



Improving understanding of groundwater flow in an alpine karst system by reconstructing its geologic history using conduit network model ensembles

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10 **Abstract.** Reconstructing the geologic history of a karst area can advance understanding of the system's present-day hydrogeologic functioning, and help predict the location of unexplored conduits. This study tests competing hypotheses describing past conditions controlling cave formation in an alpine karst catchment, by comparing an ensemble of modelled networks to the observed network map. The catchment, the Gottesacker karst system (Germany/Austria), is drained by three major springs and a paleo-spring, and includes the partially explored Hölloch cave, which consists of an active section whose
15 formation is well-understood, and an inactive section whose formation is the subject of debate. Two hypotheses for the formation of the inactive section are: 1) glaciation obscured the three present-day springs, leaving only the paleo-spring, or 2) the lowest of the three major springs (Sägebach) is comparatively young, so its subcatchment previously drained to the paleo-spring. These hypotheses were tested using the pyKasso Python library (built on anisotropic fast marching methods) to generate two ensembles of networks, one representing each scenario. Each ensemble was then compared to the known cave map. The
20 simulated networks generated under Hypothesis 2 match the observed cave map more closely than those generated under Hypothesis 1. This supports the conclusion that the Sägebach spring is young, and suggests that the cave likely continues southwards. Finally, this study extends the applicability of model ensemble methods from situations where the geologic setting is known but the network is unknown, to situations where the network is known but the geologic evolution is not.

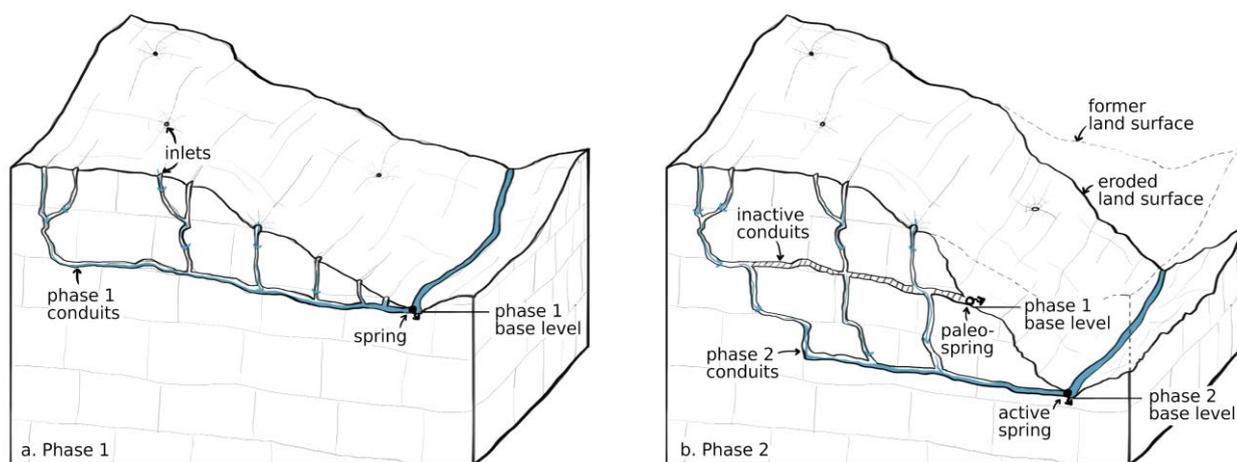
1 Introduction

25 Mapping subsurface cave and conduit networks in karst systems provides crucial information for water resource management, hazard prevention, archaeological research, and the protection of important ecological and cultural sites. Even small-diameter conduits play an important role in groundwater flow. However, mapping conduit networks is a challenging task (Trimmis, 2018, Sellers and Chamberlain, 1998). Conduits can extend far below the surface, and speleologists attempting to explore them may experience dangers such as flash floods, falling rocks, hypothermia, and disorientation. Even in well-explored systems,



30 the full network is impossible to map, because small-diameter conduits (<0.5 m) are inaccessible to humans and difficult to detect by geophysical methods (Jaquet and Jeannin, 1994).

Understanding the speleogenetic processes driving cave formation and the geologic history of an area can help guide cave exploration, data collection, and groundwater resource management. Karst systems commonly evolve over time in several phases, during which different climate and geologic conditions control which parts of the system (e.g. diffuse recharge zones, focused inlets, outlets, conduits) are actively transmitting flow. Conduits or springs that see active flow during one phase of karstification may go dry when conditions change. Although these older parts of the system may become inactive, they are still open and can be reactivated during extreme flow events or when climate conditions shift again (Audra and Palmer, 2011) (Figure 1). An experienced geologist may be able to reconstruct how the karst system functioned in the past, and therefore predict the likely locations of presently inactive conduits. These predictions can sometimes be tested through targeted exploration of the locations where caves are suspected to exist. Some hypothesized conduits, however, may be inaccessible, making it impossible to confirm or refute their existence through exploration. In other cases, several different historical scenarios may appear equally plausible, making it difficult to determine how the system evolved. As a result, geologic intuition alone may not be sufficient to fully understand a karst area. This can lead to knowledge gaps or misconceptions that limit the reliability of future simulations of the system's behavior under projected climate or land use scenarios that differ from present-day conditions.



50 **Figure 1: Schematic illustration of two-phase karstification due to a drop in base level. (a) Phase 1: conduits form, connecting the active recharge zone to the active spring at base level. (b) Phase 2: as the base level drops due to erosion, the former spring and part of the phase 1 conduit system become inactive (they no longer transmit flow), replaced by new conduits and a lower-elevation spring. This is only one of numerous scenarios that can lead to karst systems composed of both active and inactive conduits and springs. Concept after Audra and Palmer (2011).**



2 Approach

This study presents a model-based approach to reconstructing the geologic processes driving cave formation. A real karst system was used as a test site, for which detailed cave maps are available in parts of the conduit network. While some portions of the explored cave system are very active, containing an accessible cave stream with open-channel flow that connects to a major karst spring, other portions of the explored system are now inactive, and are thought to have formed when past conditions were different from the present configuration of the system. However, two different explanations are possible: 1) glaciation obscured all of the present-day springs, leaving only the paleo-spring as the primary drainage point for the entire karst system, or 2) the lowest of the major present-day springs is significantly younger than the others, so the paleo-spring and the two uppermost springs jointly drained the aquifer system.

