Reply to reviewer #1

1. This paper proposes to take into account climate seasonality and water storage capacity into the Budyko framework. Two Budyko-type equations are used to mimic the water partitioning of the Ponce-Shetty model at a 4-month time interval. In comparison to the original Budyko-type equations where one 'n' parameter is required, it’s adding two parameters: a second 'n' parameter that is calibrated over all catchments, and the water storage capacity 'Sc' that is regionalized on each catchment using soil descriptors. Results highlight improved performances compared to the original Budyko-type equations. I only have a few suggestions for improvement.

Response:
Thank you very much for taking time to review our manuscript. All comments are greatly appreciated and considerably improve our manuscript. We have addressed all the comments carefully and will make revisions accordingly.

2. Despite the convincing improvement of the new formulation, it is rather unsatisfactory to advise calculating the water balance on a 4-month time scale. In the proposed formulation, no water storage/release can span over more than 4 months, and since the periods are fixed, each period is assumed to be independent (e.g. precipitation at the end of January is not influencing the hydrology of February). As recognized by the authors, for many catchments, such as catchments influenced by snow or large groundwater reservoirs, such assumption cannot be accepted. The authors solve this problem by removing these problematic catchments from their database, which I think is a good first step. However, the expression "generalised budyko equation" might be confusing, as the proposed formulation is somehow restricting the genericity of the Budyko framework. In addition, it would also have been more acceptable to relax the assumption behind the fixed periods by introducing sliding periods instead. Total annual evaporation could then have been estimated by summing the differences between two successive periods.

Response:
Thank you very much for your comments. To validate our assumption of the negligible carrying-over water storage between two consecutive time intervals, we conducted experiments A and B in an “abcd” model with monthly variations in soil moisture and groundwater (Thomas, 1981). Experiment A is used to represent natural conditions without any constraints on water storage. Mean monthly P and PE are used as input data, and parameters were calibrated by Zhang et al. (2020). After initialization for 10 years, simulated $E_a$ in experiment A shows a high correlation with observed $E_a$, indicating the capability of “abcd” model (Figure R1a). Experiment B is used to emulate our assumption, in which soil moisture and groundwater storage before each 4-month time interval are prescribed as average values for all months. Remaining parameters are the same as those in experiment A. Thus, hydrological processes between each 4-month time interval in experiment B are independent due to their identical antecedent water storage. High correlation of simulated $E_a$ between experiments A and B demonstrates that our assumption of zero carrying-over water storage will not result in notable errors (Figure R1b).

The assumption of zero carrying-over water storage is also supported by the mean annual timescale. Water storage anomaly at the end of a hydrological year needs to be equal to the value at the beginning to close the water budget. Thus, deficiencies in water storage from the 1st to the 2nd time interval need to be supplemented by excessive water storage in the remaining time intervals. As a
result, errors in evaporation due to varying water storage are likely to be closed at the mean annual timescale.

We removed some catchments with high snow fractions in this study, which may restrict the applications of our equations. However, these catchments are also viewed as outliers in the original Budyko framework (Berghuijs et al., 2020). Thus, the incorporation of climate seasonality and water storage capacity is not likely to limit its applications compared with original Budyko-type equations.

Figure R1 Validation of our assumption of zero carrying-over water storage from an “abcd” model. (a) Comparison between observed and simulated $E_a$ from experiment A. (b) Comparison of simulated $E_a$ between experiments A and B.

3. From this study, it is somewhat difficult to rank the shortcomings of the Budyko framework that this paper is addressing. Is it seasonality or water storage capacity that is the main weakness of the Budyko framework? It would be useful, I think, to disentangle the effect of these two factors with modeling that takes into account, or not, each one. Indeed, the uncertainty in the water storage capacity may suggest that the sensitivity of this parameter is lower. Since the authors say that the original Budyko framework is just a special case of their formulation, it would be useful to treat it in a stepwise fashion (also with the idea of keeping a parsimonious framework).

