

**Journal:** HESS

**Title:** Revisiting the Hydrological Basis of the Budyko Framework with the Hydrologically Similar Groups Principle

**MS No.:** hess-2022-290

**MS Type:** Research article

## Response to Vazken Andréassian

### General Comments:

**I apologize for posting this note about what you may consider as "details".**

**But:**

Response:

Thank you for your comments. Your suggestions are very useful for us to improve our research. We revised our manuscript according to your comments. The changes in our manuscript are underlined with red. We believe our manuscript improved a lot after the modification. Please see the response below.

### Comment 1:

**the formula that you identify as "Yang et al (2008)" is much older than that: Turc in 1954 and Mezentsev in 1955 published it simultaneously. If you had read Fu (1981), which you cite, you would have heard about Mezentsev, because Fu cites him.**

**the formula of Fu (1981) was previously published by a French hydrologist, Tixeront in 1964 (but this citation is more difficult to find, I acknowledge it)**

Response:

Thank you, according to your comments, we have supplemented the relevant literature in Table 1.

**Table 1. Parametric Budyko-type formulations (Pw - watershed characteristic parameter; ET - actual evaporation, R - runoff, P - precipitation, PET - potential evapotranspiration, all in mm yr<sup>-1</sup>).**

Reference	Formulation	Pw (Theoretical range)	Reference values of Pw
Budyko (1974)	$\frac{ET}{P} = \left[ \frac{PET}{P} \tanh \left( \frac{PET}{P} \right)^{-1} (1 - \exp(-\frac{PET}{P})) \right]^{0.5}$	0.5	0.5
Zhang et al. (2001)	$\frac{ET}{P} = \frac{1 + w \frac{PET}{P}}{1 + w \frac{PET}{P} + (\frac{PET}{P})^{-1}}$	w (0, ∞)	Trees – 2.0, Plants – 0.5
Turc (1954), Mezentsev (1955), Choudhury (1999), Yang et al. (2008)	$\frac{ET}{P} = \frac{1}{\left[ 1 + (\frac{P}{PET})^n \right]^{\frac{1}{n}}}$	n (0, ∞)	Field – 2.6, River basins – 1.8
Wang and Tang (2014)	$\frac{ET}{P} = \frac{1 + \frac{PET}{P} - \sqrt{(1 + \frac{PET}{P})^2 - 4\varepsilon(2 - \varepsilon)} \frac{PET}{P}}{2\varepsilon(2 - \varepsilon)}$	ε (0,1)	0.55 - 0.58

Tixeront (1964), Fu (1981), Zhou et al. (2015)	$\frac{R}{P} = \left[ 1 + \left( \frac{P}{PET} \right)^{-m} \right]^{\frac{1}{m}} - \left( \frac{P}{PET} \right)^{-1}$	$m$ (1, ∞)	Forest – 2.83, Shrub – 2.33, Grassland or cropland – 2.28, Mixed land – 2.12
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**Comment 2:**

**the seasonality index of Walsh and Lawler (1981) is extremely weak in that it only deals with rainfall, it does not address the issue of the relative seasonality of P and E (and after all, all Budyko's framework is about comparing P and E). I would suggest you have at least a look at the work we published on that topic (de Lavenne & Andréassian, 2018).**

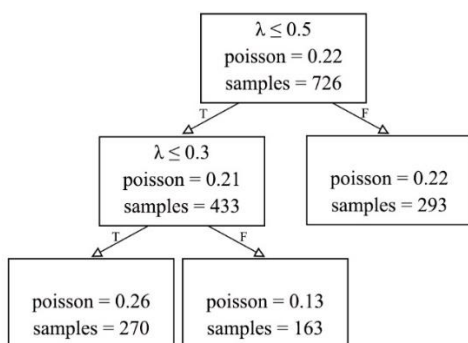
Response:

Thank you. We have carefully studied the seasonal index  $\lambda$  you proposed, and thought that was a great algorithm for the indication of seasonality. We tried to use it for our modeling. However, the results of the simulations were not better than those using the seasonality index of Walsh and Lawler (1981). The detailed process and results are as follows.

First, we used the seasonal index  $\lambda$  (De Lavenne and Andréassian, 2018) instead of the seasonal index of Walsh and Lawler (1981) for classification, and reset the model ( $\lambda\_PwM$ ) to estimate Pw. Based on the results of the Decision Tree Regressor (DTR) (Fig. S.1), we divided the  $\lambda$  into three parts ( $\lambda \leq 0.3$ ,  $0.3 < \lambda \leq 0.5$ ,  $\lambda > 0.5$ ) to represent three hydroclimatic seasonality (low, medium and high synchronicity of precipitation and potential evapotranspiration). The classifications of surface soil moisture (SM) and fractional vegetation cover (FVC) remain the same as the original. Finally, six hydrologically similar groups were classified (Table S.1).