These competing hypotheses were tested using a computationally efficient karst network model to generate probability maps of possible conduit locations under each of the proposed past scenarios. The simulated conduit networks under each scenario were then compared with the mapped portion of the inactive cave system, to determine which scenario best matched the observed network. Finally, the analysis was extended to propose possible locations of the unmapped parts of the system.

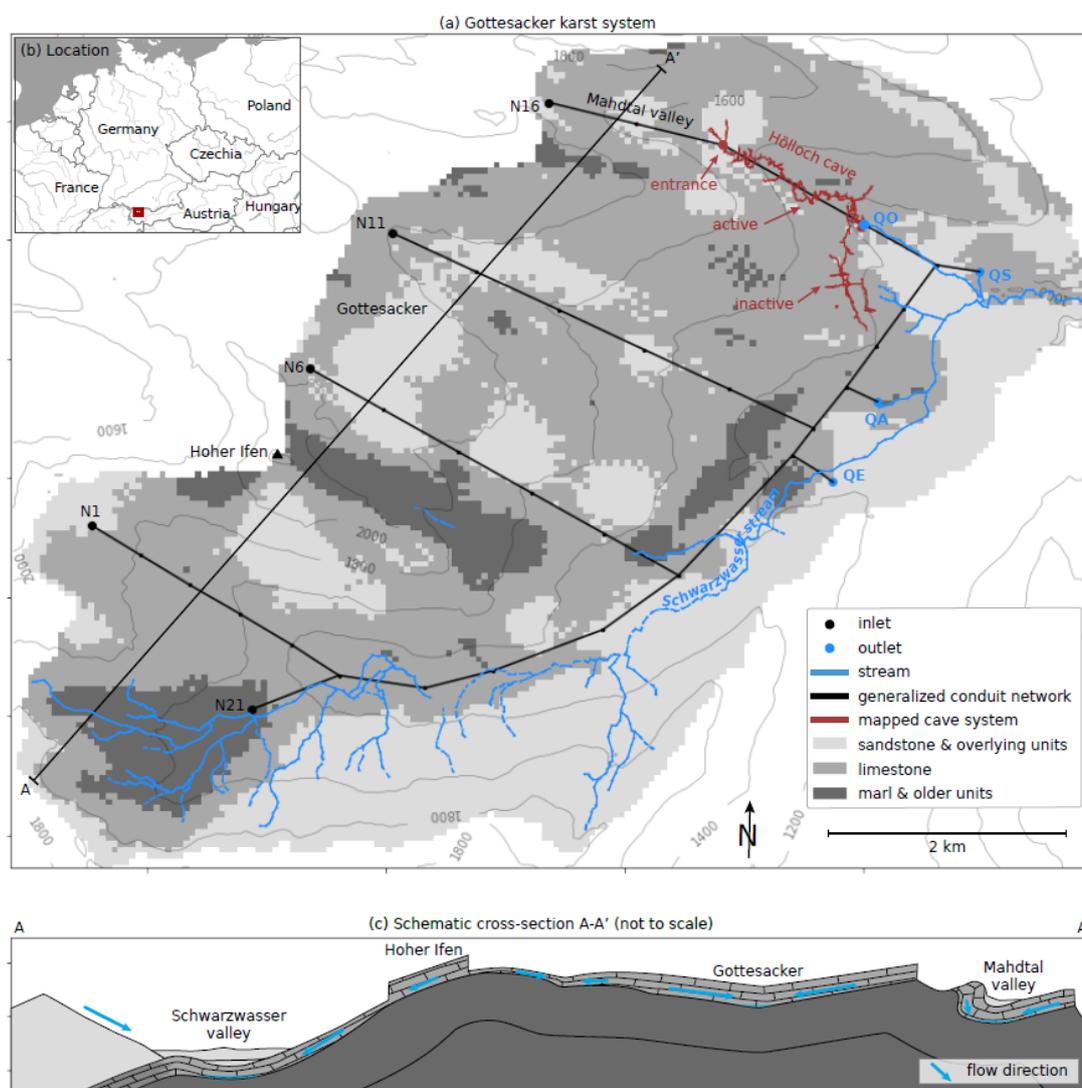
3 Field site: Gottesacker

The modeling approach demonstrated in this study was tested on the complex, extensively studied Gottesacker karst system in the German/Austrian Alps, described in Goldscheider (2005). This 35 km² catchment consists of a series of plunging synclines and anticlines draining to the Schwarzwasser valley, which cuts roughly perpendicularly across the fold axes (Figure 2). The karst aquifer lies north and northwest of the valley in a limestone unit widely exposed at the surface. It is locally overlain by sandstone and younger units, and underlain by non-karstifiable marl and older units. Three major springs drain the system: an estavelle (QE, elevation 1120 m a.s.l.), the Aubach spring (QA, elevation 1080 m a.s.l.), and the Sägebach spring (QS, elevation 1035 m a.s.l.). The estavelle acts as swallow hole during low-flow conditions but acts as a spring during high-flow conditions. A paleo-spring (QO, elevation 1190 m a.s.l.) partway up the Mahdtal valley, near Höflealpe, remains dry for timespans of years to decades, and was only recently observed to become temporarily re-activated under extreme high flow conditions. South of the main Schwarzwasser valley, non-karstifiable flysch and marl lithology prevents conduit development, and drainage occurs instead through a network of surface streams. Several other geologic units and small springs are present, but for the purposes of conduit modeling, the geology was represented using a simplified model focused on delineating the boundaries of the karstifiable limestone unit.