Response:
Thank you very much for your comments. We introduced the equations that only incorporate climate seasonality or water storage capacity. The generalized equation only accounting for climate seasonality is derived by assuming $S_c$ equals to $1e+7$, such that $S_c$ does not limit evaporation. Similarly, the equation only accounting for water storage capacity ($S_c$) is derived by assuming $P_j$ and $PE_j$ equal to one-third of $P_a$ and $PE_a$, indicating uniform distribution of P and PE throughout the year. Their performance shows that these two factors both contribute to the improvement in simulating $E_a$ over the original Budyko-type equations (Figure R2). In particular, climate seasonality plays a dominant role, which follows the higher correlation between climate seasonality index (SI) and errors in Budyko framework (Figure R2; Figure 3). Our results indicate that we may only consider climate seasonality if water storage capacity is unavailable from soil data.
Figure R2 Results from generalized Budyko equations that only consider climate seasonality (a, c) or water storage capacity (b, d). (a-b) Comparison between simulated and observed $E_a$. The bottom right corner is the $r^2$ for $E_a$. (c-d) Improvements in the absolute errors of simulated $E_a$ over original Budyko-type equation. Negative values indicate better performance and decreased absolute errors of our generalized equations.

4. The conclusion that the new formulation is a better model is based primarily on the catchments that are located on the west coast of the United States, while the eastern part appears to be a mix of improved and degraded performance. What characterizes these catchments? No physiographic or hydroclimatic descriptors are used to help us interpret under what conditions the new equations perform better or worse (or where the 4-months assumptions is more acceptable).

Response:
Thank you very much for your comments. We revised Figure 7, and calculated the improvement of our generalized equations as the difference in the absolute errors of simulated $E_a$. Catchments with degraded performance are mainly located in the lower Mississippi Valley, southern Atlantic coast states, and some catchments on western coasts. Three reasons are suggested to explain the degraded performance. First, these areas are covered by high fractions of vegetation (Addor et al., 2017), which is likely to increase water storage changes between time intervals. Assuming three independent time intervals may be inappropriate for these catchments. Second, these regions are characterized by high slopes (Addor et al., 2017), which challenge stable parameters ($n_1$ and $n_2$) that are calibrated over all catchments. Third, soil dataset includes non-negligible uncertainty due to limited depth and upscaling inadequacy, which may induce errors in our estimation (Miller and White, 1998). Thus, applying generalized Budyko-type equations deserves examinations on the 4-month assumption and reliable input data for $S_c$. Regression of two parameters ($n_1$ and $n_2$) on...
other catchment attributes is also suggested for future improvements.

5. The term "seasonality" may need to be better defined and compared to the literature on this topic. It is sometimes used to refer to the synchrony between P and PE regimes (when citing papers on this topic), and sometimes to describe the need to refine the time scale at which the water balance should be performed. I understand that both aspects are encompassed in the term "seasonality" but they are treated differently in the literature and in this paper (e.g. this work does not work well with asynchronous climate seasonality, which could be unexpected and confusing if we do not agree on what "incorporating seasonality" mean).

Response:
Thank you very much for your comments. Climate seasonality in this study refers to intra-annual distributions of climatic variables, including both in-phase and out-of-phase P and PE. Thus, the incorporation of climate seasonality in this study indicates that we need to consider the intra-annual distributions of climatic variables in simulating mean annual evaporation. We will revise and unify our definition of climate seasonality in this study.

6. 185-87: the fact that most of the hydrological response is observed within 4 months may not always mean that the travel time is less than 4 months (concept of celerity vs velocity)
Response:
We will remove the expression about travel time.

7. Table 1: why not using E, PE and P for Budyko-type equation instead of X, Xmax and Z?
Response:
We will use E, PE, and P for Budyko-type equation.

8. Table 1: Yang-Fu expression E -> Ea
Response:
This will be corrected.

9. 1264: include -> includes
Response:
This will be corrected.

10. 1336: descrese -> decreases
Response:
This will be corrected.

11. Line 410: it would be helpful to explain why does a 12-month water balance (and so without seasonality taken into account) do not tend to give similar performance to the original Budyko framework (in relation to my second main comment)
Response:
The generalized Budyko-type equation is equivalent to the original Budyko framework when 1) P and PE are uniformly distributed throughout the year and 2) water storage capacity is infinite. The 12-month equation in Figure 9 fulfills the first requirement, while still incorporates water storage
capacity. Thus, its performance is different from the original Budyko equations. If we further assume water storage capacity ($S_c$) equals to $1e+7$, both climate seasonality and water storage capacity will not be incorporated in our equations (please refer to reply to comment #3). Consequently, our generalized equations are simplified from the original equations, and the $r^2$ for $E_a$ is 0.616, the same as the original Budyko equations.

Reference: