



## Revisiting the Hydrological Basis of the Budyko Framework With the Hydrologically Similar Groups Principle

Yuchan Chen<sup>1</sup>, Xiuzhi Chen<sup>1</sup>, Meimei Xue<sup>1</sup>, Chuanxun Yang<sup>2,3</sup>, Wei Zheng<sup>1</sup>, Jun Cao<sup>4</sup>,  
5 Wenting Yan<sup>1</sup>

<sup>1</sup>Guangdong Province Key Laboratory for Climate Change and Natural Disaster Studies, School of Atmospheric Sciences, Sun Yat-sen University, Zhuhai, 519082, China;

<sup>2</sup>Guangzhou Institute of Geochemistry, Chinese Academy of Sciences, Guangzhou, 510640, China;

<sup>3</sup>University of Chinese Academy of Sciences, Beijing, 100049, China;

10 <sup>4</sup>Guangdong provincial Academy of Environmental Science, Guangzhou, 510635, China;

*Correspondence to: Xiuzhi Chen (chenxzh73@mail.sysu.edu.cn)*

**Abstract.** The Budyko framework is a simple but effective tool for watershed water balance estimation. Accurate estimation of the watershed characteristic parameter ( $P_w$ ) is critical to accurate water balance simulations using the Budyko framework. However, there is no universal quantification criterion for the  
15  $P_w$  because of the complex interactions between hydrologic, climatic, and watershed characteristic factors at global scales. Therefore, this research introduced the hydrologically similar groups principle into the Budyko framework and defined the criteria for quantifying  $P_w$  in similar environments. We classified **global watersheds** into six groups based on watershed attributes, including climate, soil moisture, and vegetation, and identified the controlling factors of the  $P_w$  in each hydrologically similar  
20 group. Our results show that the  $P_w$  is closely related to soil moisture (SM) and the power function gradually changes from positive to negative as soil moisture increases. The relationship between the  $P_w$  and fractional vegetation cover (FVC) can be described with different linear equations in different hydrologic similarity groups, except in the group with no strong seasonality and moist soils. Based on these relationships, a model for estimating the  $P_w$  ( $P_wM$ ) was established with multiple non-linear  
25 regression methods between the  $P_w$  and its controlling factors (SM and FVC). Then, we used bootstrapping and runoff reconstruction methods to verify the usability of  $P_wM$ . The validation results illustrate that  $P_wM$  overall presents a satisfactory performance through bootstrapping ( $R^2 = 0.63$ ) and runoff reconstruction ( $R^2 = 0.89$ ). Results show that the hydrologically similar groups method can quantify the  $P_w$  and the improved Budyko framework can aptly simulate global runoff, especially in



30 humid watersheds. This study lays the basis for explaining the  $P_w$  in the Budyko framework and  
improves the applicability of the Budyko framework for estimating global runoff.

## 1 Introduction

There has been an increasing interest in estimating the water balance with the Budyko framework  
(Budyko, 1974) because it is a simple and effective tool, unlike process-based models, which typically  
35 require a large number of parameters (Caracciolo et al., 2018; Lei et al., 2014). The Budyko framework  
has been used for assessing linkages and feedbacks between climate forcing and land surface  
characteristics on water and energy cycles (Zhang et al., 2001; Milly and Shmakin, 2002; Li et al., 2013;  
Xu et al., 2013), prompting a great deal of empirical, theoretical, and process-based studies (Chen and  
Sivapalan, 2020; Roderick and Farquhar, 2011; Rau et al., 2018; Goswami and Goyal, 2022). The  
40 Budyko framework is a top-down approach relating a catchment's long-term evaporative ratio (ratio  
between actual evapotranspiration and precipitation) to its aridity index (ratio between potential  
evapotranspiration and precipitation) and is rooted on a firm physical basis (Vora and Singh, 2021;  
Sivapalan, 2003; Wang and Tang, 2014).

The original Budyko equation assumes that evapotranspiration is mainly controlled by precipitation  
45 (representing the availability of water) and potential evapotranspiration (representing the availability of  
energy) (Budyko, 1974; Wang et al., 2022). Despite its solid performance, the original Budyko equation  
still produces a bias between modeled and measured evapotranspiration or runoff because it does not  
consider the effects of watershed characteristics other than climatic conditions on water balance (Kim  
and Chun, 2021; Zhang et al., 2001). As a result, hydrologists have invested considerable efforts to  
50 improve model performance by introducing parameters related to watershed characteristics into the  
original Budyko equation. Some of the introduced parametric equations include the Fu (Fu, 1981), Zhang  
(Zhang et al., 2001), Choudhury-Yang (Yang et al., 2008), and Wang-Tang equations (Wang and Tang,  
2014).

**Table 1.** Parametric Budyko-type formulations

Reference	Formulation
Budyko (1974)	$\frac{ET}{P} = \left[ \frac{ET_0}{P} \tanh \left( \frac{ET_0}{P} \right)^{-1} \left( 1 - \exp\left(-\frac{ET_0}{P}\right) \right) \right]^{0.5}$



$$\text{Fu (1981)} \quad \frac{ET}{P} = 1 + \frac{ET_0}{P} - \left[1 + \left(\frac{ET_0}{P}\right)^w\right]^{\frac{1}{w}}$$

$$\text{Zhang et al. (2001)} \quad \frac{ET}{P} = \frac{1 + w \frac{ET_0}{P}}{1 + w \frac{ET_0}{P} + \left(\frac{ET_0}{P}\right)^{-1}}$$

$$\text{Yang et al. (2008)} \quad \frac{ET}{P} = \frac{1}{\left[1 + \left(\frac{P}{ET_0}\right)^n\right]^{\frac{1}{n}}}$$

$$\text{Wang and Tang (2014)} \quad \frac{ET}{P} = \frac{1 + \frac{ET_0}{P} - \sqrt{\left(1 + \frac{ET_0}{P}\right)^2 - 4\varepsilon(2 - \varepsilon)} \frac{ET_0}{P}}{2\varepsilon(2 - \varepsilon)}$$

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These parametric equations have somewhat improved the estimation performance by taking into account the influence of watershed characteristics and thus have better estimation performance (Fu, 1981; Liu and Liang, 2015; Guan et al., 2022; Yang et al., 2008). Along with the widely used parametric equations, there has been a growing importance placed on research on the watershed characteristic parameter (Pw) as its accurate estimation is a prerequisite for the accurate simulation of evapotranspiration or runoff using the Budyko framework (Wang et al., 2022; Yao et al., 2017; Guo et al., 2019; Yu et al., 2021). Although introducing Pw improved the Budyko-type model performance, most studies failed to give a specific criterion for quantifying its value. While there is agreement that the Pw represents the integrated effects of various environmental factors (Wang et al., 2022; Liu et al., 2022; Yu et al., 2021; Gan et al., 2021), opinions differ as to what factors and effects should relate to the Pw. For instance, whether the Pw within the Budyko framework is controlled by watershed vegetation has been much debated. Some researchers advocated that vegetation plays a crucial role in the Pw, holding that there is a positive linear relationship between vegetation and the Pw (Ning et al., 2017; Zhang et al., 2018; Zhang et al., 2001). Other scholars have argued against vegetation having a strong correlation with the Pw, suggesting that most regions or some special watersheds show no significant correlation between vegetation indices and Pw (Liu et al., 2021; Li et al., 2013). Although many studies have researched the relationship between the Pw and various watershed characteristics factors, they have shown contradictory results.

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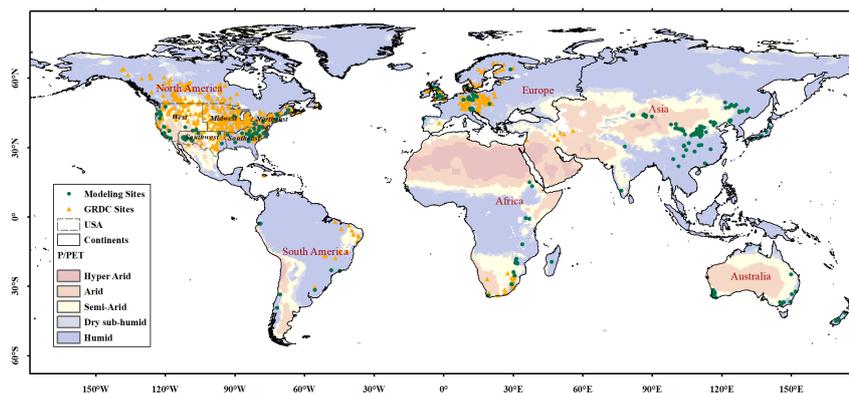
In fact, the relationships and interactions among hydrologic, climatic, and watershed characteristic  
75 factors are complicated by the great heterogeneity across space (Gao et al., 2018; Gan et al., 2021).  
Numerous studies have shown that the roles of climate and watershed characteristic factors on  
hydrological characteristics vary in different climatic regions (Li and Sivapalan, 2014; Trancoso et al.,  
2017; Singh et al., 2014). Therefore, classifying watersheds into hydrologically similar groups is essential  
80 for exploring the effect of watershed characteristics on hydrology and interpreting the physical meaning  
of the Pw within the Budyko framework. However, to date, relatively little research has been conducted  
on classifying watersheds based on the highly variable climate-Pw relationships in the Budyko  
framework. This may be an important reason for the contradictory research results on the Pw.

The purpose of this study was to investigate what factors and effects relate to the Pw based on the  
classification of hydrologically similar groups within the Budyko framework and develop a model for  
85 estimating the Pw (PwM) to simulate global runoff. We collected 726 hydrological data from globally  
published datasets and classified these watersheds into hydrologically homogenous regions applying the  
Decision Tree Regressor to measured watershed attributes. Then, we identified the controlling factors of  
the Pw from various environmental factors in each hydrologically similar group. Based on the  
relationship between the Pw and its controlling factors, the PwM was set up by multiple non-linear  
90 regression methods. This study highlights the need to account for the interactions among hydrologic,  
climatic, and watershed characteristic factors for explaining the Pw in the Budyko framework.

## 2 Data

### 2.1 Modeling data

Global hydrological data, including runoff (R) and corresponding precipitation (PRE), were  
95 collected from globally published datasets (726 samples listed in Supplementary Data 1, Fig. 1). Potential  
evapotranspiration (PET) data were downloaded from version 4.05 of the CRU TS (Climatic Research  
Unit gridded Time Series) climate dataset (<https://doi.org/10.6084/m9.figshare.11980500>), which is  
produced by the CRU at the University of East Anglia. For consistency, we used PET values extracted  
100 from the CRU TS dataset of all watersheds listed in Supplementary Data 1, even for studies with PET  
values reported.



**Figure 1.** Location of the observation sites for modeling (green dots) ( $n = 726$ ) and the GRDC (Global Runoff Data Centre) sites (orange triangles) ( $n = 545$ ) for validation. Background colors represent P/PET.

105 The datasets of other watershed characteristic factors were extracted from remote sensing data. All datasets were aggregated at the same spatial resolution (0.5 degrees). The sources of datasets are summarized in Table 2.

**Table 2.** Data sources for watershed characteristic factors

Parameter	Full name	Data source/version	Spatial/temporal resolution	Reference
SM	Soil moisture 15cm	GLDAS Noah Land Surface Model L4	0.5°/monthly	Rodell et al. (2004)
FVC	Fractional vegetation cover	GLASS FVC V4	0.5°/monthly	Liang et al. (2021)
SI	Seasonality Index	CRU TS dataset version 4.03	0.5°/ multi-year average	Walsh and Lawler (1981);Feng (2019)

## 2.2 Validation data

110 Observed river discharge data for validation were obtained from the Global Runoff Data Centre (GRDC, [https://www.bafg.de/GRDC/EN/02\\_srvcs/21\\_tmsrs/riverdischarge\\_node.html](https://www.bafg.de/GRDC/EN/02_srvcs/21_tmsrs/riverdischarge_node.html)). The PET and PRE values corresponding to selected sites of GRDC were extracted from remote sensing data. PET values were extracted from the CRU TS dataset. PRE values were extracted from Global Precipitation Climatology Centre (GPCC) Precipitation Total Full V2018 (0.5×0.5) data provided by the  
 115 NOAA/OAR/ESRL PSL, Boulder, Colorado, USA (<https://psl.noaa.gov/data/gridded/data.gpcc.html>).



### 3 Methods

#### 3.1 Budyko framework

This study employed the new Fu's formula (Zhou et al., 2015), a Budyko-type equation derived from Fu's equation, to analyze Pw in the Budyko framework. Within the new Fu's model, the ratio (R/P) of annual water yield (R) to precipitation (P) is determined by two variables: an aridity index (precipitation/potential evapotranspiration; P/PET), and Pw ( $m$ ). The formula is expressed as:

$$\frac{R}{P} = \left( 1 + \left( \frac{P}{PET} \right)^{-m} \right)^{\frac{1}{m}} - \left( \frac{P}{PET} \right)^{-1} \quad (1)$$

where  $m$  is a dimensionless integration constant varying between 1 and infinity.

Based on the randomly selected 726 samples from global hydrological studies, we derived the Pw ( $m$ ) values for each sample.

#### 3.2 Classification of watersheds into hydrologically similar groups using watershed attributes

A hydrologically similar group (hydrologically homogeneous region) is defined as a group of drainage basins whose hydrologic responses are similar (Kanishka and Eldho, 2020). Therefore, the relationship between Pw and a variable does not change substantially in a hydrologically similar group. However, when that relationship between Pw and the variable changes as certain boundaries are crossed, the corresponding watersheds are divided into different groups by these boundaries.

Three watershed characteristic variables — soil moisture (SM), rainfall seasonality index (SI), and fractional vegetation cover (FVC) — were selected for classification. For SM and FVC, the bounded intervals of the variables were given by the Decision Tree Regressor (DTR). The locations of splits in DTR were used as dividing intervals. The Scikit-learn library (Pedregosa et al., 2011) in Python provides the DTR used in this study. Based on Walsh and Lawler (1981), we divided the SI into three parts ( $SI \leq 0.4$ ,  $0.4 < SI \leq 0.8$ ,  $SI > 0.8$ ) to represent three hydroclimatic seasonalities (precipitation spread throughout the year, marked seasonality with a short drier season, extreme seasonality with a long drier season).

Six hydrologically similar groups are detailed in Table 3.

**Table 3.** Classification of watersheds

Soil moisture classifier	Water soil regime	Seasonality index classifier	Seasonality precipitation regime	Fractional vegetation cover classifier	vegetation cover regime	Name of the group
$SM \leq 20$	Dry soil	—	—	—	—	IN <sub>D</sub>



		SI ≤ 0.4	Seasonless	—	—	IN <sub>WP</sub>
SM > 20	Wet soil	0.4 < SI ≤ 0.8	Marked seasonality	FVC ≤ 0.2	Low density	IN <sub>WMS</sub>
				0.2 < FVC ≤ 0.5	Middle density	IN <sub>WMM</sub>
		FVC > 0.5	High density	IN <sub>WML</sub>		
		SI > 0.8	Extreme seasonality	—		IN <sub>WE</sub>

### 3.3 Setup of PwM

We performed regression analysis between the Pw and watershed characteristic variables to determine the PwM. The variables whose R<sup>2</sup> of the regression model was greater than 0.1 were selected as input variables. Then we used a polynomial as the basic model form. Each term of the polynomial depends on the regression model of the corresponding variable and the Pw. For each hydrological group, the PwM is modeled as a function as:

$$m = \sum \beta_i \times f(x_i) \quad (2)$$

where  $m$  represents the value of the Pw;  $x_i$  represents the input variables;  $f$  corresponds to the function derived from the regression of  $m$  on  $x_i$ ;  $\beta_i$  represents the empirical coefficient fitted by multiple non-linear regression (MNR).

### 3.4 Model validation

#### 3.4.1 Performance metrics

Three performance metrics were used to assess the accuracy of the PwM. The term  $N$  is the number of observations,  $i$  is the  $i^{\text{th}}$  value to be simulated, and  $y_s$  and  $y_o$  are the simulated and observed series, respectively.

The relative bias (RelBIAS) represents systematic errors. A positive (negative) value indicates a general overestimation (underestimation), and the perfect agreement is achieved when RelBIAS is equal to zero. RelBIAS is defined as:

$$RelBIAS = \frac{mean(y_s - y_o)}{mean(y_o)} \quad (3)$$

The coefficient of determination (R<sup>2</sup>) assesses how strong the linear relationship is between the simulated and the observed series. It is represented as a value between 0.0 and 1.0. The optimal value is 1 and indicates a perfect fit. It is defined as:

$$R^2 = \left\{ \frac{\sum_{i=1}^N (y_o^i - \bar{y}_o)(y_s^i - \bar{y}_s)}{[\sum_{i=1}^N (y_o^i - \bar{y}_o)^2]^{0.5} [\sum_{i=1}^N (y_s^i - \bar{y}_s)^2]^{0.5}} \right\} \quad (4)$$



The Nash–Sutcliffe efficiency (NSE) (Nash and Sutcliffe, 1970), a goodness-of-fit index, is usually  
165 used to assess the accuracy of the model. When  $NSE = 1$ , the model predictions perfectly match the  
observed data. A value lower than 0 indicates that the observed mean is a better predictor than the model.  
It is defined as:

$$NSE = 1 - \frac{\sum_{i=1}^N (y_s^i - y_o^i)^2}{\sum_{i=1}^N (y_o^i - \bar{y}_o)^2} \quad (5)$$

### 3.4.2 Bootstrapping validation

170 The available data were split into training and test sets for the purpose of bootstrapping validation.  
A subset of 60% of the data was randomly selected without replacement for training PwM. The trained  
PwM was used to estimate the remaining 40% of the runoff data set, and then the performance metrics  
were used to evaluate the difference between the estimated and observed values. The process was  
repeated randomly 10000 times. We documented the model skill for each validation and showed them in  
175 a violin plot.

### 3.4.3 Runoff reconstruction validation

#### (1) The runoff reconstruction by using the PwM

To assess the accuracy of the PwM, runoff reconstructions were generated using the Budyko  
framework in which the value of Pw is derived from the PwM simulation.

#### 180 (2) Selection of GRDC stations and conversion of flow volumes to runoff rates

To evaluate the estimates of runoff reconstructed by the PwM, only the GRDC stations meeting the  
following criteria were selected for further analysis.

1) The time series has observations within the period 2000–2016 (when corresponding SM, FVC,  
and SI were available).  
185 2) The drainage area reports can be found in the original data. This criterion is designed to provide  
area parameters for converting original flow volumes to runoff rates.

3) The geographical coordinates reports can be found in the original data and the shape of the  
drainage area can be found in the GRDC Watershed Boundaries (2011). This choice was made to retrieve  
the geographic location of the station and then extract the corresponding required values from remote  
190 sensing data.



4) Time series with unrealistic runoff rates are removed. It is generally agreed that in the Budyko framework, runoff is maximum (minimum) when  $m = 1$  (10). Observations out of range are considered unrealistic. This criterion has been adopted to eliminate observations that are physically extremely unlikely.

195 Based on these criteria, 545 GRDC stations were selected for validation (Fig. 1).

Then, the flow volumes of selected sites were converted to runoff rates. The average year of catchment runoff can equal the annual streamflow measured at the outlet divided by the watershed area, provided other water losses are minimal (Ghiggi et al., 2019). Thus, runoff rates are obtained as:

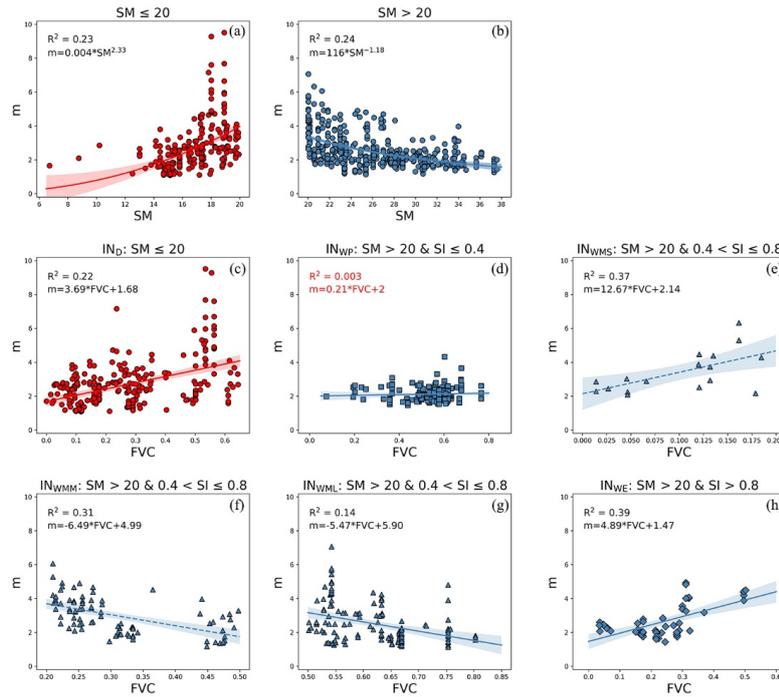
$$R_{(GRDC)} = \frac{Discharge_{(GRDC)}}{Area_{(GRDC)}} \times \frac{1}{1000} \quad (6)$$

200 where  $R_{(GRDC)}$  is the GRDC annual runoff rate ( $\text{mm yr}^{-1}$ );  $Discharge_{(GRDC)}$  is the GRDC annual flow volume ( $\text{m}^3 \text{ yr}^{-1}$ );  $Area_{(GRDC)}$  is the drainage area ( $\text{km}^2$ ); 1000 is the conversion factor.

## 4 Results

### 4.1 Model for estimating Pw

205 Figure 2 shows the results of the regression between  $m$  and watershed characteristic variables for the studied watersheds within new Fu's formula and helps assess the relationship between the Pw and watershed characteristic variables.



210 **Figure 2.** Regression between  $m$  with (a-b) SM (soil moisture) and (c-h) FVC (fractional vegetation cover). Symbol colors represent dry (red) and wet (blue) soil moisture. Symbol shapes indicate seasonless (square), marked seasonality (triangle), and extreme seasonality (diamond). The equation in red indicates that the input parameter is rejected in the corresponding group. The groups are defined in Table 3.

We found that the relationship between  $m$  and SM shows a positive power function for SM values from 0 to 20 (Fig. 2a), while there is a negative power function with SM values from 20 to 100 (Fig. 2b). The relationship between  $m$  and FVC shows different situations in different groups (Fig. 2c-h). The relationship between  $m$  and FVC can be described as a positive linear equation in the  $IN_D$  group, the  $IN_{WSS}$  group, and the  $IN_{WE}$  group. The relationship can be described as a negative linear equation in the  $IN_{WMM}$  group and the  $IN_{WML}$  group. However, in the  $IN_{WP}$  group, the relationship between  $m$  and FVC is not significant. Therefore, FVC was rejected as the input variable in the  $IN_{WP}$  group.

Finally, the developed PwM is given by:

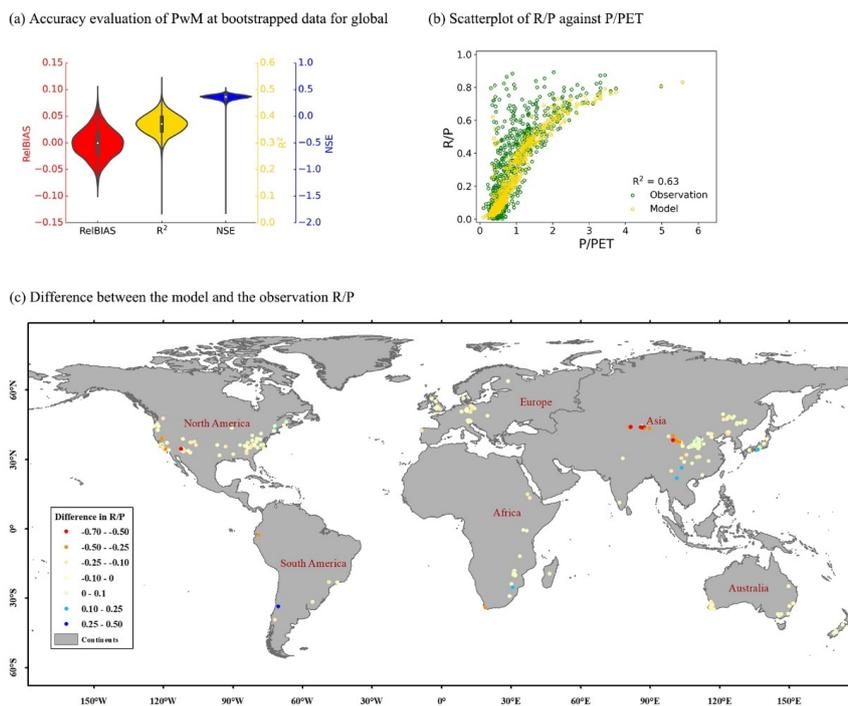
$$220 \quad m = \begin{cases} 0.91 \times SM^{0.38} + 1.48 \times FVC & (IN_D, SM \leq 20) \\ 28.72 \times SM^{-0.76} & (IN_{WP}, SM > 20, SI \leq 0.4) \\ 39.03 \times SM^{-0.96} + 11.82 \times FVC & (IN_{WMS}, SM > 20, 0.4 < SI \leq 0.8, FVC \leq 0.2) \\ 33.76 \times SM^{-0.71} - 1.47 \times FVC & (IN_{WMM}, SM > 20, 0.4 < SI \leq 0.8, 0.2 < FVC \leq 0.5) \\ 20.41 \times SM^{-0.42} - 4.221 \times FVC & (IN_{WML}, SM > 20, 0.4 < SI \leq 0.8, FVC > 0.5) \\ 3078 \times SM^{-2.43} + 3.53 \times FVC & (IN_{WE}, SM > 20, SI > 0.8) \end{cases}$$



where  $m$  is the value of Pw;  $SM$  is soil moisture ( $\text{kg m}^{-2}$ );  $FVC$  is fractional vegetation cover ( $\text{m}^2 \text{m}^{-2}$ ).

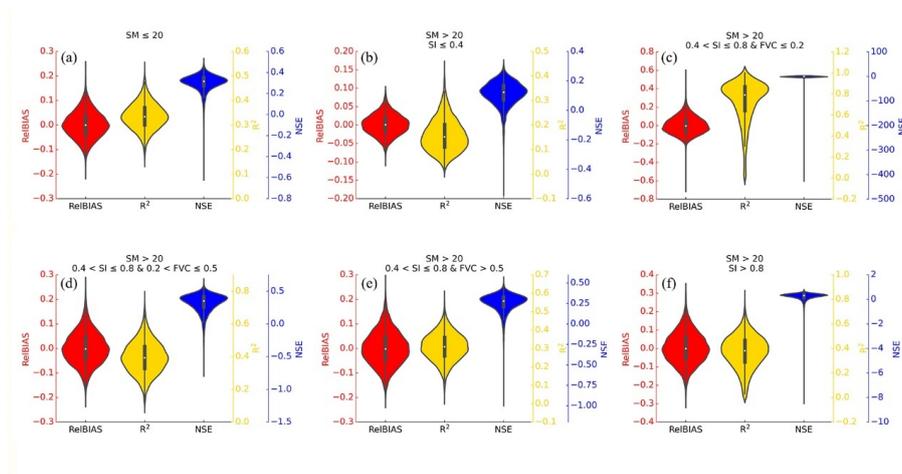
#### 4.2 Model validation

Figure 3 helps evaluate the performance of PwM by showing the results of the global bootstrapping validation. Overall, the PwM performs well, as indicated by satisfactory skill scores (Fig. 3a). On average, the ensemble RelBIAS of the  $m$  simulated by the model is slightly negative, indicating a weak tendency to underestimate the values of Pw, but the maximum relative bias is less than 0.1. The interquartile range of  $R^2$  for the PwM is from 0.35 to 0.40, with a median of 0.37. The scores of  $R^2$  are higher than 0.3 in more than 95% of the global bootstrapping events. The global NSE skill scores show that in most bootstrapping events, the estimation error estimated variance for the PwM is less than the variance of the observations ( $\text{NSE} > 0$ ), with the interquartile range from 0.33 to 0.39. Figure 3b compares the published R/P observations against those simulated by the PwM. The  $R^2$  between the observed and the simulated values is higher than 0.63. The model performs well in arid and semi-arid regions. The main underestimated regions are the dry sub-humid regions and humid regions with Aridity Index values less than 1. In terms of the distribution of simulated and observed differences (Fig. 3c), the global R/P simulations are dominated by weak underestimations, of which larger underestimations occurred in western America and northwest China.



**Figure 3.** Global accuracy evaluation of the PwM. (a) Violin plot of skill scores for global bootstrapping. A violin represents the distribution of the considered skill scores of the bootstrapping validation. The white dot on the violin plot represents the median. The black bar in the center of the violin represents the interquartile range. Colors distinguish three performance metrics: Red (RelBIAS), yellow ( $R^2$ ) and blue (NSE). (b) Scatter plots between the R/P simulated by PwM and P/PET (yellow) and those from published data and P/PET (green). (c) Difference between the R/P values from the PwM and the published observations.

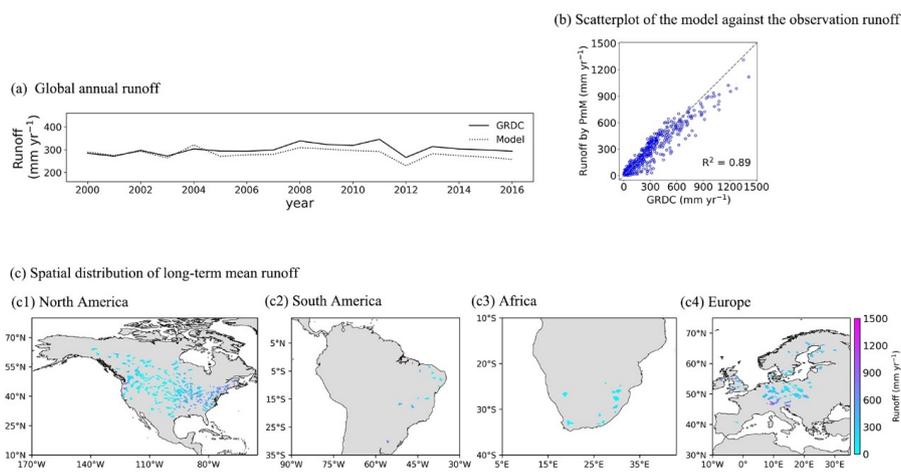
The skill scores of six intervals (Fig. 4) show more variability. Though the overall RelBIAS of the PwM is negative, the PwM tends to overestimate values of Pw in the  $IN_{WP}$  group (the median of RelBIAS is positive).  $R^2$  scores vary widely between groups. The  $IN_{WMS}$  group scores highest in  $R^2$ , with a median of 0.73, and the lowest in the  $IN_{WP}$  group with a median of 0.16. The grouped NSE scores show more uncertainty than the overall, especially in the  $IN_{WMS}$ , although the value of the lower adjacent larger than zero indicates more skill than the mean of observations, and the outliers are far below zero. The low NSE value may be due to the low number of watersheds sampled in this interval, which increased the inconclusive results.



**Figure 4.** Accuracy evaluation of PwM at bootstrapped works for (a) IN<sub>D</sub>, (b) IN<sub>WP</sub>, (c) IN<sub>WMS</sub>, (d) IN<sub>WMM</sub>, (e) IN<sub>WML</sub>, and (f) IN<sub>WE</sub>.

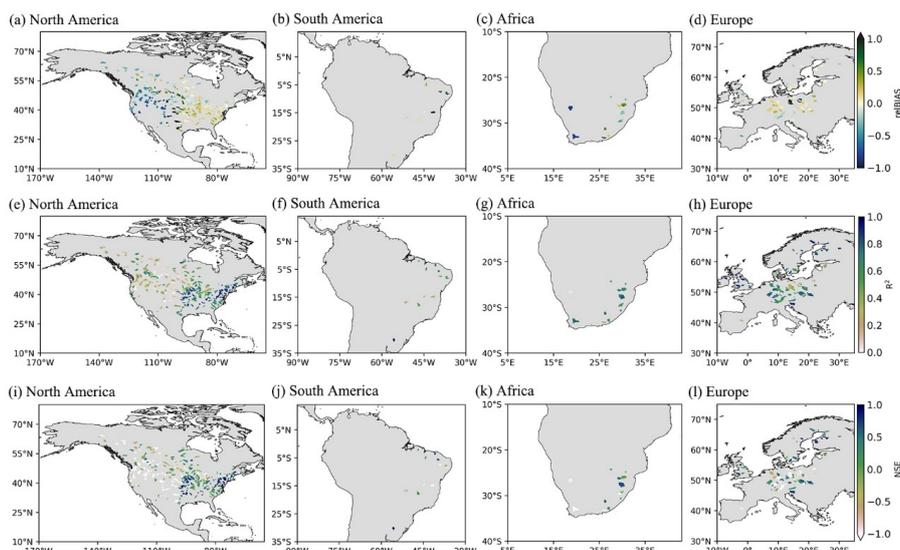
#### 255 4.3 Runoff reconstruction validation

The runoff reconstruction results are shown in Fig. 5. The global annual runoff estimated by the PwM ranges from 229.84 to 320.34 mm, which is slightly lower than the observed range of GRDC (265.82 ~ 345.50 mm yr<sup>-1</sup>) (Fig. 5a). Overall, the temporal evolution of runoff is captured well in the period 2000-2010. However, since 2011, the consistency between reconstructed runoff and GRDC runoff has decreased, and the reconstruction results are constantly lower than the GRDC observations. Influenced by the underestimations in 2011-2016, the reconstructed global long-term mean runoff also shows a slight underestimation (Fig. 5b). The spatial patterns of long-term mean runoff are shown in Fig. 5c. The global estimated runoff shows lower values in the west of the United States and south of Africa, and higher values in the northeastern United States and the European Mediterranean area. Overall, the reconstructed spatial patterns are compatible with other reported findings (Ghiggi et al., 2019).