Numerous previous tracer studies (Goldscheider, 2005, Goepfert and Goldscheider, 2008, Goepfert et al., 2020) have yielded a good understanding of the general configuration of the conduit network, which has been used successfully for previous groundwater flow modeling efforts (Chen and Goldscheider, 2014, Chen et al., 2017, Chen et al., 2018, Fandel et al., 2021, Fandel et al., 2022). The major karst conduits were determined to follow synclinal axes before merging into a primary conduit that parallels the surface stream in the Schwarzwasser valley.



This karst system is also the site of one of the longest caves in Germany, the 12 km Hölloch cave in the Mahd tal valley, which
85 has been the subject of avid exploration and of several books and documentary films since the early 1900s (Höhlenverein
Sonthofen, 2006). The local caving club has created and shared a detailed map of the explored portion of the cave. The northern
part of the cave (trending SE) is active and contains a cave stream, while the southern part (trending NNW) is inactive and
does not have any continuous underground drainage or cave streams.



90 **Figure 2: Overview of the Gottesacker karst system. (a) Simplified geologic model discretized into 50 m x 50 m pixels. The karst**
aquifer is located in the limestone unit, and drains to the three springs in the lower part of the system (QE, QA, QS). The paleo-
spring (QO) is only active during extreme high-flow conditions. All springs flow into the Schwarzwasser stream, shown in blue.
Several small tributaries also feed the stream from the south. Note: the sandstone patches are overrepresented spatially due to minor
superficial modeling artifacts, which are not significant for the purposes of this study (see Fandel et al. 2021 for a detailed geologic
map). Outlets correspond to mapped karst springs after Goldscheider (2005). Conduit network and inlet names are model
95 **representations after Chen et al. (2018) (b) Location within Europe. Basemap: ESRI. (c) Schematic cross-section along line A-A',**
adapted from Goldscheider (2005).



4 Hypotheses

Two different hypotheses are currently under consideration to explain the formation of the inactive part of the Hölloch cave system:

Hypothesis 1: The Schwarzwasser valley was covered by a glacier, overlying the karstifiable units up to the elevation of the paleo-spring (QO), which was the primary spring draining the entire system.

Geologic evidence: Widespread moraines and other geomorphologic evidence indicate the presence and extent of the glacier in the valley.

Hypothesis 2: Low-permeability sedimentary formations covered the Sägebach spring (QS), so the paleo-spring (QO) drained the Mahdtal valley and connected to the Aubach spring (QA), which was at the time the lowest spring in the system.

Geologic evidence: The Sägebach spring (QS) is located in a narrow depression at a fault, where the top of the karst aquifer limestone unit is exposed only very locally, and is surrounded and overlain by low-permeability sandstone and marl. This geologic setting suggests very young (probably post-glacial) exposure by erosion along the fault. By comparison, the other springs (QE and QA) are located in fully exposed karst limestone, indicating that they have been in existence for a longer time.

A third possibility also exists, in which some combination of the above scenarios occurred, with glaciation and low-permeability sediments covering the valley and the Sägebach spring (QS), then the glacier retreating before the Sägebach spring was uncovered. Since both hypotheses are supported by geologic evidence, it is quite likely that all of these geologic events occurred, but there remains the question of which events had the strongest influence on cave formation, which depends both on the temporal order and on the duration of the different processes at work.

5 Method: Anisotropic fast marching to generate probability maps

To test these hypotheses, many possible network configurations were modelled using anisotropic fast marching methods, implemented in the Python karst modelling package pyKasso (Fandel et al., 2022). Anisotropic fast marching algorithms calculate the optimal path from one point to another through a medium, in which the ease of travel varies both spatially and directionally. Karst conduits can be simulated using this type of algorithm based on the assumption that a conduit represents the fastest path from an inlet (such as a doline or swallow hole) to an outlet (a spring). The travel medium represents the geologic setting, in which some rock units are more soluble than others (i.e. easier for conduits to travel through). Conduits are also assumed to form preferentially in the direction of the maximum downward hydraulic gradient. For shallow, unsaturated karst systems such as the example presented here, the hydraulic gradient roughly coincides with the topographic gradient, which is significantly simpler to calculate. In this study, the topographic gradient was therefore used as a proxy for the hydraulic gradient.



130 Stochasticity can be introduced to the simulations through the generation of a unique fracture network for each model realization, based on the statistical distribution of fracture families observed in the field. The statistical metrics describing the fracture network are the density, the minimum and maximum strike, and the minimum and maximum length. For a table of fracture statistics, see Fandel et. al. (2022). Fractures are assumed to be easier for conduits to travel through than the surrounding rock.

135 Additional controls on the configuration of the simulated network are possible by iterating over multiple phases of karst development, and over multiple outlets. The conduits leading to each outlet can be simulated in separate iterations, to represent springs of different ages. Existing conduits are easier to travel through than the surrounding rock, and therefore attract younger conduits forming in later iterations, resulting in a branching network structure commonly observed in real karst systems. The inlets to the system can also be divided into groups and assigned to separate outlets, representing different subcatchments
140 within the larger system. All these aspects can be controlled in a deterministic or stochastic manner depending on the available information for a given site.

The primary advantages of fast marching methods are their low data requirements and their computational efficiency compared to other conduit network models. These characteristics allow the rapid simulation of hundreds of network realizations for a single site (for this study, 100 simulations ran in under two minutes on a laptop with a 2.7 GHz dual-core i7 processor and 16
145 GB RAM). However, this approach does not represent the actual physical and chemical processes driving speleogenesis. Unlike more complex speleogenetic models, each individual pyKasso model realization, because it is based on very little information, is unlikely to capture the true network configuration. Instead, this method is more appropriate for rapidly exploring various scenarios and generating probability maps: visual representations of a model ensemble indicating the likelihood of a simulated conduit being present at any given location.

150 **5.1 Quality check: accuracy of network simulations**

To represent the Gottesacker karst system with pyKasso, outlet coordinates were assigned based on mapped spring locations (Goldscheider, 2005) and inlet coordinates were assigned based on locations inferred from tracer test evidence and geologic structure in previous modeling efforts by (Chen and Goldscheider, 2014). For the medium through which conduits form, a three-dimensional geologic model was created using the Python package GemPy (de la Varga et al., 2019). For a detailed
155 description of the geologic model, see Fandel et al. (2021). To avoid edge effects, the initial geologic model extends beyond the study area, and was then cropped to the catchment boundaries. For use with pyKasso (currently only available in 2D), the 3D model was sliced using the surface topography, because, in most of the study area, the karstifiable unit is exposed at the land surface. The resulting 2D geologic map has a resolution of 181 x 141 cells, with 50 m x 50 m pixels. This is not fine enough to capture the detailed layout of the Hölloch cave, but is suitable for representing the general location and orientation
160 of conduits.

A pyKasso-generated ensemble of 100 realizations with the simplified inputs described above, random inlet/outlet assignments, and random spring ordering yielded a probability map in which many of the likely conduits roughly follow



165 expected paths (Figure 3a). However, some simulated conduit paths were highly geologically improbable (e.g. conduits
climbing up and over anticlines). These are largely attributable to the random inlet/outlet pairings, which sometimes assign an
inlet to an outlet located in a different subcatchment, thereby forcing unrealistic model behaviors. For a full description of the
conduit model parameters, results, and limitations, see Fandel et al. (2022). When the inlet/outlet assignments were fixed, the
resulting probability map closely matched the branching pattern of the expected network (Figure 3b), despite the limitations
of using a low-resolution, simplified, 2D geologic model as the travel medium and the land surface gradient as a proxy for the
hydraulic gradient. These results support placing confidence in the ability of pyKasso-generated ensembles to simulate the
170 general configuration of real karst conduit networks in this system, particularly when the inlet/outlet assignments are fixed.

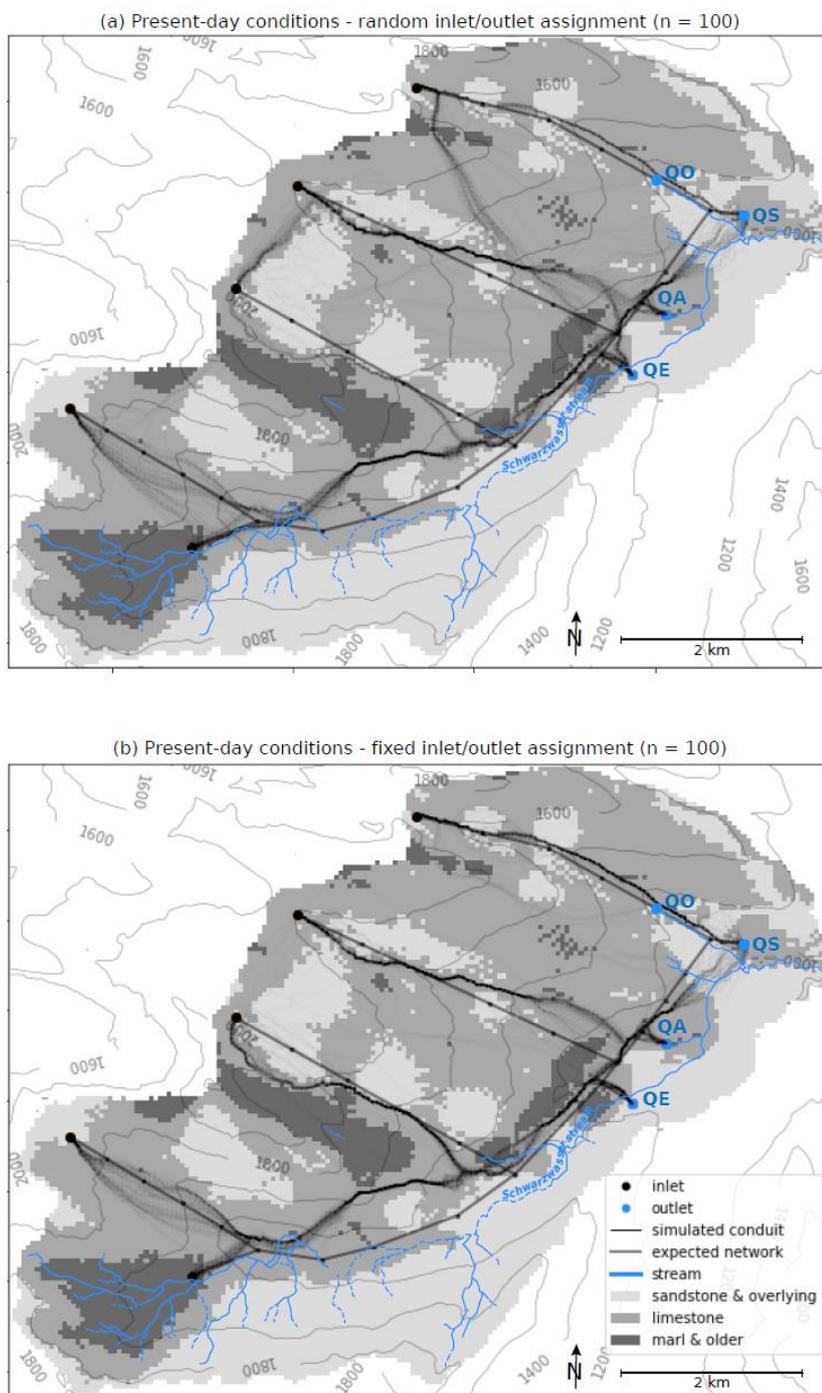


Figure 3: Probability maps of the conduit network under present-day conditions. (a) When the inlet/outlet pairings and the outlet iteration order are randomly shuffled, some simulated conduits follow expected paths while others take geologically improbable routes, such as crossing anticlines. (b) When the inlet/outlet pairings and the outlet iteration order are fixed based on tracer test results (Goldscheider 2005), stochasticity arises from the fracture network only, and the simulated conduits generally match the expected network configuration. Adapted from Fandel et al. (2022).



5.2 Hypothesis testing

To test the two hypotheses for the past geologic conditions controlling conduit formation, 100 network realizations were run for each scenario using pyKasso, at the same model resolution previously used to simulate present-day conditions (50 m x 50 m cells).

To simulate Hypothesis 1 (glaciation closing all springs except QO), the general extent of glaciation was first reconstructed by the authors, based on mapped glacial deposits and on the elevation of the paleo-spring (Figure 4a). The glacier was assumed to have filled the Schwarzwasser valley up to the level of the spring, located approximately 200 m above the valley floor, and to have covered the rest of the valley with roughly the same thickness of ice. Geomorphological evidence indicates that two smaller glaciers also existed in the northern part of the catchment, one in the upper portion of the Mahdtal valley and one in the breached syncline northeast of Hoher Ifen. The model cells covered by the glacier were assigned a very high travel cost, discouraging conduits from crossing the glaciated area. The inlets and outlets covered by the glacier (inlets N21 and N16 and outlets QE, QA, and QS) were removed from the model, and inlet N16 was replaced with an inlet at the entrance to the Hölloch cave. Under this scenario, inlet N1 was separated from the rest of the karst system by the glacier, and was assumed to have drained to a paleo-spring near Ifersguntentalpe (QI). However, the extent of the glacier in this subcatchment is uncertain, so the focus of this simulation remains on the eastern portion of the karst network connected to the paleo-spring (QO), which drained all remaining inlets not covered by the glacier (Figure 4a).

To simulate Hypothesis 2 (QS is much younger than the other springs), the Sägebach spring was removed from the list of system outlets and replaced with the paleo-spring (Figure 4b). The existing inlets remained the same as the present-day configuration. Under this scenario, the Aubach spring (QA), which is lower in elevation than the paleo-spring, was assumed to have served as an “attractor” receiving flow from the entire system, including the Mahdtal valley. An additional inlet was therefore co-located with the paleo-spring and assigned to the Aubach spring (Figure 4b).

For both scenarios, the inlet/outlet pairings and iteration order were kept constant. A new fracture network was stochastically generated for each realization, using the same input statistics as the simulations of the present-day network. All other parameters were held constant.

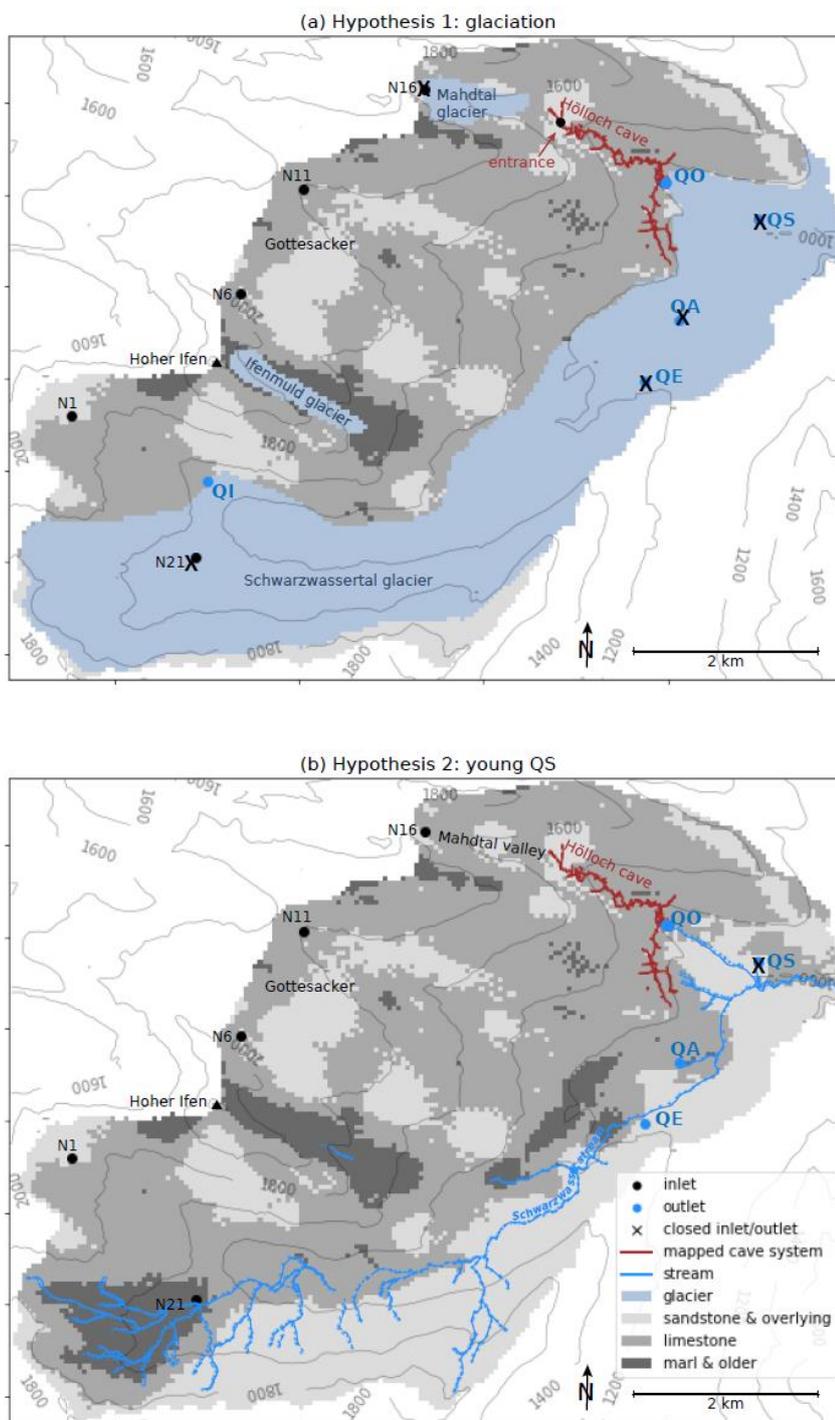


Figure 4: Two competing hypotheses for past geological conditions controlling cave formation. (a) A glacier filled the Schwarzwasser valley, obscuring the karstifiable unit up to the elevation of the paleo-spring (QO). (b) Low-permeability sediments covered the Sägebach spring (QS), making the paleo-spring (QO) and the Aubach spring (QA) the primary outlets for the system. A third hypothesis (not shown) is that some combination of the above occurred.



6 Findings

The modeled probability maps support Hypothesis 2 (a comparatively young QS) (Figure 4). The simulated networks matched the location and orientation of the mapped cave system better when the Sägebach spring was removed from the list of outlets and replaced with the paleo-spring. In fact, the mapped conduits lie in an envelope of possibilities visible in the probability map (Figure 4b). By comparison, the simulated networks under Hypothesis 1 (glaciation obscuring the valley bottom) only matched the location and orientation of the mapped conduits in the active upper portion of the cave system, not in the inactive lower portion which is the focus of inquiry (Figure 4a). These results support the conclusion that Hypothesis 2 is the most likely representation of the past geologic conditions that led to the formation of the inactive portion of the Hölloch cave system. The Sägebach spring was likely covered by overlying non-karstifiable units until relatively recently, at which point the older conduits were abandoned in favor of the more direct present-day connections confirmed by tracer tests.

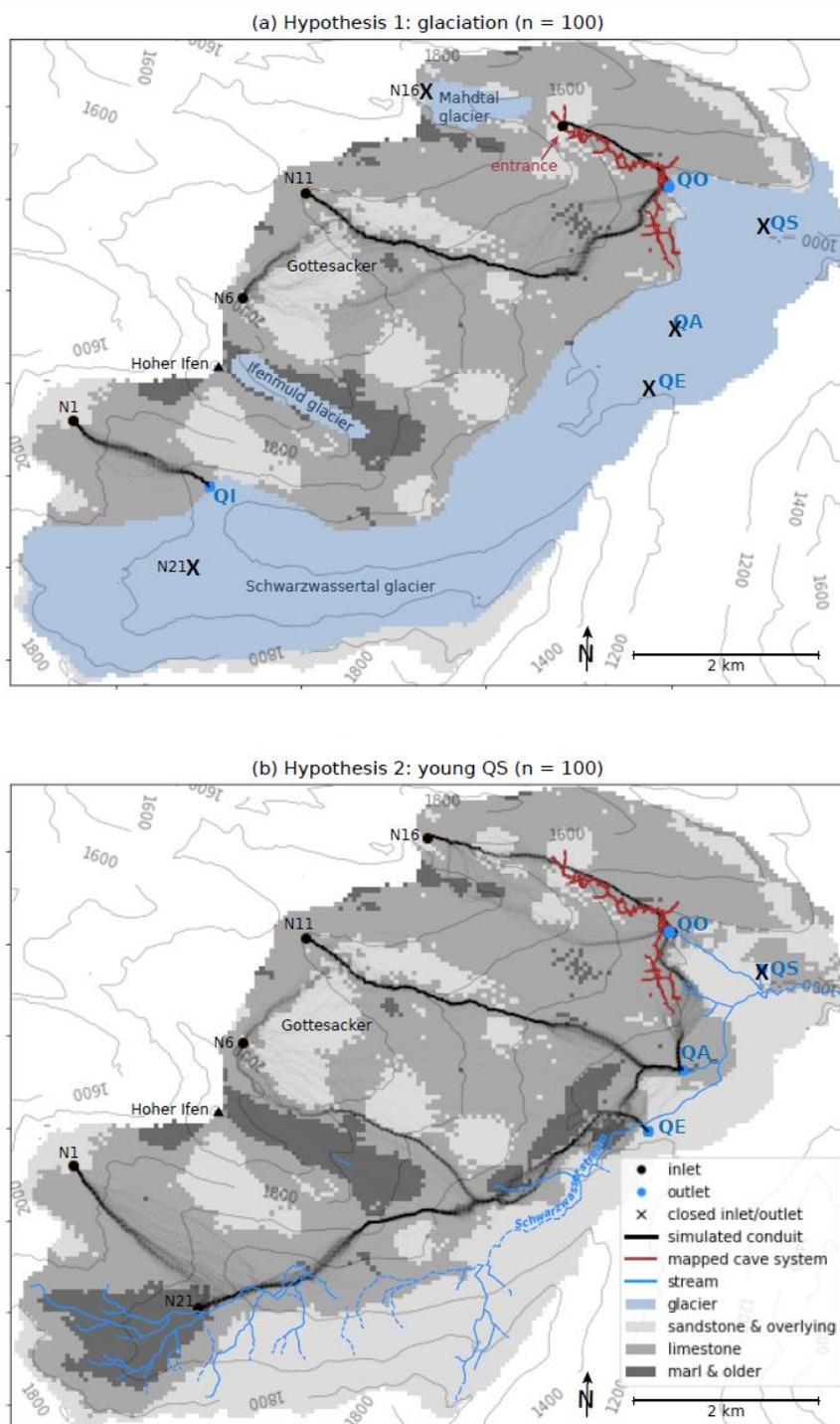


Figure 5: Probability maps for 100 simulations under each hypothesis, compared to the observed cave map. (a) Hypothesis 1 diverges from the mapped cave system. (b) Hypothesis 2 more closely matches the inactive cave network than hypothesis 1, suggesting that the Sägebach spring is relatively young.