**Table S.1** Classification of watersheds

Soil moisture classifier	Water soil regime	Seasonality index classifier	Seasonality regime	Fractional vegetation cover classifier	vegetation cover regime	Name of the group
SM $\leq$ 20	Dry soil	—	—	—	—	IN <sub>D</sub>
		$\lambda \leq 0.3$	Low synchronicity	—	—	IN <sub>WL</sub>
SM $>$ 20	Wet soil	$0.3 < \lambda \leq 0.5$	Medium synchronicity	FVC $\leq$ 0.2	Low density	IN <sub>WMS</sub>
				$0.2 < FVC \leq 0.5$	Middle density	IN <sub>WMM</sub>
		$\lambda > 0.5$	High synchronicity	FVC $>$ 0.5	High density	IN <sub>WML</sub>
				—		IN <sub>WH</sub>

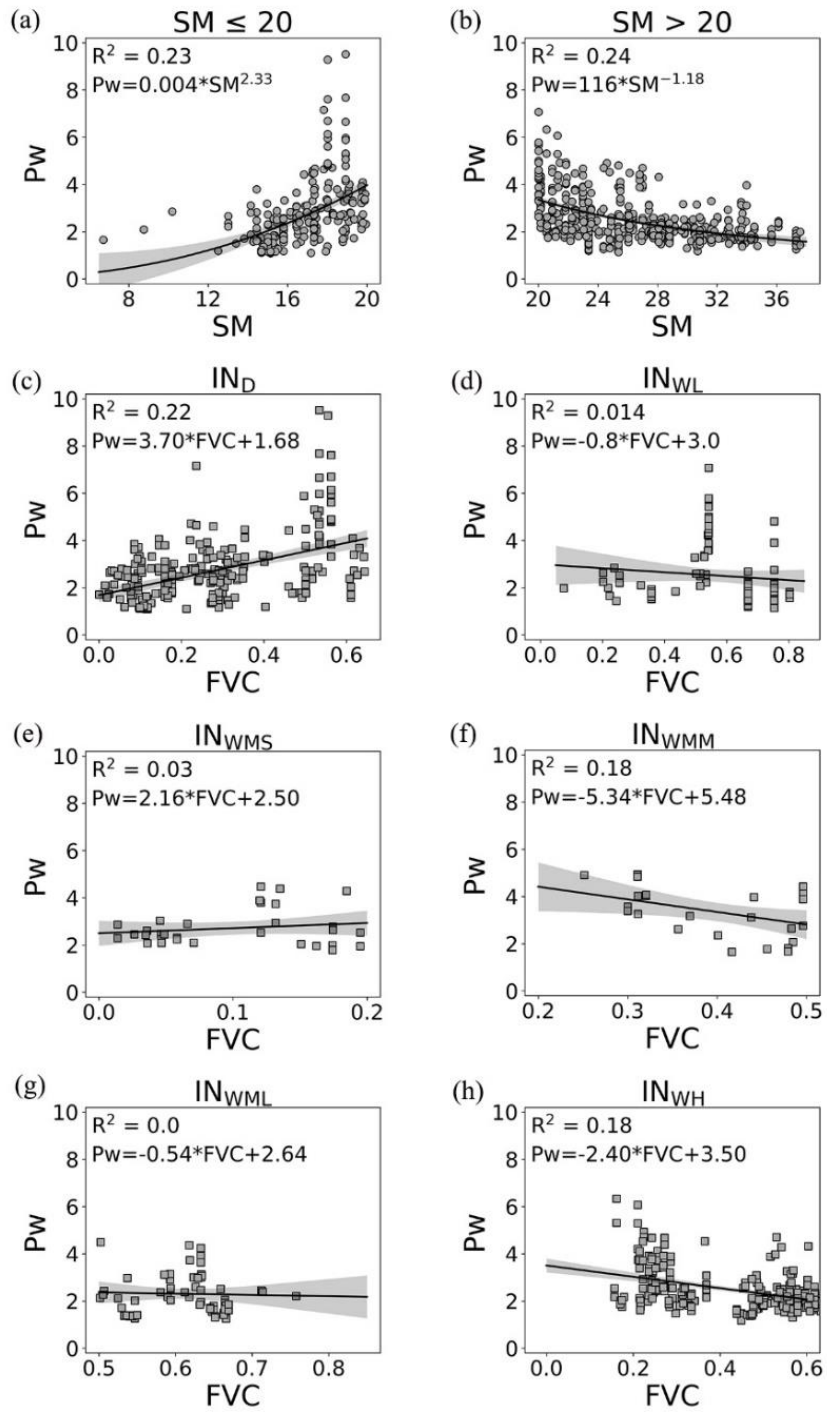


**Figure S.1** The results of the Decision Tree Regressor for ensemble seasonal index  $\lambda$ . The “poisson” indicates the value of Poisson deviance, “samples” indicates the number of samples, “T” means True, and “F” means Fales.

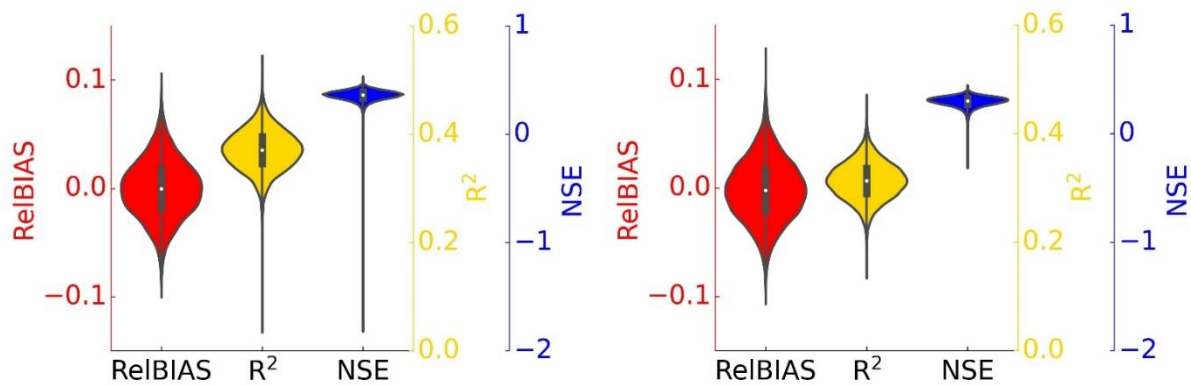
The regressions between Pw in Fu's formula and watershed characteristic variables collected from globally published datasets are shown in Fig. S.2. The variables whose  $R^2$  of the regression model was greater than 0.1 were selected as input variables. Therefore, in the proposed  $\lambda\_PwM$ , SM and FVC were selected as input variables for all the groups, except that FVC was rejected in the  $IN_{WL}$ ,  $IN_{WMS}$ , and  $IN_{WML}$  group. The formula in  $\lambda\_PwM$  for calculating the Pw is modeled as the sum of a power function of SM and a linear function of FVC, given by Equation S.1.

$$\lambda\_Pw = \begin{cases} 0.91 \times SM^{0.38} + 1.48 \times FVC & (IN_D, SM \leq 20) \\ 0.0003 \times SM^{-3.02} & (IN_{WL}, SM > 20, \lambda \leq 0.3) \\ 7.82 \times SM^{-0.36} & (IN_{WMS}, SM > 20, 0.3 < \lambda \leq 0.5, FVC \leq 0.2) \\ 31.14 \times SM^{-0.67} - 0.59 \times FVC & (IN_{WMM}, SM > 20, 0.3 < \lambda \leq 0.5, 0.2 < FVC \leq 0.5) \\ 73.15 \times SM^{-1.06} & (IN_{WML}, SM > 20, 0.3 < \lambda \leq 0.5, FVC > 0.5) \\ 24.86 \times SM^{-0.67} - 0.85 \times FVC & (IN_{WH}, SM > 20, \lambda > 0.5) \end{cases} \quad (S.1)$$

The performance of the PwM and  $\lambda\_PwM$  were cross-validated based on the data collected from globally published literatures using the bootstrap sampling method (Fig. S.3). On average, the maximum relative bias of the Pw simulated by the PwM is 0.09. The interquartile range of  $R^2$  for the PwM is from 0.35 to 0.40, with a median of 0.37. The NSE interquartile range from 0.33 to 0.39, with a median of 0.36. In comparison, the maximum relative bias of the Pw simulated by the non\_PwM is 0.12, the median of  $R^2$  is 0.31, and the median of NSE is 0.30. Overall, the cross-validations show that the performance of the PwM is better and more stable than the  $\lambda\_PwM$ . Therefore, in this study, we still use the seasonality index of Walsh and Lawler (1981).



**Figure S.2** Regression between Pw in Fu's formula and (a) SM ( $SM \leq 20$ mm), (b) SM ( $SM > 20$ mm), (c) FVC ( $IN_D$ ), (d) FVC ( $IN_{WL}$ ), (e) FVC ( $IN_{WMS}$ ), (f) FVC ( $IN_{WMM}$ ), (g) FVC ( $IN_{WML}$ ), and (h) FVC ( $IN_{WH}$ ). Symbol shapes indicate SM (dot) and FVC (square).



**Figure S.3** Cross-validation results of (a) PwM and (b)  $\lambda$ \_PwM. A violin represents the distribution of the considered skill scores. The white dot on the violin plot represents the median. The black bar in the center of the violin represents the interquartile range. Colors distinguish three performance metrics: Red (RelBIAS), yellow ( $R^2$ ) and blue (NSE).

## References

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