**Figure 5.** Runoff reconstruction results based on selected GRDC stations.

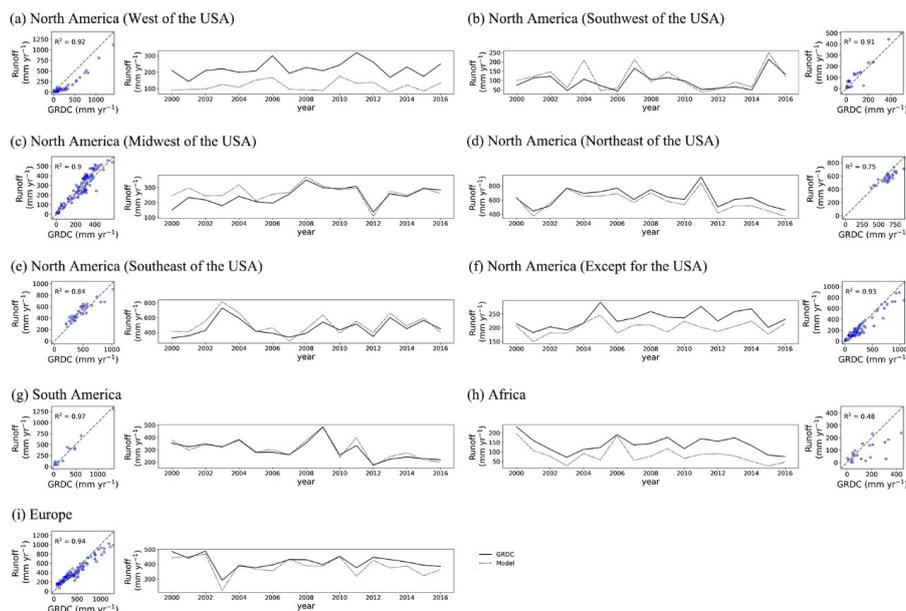
Figure 6 displays the considered skill scores of the reconstructed runoff obtained from each  
270 watershed of the selected GRDC ensemble from 2000-2016. It can be seen that the result of  
reconstruction with PwM, in general, is satisfactory, as indicated by the RelBIAS close to 0. The main  
area of underestimation is in the high mountains of the western United States. In the lower part of the  
runoff rate distribution, the runoff tends to be underestimated. Humid regions such as the northeastern  
United States and the European Mediterranean area have quite high  $R^2$  values, while lower values are  
275 observed in semi-arid and dry sub-humid regions, which are mainly found in the western and midwestern  
United States. The low NSE scores tend to correspond to the watersheds where runoff is unusually under  
or over-estimated. Especially in the western United States, the model performance indicated by NSE  
decreases when runoff is underestimated.



280 **Figure 6.** Spatial distribution of the skill scores of the reconstructed runoff.

We divided the world into nine geographic regions (Fig. 1) to further evaluate the performance of PwM on a global scale. Figure 7 shows the observational agreement of runoff time series and long-term mean for nine geographic regions. The temporal evolution of runoff is, in general, well captured, except in the western United States, where runoff was consistently underestimated. In addition, the runoff estimated by PwM is underestimated in 2011 to a greater extent than in other years. The regions where runoff was underestimated include the western United States and high latitudes in North America, and the runoff underestimation is more severe in the arid western United States than in the relatively wet northwest of North America. We considered that glacial meltwater might be the main cause of runoff underestimation. On the one hand, the spatial pattern of runoff underestimation almost coincides with that of glaciers. In glacier-covered areas, glacial snowmelt may play a more important role as a water input in arid regions than in wet ones. Therefore, the underestimation of runoff in the western United States is greater than in the northwest of North America. On the other hand, 2011 was a year in which the world generally experienced record high temperatures. The abnormal temperature might have accelerated glacier melting and altered watersheds' natural runoff yielding. The widespread underestimation in 2011 is consistent with the effect of glaciers.

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**Figure 7.** Observed versus reconstructed time series (line plots) and long-term mean (scatter plots) runoff values. The globe was divided into nine geographic regions (Fig. 1): North America ((a) west, (b) southwest, (c) midwest, (d) northeast, (e) southeast, (f) except of the USA), (g) South America, (h) Africa, and (i) Europe.

## 300 5 Discussion

Zhou et al. (2015) provided a Budyko equation derived from Fu’s equation and confirmed that this is a valid framework for studying hydrological responses. However, the physical meaning of parameter  $m$ , a  $P_w$  in the Budyko equation, has remained unknown. In this paper, we selected the new Fu’s equation and developed  $P_wM$ , a universal framework for estimating  $P_w$ , and exploring its physical meaning. The results show that, to a large extent,  $P_wM$  can estimate  $P_w$  with soil moisture and fractional vegetation. As important hydrological watershed characteristics, soil moisture and fractional vegetation cover strongly control the  $P_w$  and affect runoff by the Budyko framework.

The universal framework  $P_wM$  for the derivation of  $P_w$  presented in the paper is built on empirically-based power relationships between  $P_w$  and soil moisture and linear relationships between  $P_w$  and fractional vegetation cover. Concerning the power relationship between  $P_w$  and soil moisture, our findings seem to confirm those of Chen and Sivapalan (2020). However, the observed power relationship showed an evident soil moisture threshold—a positive power function appeared in the interval of 0 to 20  $\text{kg m}^{-2}$  (Fig. 2a), while a negative power function was more appropriate from 20 to 100  $\text{kg m}^{-2}$  (Fig. 2b).



The possible reason for the threshold may be that transpiration increased as the relative extractable soil  
315 water increased until reaching a soil moisture threshold value. Once the soil moisture threshold was  
exceeded, the acceleration of transpiration from soil moisture slowed down, and excess soil moisture  
provided conditions for high runoff ratios. These findings are largely in line with previous studies  
(Havranek and Benecke, 1978; Jiao et al., 2019; Cavanaugh et al., 2011; Ducharme et al., 1998), although  
the threshold of soil moisture varied in these studies (e.g., the results of Ducharme, Cavanaugh and Jiao  
320 show that the threshold value is 0.25, 0.10 and 0.20 m<sup>3</sup> m<sup>-3</sup>, respectively).