220 7 Discussion

The model ensembles in this study clearly support one hypothesis over the other: the probability map for Hypothesis 2 suggests that the inactive portion of the Hölloch cave continues southeast for several hundred meters before turning first southwest, then due south to connect to the Aubach spring (Figure 5b).

These results suggest two additional ways that these hypotheses could be further tested by fieldwork:

- 225 1) Additional speleological explorations, focusing on the passages trending in the directions projected by the probability map – if the hypothesis is correct, these passages will continue rather than terminating in dead-ends. This information may be helpful in guiding future speleological exploration.
- 2) A tracer injection under extreme high flow conditions at the Hölloch cave entrance in the Mahdtal valley with sampling at the Sägebach spring (QS) and the Aubach spring (QA). When, under high flow conditions, the water
230 level in the cave system is greater than the elevation of the Aubach spring (QA), the paleo-spring (QO) becomes active. Previous tracer tests under normal flow conditions found a connection only to the Sägebach spring, but under extreme high flow conditions, the normally inactive conduits projected to connect to the Aubach spring may reactivate. Currently, no conduits have been found along the path expected if glaciation had controlled conduit formation. Unfortunately, the logistical challenges of waiting for such an extreme event to occur and then achieving
235 a high-quality tracer test on short notice are significant.

A third hypothesis is also possible, a combination of the two scenarios explored in this study: that an initial phase of conduit formation occurred dominated by the influence of the glacier covering the valley, another phase occurred after the glacier had retreated but before the Sägebach spring was exposed, and the most recent phase developed with all three major springs (QE, QA, and QS) active. If this were to have occurred, two sets of inactive conduits should be present, corresponding to the two
240 past phases of karstification. However, exploration of the Hölloch system is incomplete. If future exploration reveals new conduits along the path simulated under the glaciation scenario, this hypothesis of two-phase karstification could also be supported.

Another possible scenario that we did not explore in depth is that some combination of different springs were cyclically occluded then exposed over the course of several phases of glacial and inter-glacial periods (with some springs active during
245 the inter-glacials but covered during the glaciations).

While the model results provide insight into this area's geologic past, some questions and limitations remain. In both scenarios, the underlying geologic model is a limitation. A 2D geologic map simplifies the complex three-dimensional reality. In this example, patches of sandstone appear in the middle of the Gottesacker plateau when in fact, almost the entire plateau consists
250 of exposed limestone (Figure 6). This is because the upper contact between the limestone and the overlying sandstone closely coincides with the land surface, so when the 3D model is sliced using the surface topography, cells near the contact are frequently mis-assigned. This results in conduits curving around or crossing fold axes in geologically implausible ways to



255 avoid patches of hard-to-traverse geologic units in 2D, when in fact in 3D the conduit paths should be able to pass unobstructed
beneath the shallow sandstone (particularly for the conduits departing from inlet N6). These issues could be resolved with a
3D geologic model (currently in development for pyKasso). However, the hypothesis testing presented here focuses on a small
area within the catchment (the lower part of the Hölloch cave), in which the geologic model is fairly representative, with few
slicing artifacts, so this is not a major issue for the purposes of this study.

260 Another question arises from the observation that, although the simulated conduits under Hypothesis 2 (a young QS) are much
more similar to the inactive portion of the Hölloch cave than under Hypothesis 1 (glaciation), they do not exactly match the
mapped cave system. The simulated conduits tend to lie slightly to the east of the mapped conduits, skirting the edge of the
overlying sandstone unit. This is likely a limitation of the model's ability and our simplified assumptions to predict exact
conduit locations rather than general orientations and connections. However, it may also indicate a conceptual gap in our
understanding of the system. A few examples of factors not considered in this study that could affect conduit formation include:

- 265 1) The presence of an unmapped tectonic feature with higher or lower conduit-forming propensity than the
surrounding rock. This would attract or repel conduits in the model.
- 2) A difference between the shape of the land surface topography and the topography of the base of the
karstifiable limestone unit. The conduits in this study are generated under the assumption that the land surface and
the base of the limestone unit are parallel, so if they are in reality not parallel, there would be some error in the
anisotropy field.
- 270 3) An eastward (downgradient) shift in the location of the contact between the karstifiable limestone and the
thin overlying sandstone unit as the sandstone erodes over time. The modeled conduits in this specific location tend
to follow the contact, so the real conduits being slightly westward of the modeled conduits could be explained if the
contact were previously also slightly westward of its current location.
- 275 4) More phases of karstification could have occurred, in various orders, resulting in a conduit network that is
partially but not fully explained by the hypotheses presented here.

