Response to comments of Anonymous Referee 1

We would like to thank the referee for the valuable comments and suggestions, which improved the quality of the paper. Below is our response to the comments and suggestions.

Comment on hess-2022-298

I have tried to read this paper multiple times now and every time find myself frustrated. I am highly litterate in terms of mathematically dense papers, but I found this paper next to impossible to make my way through. I do not usually write grumpy reviews, but this will be one. I have three major concerns that lead me to recommend that this paper be rejected.

(1) My first and likely biggest issue is going from equation (1) to (2). Any time you average and ADE equation like the one the authors have you will have a mean and fluctuation of the things that vary. In this case concentration, velocity and depending on the nature of the dispersion coefficient that also. Where are all of these gone? They don't just disappear as it seems that they do in equation (2) - i.e. it's fine to say that the average of the fluctuation of concentration is zero, but the average of the product of concentration and velocity fluctuations is not. Indeed this is exactly what leads to things like macrodispersion and deviations from standard behaviors. Where have these gone here? There is no discussion of them and none of the assumptions I see in the problem setup suggest they do not exist or are negligible. This is the starting point of the paper and frankly makes me feel like the authors are departing from a faulty point from the getgo.

Response

a. The derivation of Eq. (1) to Eq. (2) was presented in Holly (1975). Equation (2) also appears in the textbook by Fischer et al. (1979). In addition, Eq. (2) has been widely used to analyze problems related to solute transport by fluid flow (e.g., Zerihun et al. 2005, Baek et al. 2006, Chavez et al. 2014).

Zerihun, D., Furman, A., Warrick, A. W., and Sanchez, C. A.: Coupled surface-subsurface solute transport model for irrigation borders and basins. I. Model development, J. Irrig. Drain. Eng., ASCE 131(5), 396-406, 2005.

Baek, K. O., Seo, I. W., and Jeong, S. J.: Evaluation of dispersion coefficients in meandering channels from transient tracer tests. J. Hydraul. Eng., ASCE 132

(10), 1021-1032, 2006.

Chavez, C., Fuentes, C., Brambila, F., and Castañeda, A.: Numerical solution of the advection-dispersion equation: Application to the agricultural drainage, J. Agric. Sci. Technol., 16(6), 1357-1388, 2014.

b. We apologize for not mentioning Holly's (1975) idea in developing Eq. (2) regarding the average of the product of concentration and velocity fluctuations. Holly (1975) considered the mixing of the contaminant plume over depth in natural channels to be complete, so that the fluctuations around the depth-averaged concentration are relatively small. Then the average of the product of concentration and velocity fluctuations can be considered to be absorbed into the gradient transport terms in Eq. (2).

A note is added to the manuscript mentioning this as follows (Line 446 on page 26):

"In developing Eq. (A2), it is assumed that the contaminant plume in confined aquifers is well mixed over depth, so that variations around the depth-averaged concentration are relatively small (Holly, 1975). Then the average of the product of concentration and velocity fluctuations can be assumed to be absorbed in the gradient transport terms in Eq. (A2)"

c. It can be clearly seen (or verified) that Eq. (2) for flow in aquifers of uniform thickness (i.e., $B(x_1,x_2)$ = constant) reduces to the traditional two-dimensional advection-dispersion equation for solute transport in confined aquifers, with the flow fields characterized by the aquifer transmissivity fields instead of the hydraulic conductivity fields.

(2) As I noted I am someone who writes and reads a lot of papers with pretty dense and complex mathematics in it, but I found a lot of what the authors present extremely hard to follow, where in some places there is abundant detail and in others serious gaps.

Response

The structure of the manuscript was fundamentally changed to make it clear and readable.

a. A brief preview of this work is added on page 5 (Line 74) as

"In the present work, the convection velocity of solute particles is first developed based on the relationship between the two-dimensional depth-averaged solute mass conservation equation and the Fokker-Planck equation, so that the convection velocity can explicitly reflect the effects of hydraulic conductivity and aquifer thickness. Using the perturbation approach to solute convection velocity, the covariance function of solute convection velocity is then developed, which allows a general expression for the variance of the displacement of a solute particle in the mean flow direction to be developed. A closed-form expression for the solute displacement variance is also developed for the case where solute transport is dominated by advection and the random fields of log conductivity and log thickness of the confined aquifer are second-order stationary. Finally, the influence of variations in log hydraulic conductivity and log aquifer thickness on the variability of solution displacement is analyzed."

b. To facilitate understanding, we have restructured the manuscript so that the main text of the manuscript focuses on the step-by-step development of the variance of the solute displacement, while many details of the mathematical derivations related to the flow fields have been moved to Appendices A and B, such as the detailed solute convection velocity derivation and the cross-covariance and covariance functions of the flow velocity fields. **For details, please see the revised manuscript.**

c. Further insight is provided to better understand the idea behind Figures 1-2 (Line 280 on page 17):

"When taking samples from a field, one obtains a histogram from which a certain value of the variance can always be calculated. However, for many phenomena, the experimental variance is actually a function of the field. In particular, it increases as the field increases, i.e., many phenomena have an almost unlimited capacity of dispersion and cannot be adequately described by ascribing to them a finite a priori variance. In this case, the use of the semivariogram is an appropriate way to measure the variability of the variation."

(3) Last but not least, even if everything is right (which I cannot verify) I struggle to see the real importance of this paper and thus am hesitant to see it published in such a high level journal such as HESS which is one of the top journals in our field. Much of the paper feels a little archaic in nature and while I love theoretical papers with full mathematics I also feel that something clear should be gained by ellaborating it and I just do not see that here.

Response

a. To make it clear that this is a new and original work and the results of this work are important, the novelty of this study is added in the **Introduction** section (Line 59 on page 5) as follows:

“The traditional approach to regional groundwater flow problems introduces the transmissivity parameter to describe the ability of a confined aquifer to transmit water throughout its saturated thickness. The effect of the thickness of the aquifer is implicitly reflected in the transmissivity parameter. It is very difficult to assess the effect of thickness on the flow field and thus on solute transport at a regional scale. The stochastic approach presented here provides an efficient and rational way to analyze flow and solute transport fields affected by the non-uniform thickness of confined aquifers, which has not been previously presented in the literature. This work shows that variability in aquifer thickness can lead to nonstationarity in hydraulic head fields and thus to nonstationary flow velocity fields and anomalous longitudinal dispersion. This implies that neglecting the variability of aquifer thickness when predicting the longitudinal displacement of solutes at large times can lead to a significant underestimation of longitudinal dispersion. The stochastic theory presented here improves quantification of the variance of the solute displacement in natural confined aquifers of random thickness fields.”

b. To our knowledge, the analysis of the influence of the variability of the thickness of the aquifer on the longitudinal displacement of the solute within the framework of stochastics has not yet been presented in the literature.

We believe that the manuscript is of the quality required for publication and should be of interest to many readers in *Hydrology and Earth System Science*.