parameters, experiment, and case study is available in the supplement.

#### 10 **Author contributions**



1        XL built and calibrated the models, analyzed the simulated results, and wrote the paper. ZZ  
 2        provided the experimental data and instructed the experiment section writing. AF instructed model  
 3        development and revised the paper. All authors read and checked the paper.

## 4        **Competing interests**

5        The authors declare that they have no conflict of interest.

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 9        Cooperation Program (project number: 02WIL1451A/WT1601), German Federal Ministry of  
 10        Education and Research (BMBF), and Israeli Ministry of Science, Technology and Space (MOST).  
 11

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