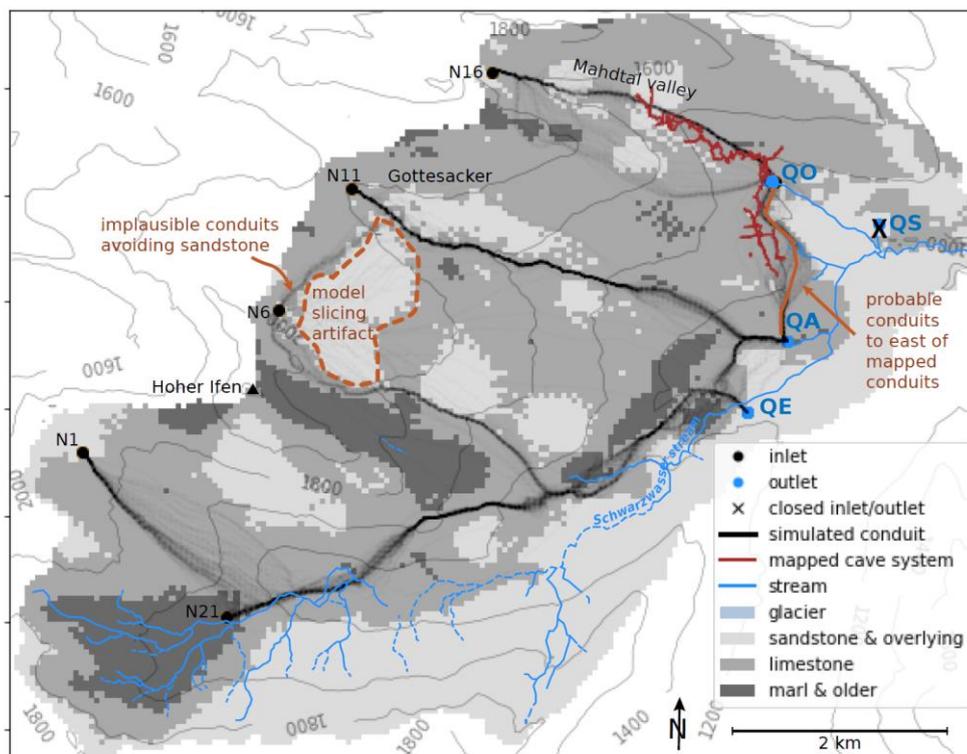


Figure 6: Inaccuracies in the simulated conduit networks, highlighting limitations of the anisotropic fast marching approach.

7.1 Applicability to other situations

The advantages of the approach presented in this study are that it requires very little data (inlet and outlet coordinates, fracture distribution statistics, and a geologic map), and is computationally efficient. This allows for rapid exploration of different scenarios and quick hypothesis testing.

The disadvantages of this approach are that the probability maps produced by running multiple model realizations must be interpreted with care. They represent the probability of a conduit simulation being present within the framework of the set of assumptions and available (often incomplete) data used to construct the model. If the underlying model inputs and assumptions are flawed or incomplete, the resulting ensemble of simulated networks may fail to capture the actual network configuration.

There are several particularly significant assumptions that could influence the model results: that the system has distinct inlets with known locations rather than diffuse recharge across a general area, that a simplified 2D geologic model can adequately represent the subsurface geologic structure, and that the topographic gradient is an adequate proxy for the hydraulic gradient. These assumptions are taken to be valid in the well-studied Gottesacker system, because the shallow karstifiable unit is generally exposed at the land surface, its structure roughly paralleling the topography. The close fit between the expected conduit network as determined from tracer tests and the simulated networks in the present-day scenario model ensemble further supports the validity of the anisotropic fast marching approach for this system, and increases our confidence in the historical



simulations. However, these assumptions may not be appropriate in other karst systems, and the simulated conduits represent only the general network configuration, not the detailed local structure of the conduits. It is therefore crucial when transferring
295 this approach to other locations first to consider whether it is well-suited to the karst system in question, and second, to pair modeling efforts with supporting fieldwork in less well-studied sites.

8 Conclusion

This study demonstrates the application of anisotropic fast marching methods for rapid model ensemble generation to test competing hypotheses describing the past geologic conditions that controlled karst conduit evolution in a real catchment. It
300 extends the applicability of these methods from situations where the goal is to generate a map of an unknown conduit network, to situations where the goal is to understand how an already-mapped conduit network was formed.

For the field site investigated in this study, the Gottesacker karst system, comparing the conduit network under different past scenarios (based on model-generated probability maps of conduit locations) to the actual conduit network (based on maps of the explored present-day cave system) allowed the identification of the past conditions with the greatest influence on cave
305 development. The model results indicate that the most probable scenario is that the conduits of interest formed before the lowermost spring in the catchment came into existence, and drained to the other two major springs in the system (as opposed to conduit formation being dominantly influenced by glaciation occluding all three major present-day springs). The modeling results also enabled making recommendations to guide cave exploration efforts and future data collection that could further test the conduit formation hypotheses.

310 The strength of the anisotropic fast marching method is its comparatively low cost in terms of computational resources and quantity of initial input data required. This allows for rapid iteration and exploration of many different scenarios, which makes it especially well-suited for ensemble modeling. The model ensemble approach demonstrated in this study could be applied to other karst systems with mapped conduit networks, both to better understand the past geologic conditions that influenced the conduit network development, and to better target future mapping and data collection efforts to answer outstanding questions
315 about the system. It also holds potential for exploring competing conceptual understandings of a karst system, and identifying possible conceptual gaps, both of which are significant sources of uncertainty in predicting karst systems' response to anthropogenic stresses.

Code availability

The code and supporting data used to generate the figures in this paper are presented in a Jupyter Notebook available in a
320 public GitHub repository here:

[https://github.com/randlab/pyKasso/blob/replace-fast-marching-with-HFM/notebooks/Fandel et al 2022_HESS.ipynb](https://github.com/randlab/pyKasso/blob/replace-fast-marching-with-HFM/notebooks/Fandel_et_al_2022_HESS.ipynb)



Author contribution

NG developed the concept for the paper and the hypotheses, and provided data on the study site. FM, CF, and PR developed and debugged the pyKasso modeling package. CF implemented the model and performed the simulations. TF provided
325 research supervision and guidance in the use of model ensemble approaches. CF created the figures and wrote the manuscript draft, and NG and PR reviewed and edited the manuscript.

Competing interests

The authors declare that they have no conflict of interest.

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