Our study found a close linear relationship between Pw and fractional vegetation cover, and a similar  
linear relationship has been reported in previous studies. For example, Li et al. (2013) found that the  
spatial pattern of the Pw was linearly correlated with the spatial pattern of vegetation cover fraction.  
However, these reports were mostly from studies in large watersheds or non-humid watersheds. At the  
325 global scale, including small and wet watersheds, vegetation was considered weakly correlated with the  
watershed characteristic parameter of the Budyko framework (Liu et al., 2021). The classification of  
watersheds might provide some insights for explaining this paradox. The findings in this paper show that  
there were different relationships between fractional vegetation cover and Pw in different hydrological  
similarity groups. In dry soil watersheds (IN<sub>D</sub>), the relationship between Pw and fractional vegetation  
330 cover followed a positive linear function. This finding was consistent with the majority view that  
vegetation transpiration increases (reflected by the increased Pw) with increasing vegetation in regions  
with insufficient soil moisture (Wang et al., 2012; Yao et al., 2016; Schwarzel et al., 2020). In wet soil  
watersheds, the relationship between vegetation and Pw also depends on the seasonality of precipitation  
and the size of vegetation: the relationship between the Pw and FVC could be described as a positive  
335 linear equation in the IN<sub>WSS</sub> and the IN<sub>WE</sub> groups. In contrast, a negative linear equation is needed in the  
IN<sub>WMM</sub> and IN<sub>WML</sub> groups. This confirms that climate, soil moisture, and vegetation are not independent  
factors affecting the water balance, and the physiological characteristics of vegetation greatly depend on  
climate and soil moisture (Gan et al., 2021; Yang et al., 2009). When vegetation was coupled with other  
catchment properties, the watershed characteristic parameter exhibited greater variations (Gan et al.,  
340 2021). Therefore, the classification of watersheds is crucial and supports the hypothesis that watersheds  
in the same class would function in a similar climate, soil moisture, and vegetation environment  
(Kanishka and Eldho, 2017; Sinha et al., 2019). The relationship between watershed characteristic  
variables and Pw may be confused without watershed classification.



345 Although the validation showed that the overall performance of PwM was satisfactory, we noted  
that the accuracy of the runoff simulated by the Budyko framework in some regions was likely not  
optimal. Because the Pw was only forced with soil moisture, seasonality index and fractional vegetation  
cover, the estimated runoff could not clearly account for the effects of temperature anomalies and excess  
glacial meltwater on the hydrological regimes. This may be one of the reasons for the severe  
underestimation of runoff in western North America and southern Europe. The time series and spatial  
350 distribution results of runoff validation also point to these reasons. However, the spatial resolution of the  
considered remote sensing data did not allow to capture the variability of snowmelt volume governed by  
the unusually high temperatures. Perhaps future research could examine the relationship between  
watershed characteristic parameters and glacier melting caused by temperature anomalies and further  
improve the accuracy of runoff simulation based on the Budyko framework.

## 355 **6 Conclusions**

This research developed PwM, a universal model for estimating the Pw and exploring its physical  
meaning. The development of PwM using global hydrological data collected from globally published  
datasets and validated using GRDC observational data provides confidence in PwM. The results show  
that the overall performance of PwM is satisfactory. Moreover, the findings indicated that the Pw is  
360 closely related to soil moisture and fractional vegetation cover, and the relationship varies across specific  
hydrologic similarity groups.

Due to the complexity of hydrological processes, the PwM could not fully account for all the  
dynamic impacts of watershed characteristics, such as temperature anomalies and excess glacial  
meltwater, which might result in an underestimation of runoff in regions with glaciers. Therefore, the  
365 interactions of climate and glaciers should be explicitly incorporated into a future Budyko framework.  
To achieve this, detailed hydrological and glacial melt datasets at fine spatial and temporal scales are  
also needed.

The positive findings lay a sound basis for explaining the Pw in the Budyko framework. They could  
also be applied to improve global runoff estimations. We hope it will improve water balance estimates,  
370 pave the way for future hydrology research, and help consolidate water resources management studies.



*Code availability.* The pieces of code that were used for all analyses are available from the authors upon request.

*Data availability.* All data used in this study are publicly available. PET data are available from CRU TS  
375 (<https://doi.org/10.6084/m9.figshare.11980500>), SM data are available from GLDAS  
([https://disc.gsfc.nasa.gov/datasets/GLDAS\\_NOAH025\\_M\\_2.1/summary?keywords=GLDAS](https://disc.gsfc.nasa.gov/datasets/GLDAS_NOAH025_M_2.1/summary?keywords=GLDAS)), FVC  
data are available from GLASS (<http://www.glass.umd.edu/05D/FVC/>), SI data are available from  
HydroShare (<http://www.hydroshare.org/resource/ff287c90c9e947a78e351c8d07d9d3f3>), PRE data  
used to model validation are available from GPCC (<https://psl.noaa.gov/data/gridded/data.gpcc.html>),  
380 and observed river discharge data are available from GRDC  
([https://www.bafg.de/GRDC/EN/02\\_srvcs/21\\_tmsrs/riverdischarge\\_node.html](https://www.bafg.de/GRDC/EN/02_srvcs/21_tmsrs/riverdischarge_node.html)).

*Author contributions.* YC and XC designed the study and proposed the scientific hypothesis. YC  
implemented the experiments, conducted the analysis and wrote the paper. MX helped with data  
collection, and checked the technical adequacy of the experiments. CY and WZ helped with data  
385 processing. CY, WZ, CJ and WY reviewed and edited the manuscript. XC oversaw the study and  
conducted manuscript revision as a mentor.

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