



1 **Resistance parameters and permissible velocity from cohesive channels**

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11

12 **ABSTRACT**

13

14 The forces determining erosion resistance in cohesive channels are not yet completely  
15 understood. Therefore, this study aimed to evaluate the resistance parameters and obtain  
16 a flow velocity equation for such channels. Experimental data were obtained from  
17 cohesive channels with a 60% clay proportion and under increasing flow levels. The soil  
18 detachment rates were inversely proportional to the applied shear stress, and the obtained  
19 critical shear stress and soil erodibility values were as high as 120 Pa and 0.00003 kg N<sup>-1</sup>  
20 s<sup>-1</sup>, respectively. Under the highest applied flow, the yield stress was significantly  
21 influenced by the geometry variation, flow velocity, and sediment concentration. The  
22 shear stress generated by the applied flows remained below the critical shear stress of the  
23 cohesive bed channels. Using the Buckingham theorem, we developed an equation to  
24 predict the permissible flow velocity in cohesive channels. this will help engineers design  
25 and manage river structures more effectively.

26

27 Keywords: dimensional analysis, shear stress, cohesive erodibility, resistance to direct  
28 shear stress, critical shear stress, yield stress, cohesive beds.

29 **1. Introduction**

30 The permissible velocities applied in conventional methods predict the constant  
31 shear stress and flow velocity using practical engineering principles that can vary between  
32 projects (Qasem et al., 2017). Furthermore, Utley and Wynn (2008) predicted erosion



33 rates under cohesive conditions by using the interactions between the water contained in  
34 the porous soil and the eroding flow, as well as the cohesive properties of the soil.

35 Detachment processes can occur in cohesive sediment within channels, earthen  
36 dams, and spillways; these are modeled using an linear approach to determine the shear  
37 stress. Hence, engineers require methods to quantify water erosion under cohesive  
38 conditions (Khanal et al., 2016). Partheniades (1965) first modeled cohesive erosion rates  
39 using the excess shear stress:

$$40 \quad \varepsilon_r = K_d (\tau - \tau_c)^a. \quad (1)$$

41 Here,  $\varepsilon_r$  is the erosion rate,  $K_d$  is the erodibility coefficient,  $\tau$  is the flow shear stress,  $\tau_c$  is  
42 the critical shear stress, and  $a$  is the exponent, which is taken as unity. In this model, the  
43 erosion rates are proportional to the difference between the applied shear stress and  
44 boundary critical shear stress.

45 The erosion rates of overland flow on rangelands tend to be relatively low;  
46 however, where the flow is concentrated, soil loss can be significant. Therefore, a  
47 cropland site can be susceptible to concentrated flows when excess shear stress is placed  
48 on the soil particles. This concept has been applied to crops in agricultural areas and  
49 hydrological soil erosion events around the world (Al-Hamdan et al., 2013).

50 Concentrated flows consist of storm water flowing within a confined geomorphic  
51 landscape feature such as a rill, channel, or river. De Baets et al. (2006) reported that in  
52 the 1990s, important advances were made toward understanding concentrated flow  
53 erosion and its hydraulic behavior under environmental conditions that may result in the  
54 formation of rills and gullies.

55 Aliev (1985) stated that when erosion processes occur in cohesive channels,  
56 detachment occurs for all stretches of the drain system and continues throughout the  
57 channel. In this situation, the transport capacity is incomplete, in contrast to the  
58 deformation processes in the sand channels.

59 Because cohesive sediments feature a large specific area (owing to the small sizes  
60 of clay particles), physicochemical forces act as cation and hydrogen bonds to ensure  
61 cohesion between clay particles; however, these factors have not been widely studied in  
62 situations where the soil undergoes applied shear stress and increased soil moisture  
63 (Ansari et al., 2003).



64 For cohesionless sediments, the primary resistance to erosion is provided by the  
65 submerged weight of the sediment. In cohesive beds, the net attractive interparticle  
66 surface forces, frictional interlocking of grain aggregates, and electrochemical forces all  
67 control the resistance to erosion and detachment. These forces vary with the type of clay,  
68 antecedent moisture conditions, type of shear applied, and drainage conditions (Ansari et  
69 al., 2003). Therefore, these forces are not completely understood. The main mechanisms  
70 that cause sediment to move in flowing water are the flow velocity, shear, and normal  
71 stress resulting from flow turbulence (Jain and Kothiyari, 2009).

72 Sekine et al. (2008) reported that only minimal information is available regarding  
73 the erosion rates of cohesive sediments via water surfaces, and the erosion mechanisms  
74 of cohesive sediments are not entirely understood. However, researchers have asserted  
75 that clay particles combine owing to the complicated mechanism of cohesive force  
76 applied to their surfaces. Engineers must know the quantity of deposited cohesive  
77 sediment that can be detached or transported under specific shear stresses, to facilitate  
78 effective management of river structures (e.g., dams and sluice gates) and water transport  
79 facilities and the proper design of stable channels in cohesive sediment.

80 Mirtskhoulava (1991) stated that water erosion is an extremely relevant aspect of  
81 hydraulic design: appropriate designs can restrict flow velocities to below the permissible  
82 level, to prevent water erosion; such designs represent a target of fluvial hydraulics.  
83 However, because of their mineralogical and chemical characteristics (and subsequent  
84 mechanical and physical behaviors), predicting the resistance of cohesive soils is more  
85 complicated than that of sandy soils, for the resistance under these conditions depends  
86 solely on the particle weight. This complexifies the designing of hydraulic structures in  
87 cohesive soils.

88 To understand the incipient motion of cohesive sediments, several factors  
89 pertaining to the flow acting on the boundary and the cohesive material's boundary must  
90 be understood. Zhang and Yu (2017) reported that cohesive sediment transport depends  
91 on the rheological properties of the sediment, and they introduced the yield stress ( $\tau_y$ )  
92 concept. This stress is produced when a shear stress acts on a soil or sediment sampler,  
93 thereby changing its state from solid to liquid and causing it to flow. The stress is  
94 proportional to the interparticle interactions. Yield stress can also occur when a sample is  
95 subjected to normal stress in a direct shear test that produces shear stress. Zhang et al.



96 (2017) claimed that the primary factor determining cohesive sediment erodibility is yield  
97 stress.

98 Zhang and Yu (2017) presented an empirical expression for the onset of  
99 movement for a cohesive sediment ( $\Theta_{cr}$ ), which they developed via dimensional analyses  
100 to produce a dimensionless yield stress parameter ( $\tau_r$ ), expressed as

$$101 \quad \tau_r = \frac{\tau_y}{\rho_s (vg)^{\frac{2}{3}}}, \quad (2)$$

102 where  $\rho_s$  is the particle density ( $\text{kg m}^{-3}$ ),  $g$  is the gravitational acceleration ( $\text{m s}^{-2}$ ),  
103  $\nu$  is the kinematic viscosity of flow ( $\text{m}^2 \text{s}^{-1}$ ), and  $\tau_y$  is the yield stress ( $\text{N m}^{-2}$ ). Zhang and  
104 Yu (2017) assumed that ( $\Theta_{cr}$ ) corresponded to an exponential function of the yield stress  
105 ( $\tau_y$ ) and dimensionless particle diameter ( $D_*$ ); thus, the increase of ( $\Theta_{cr}$ ) became  
106 progressively weaker; as a result, the rheological term reflected the yield stress influence  
107 on  $\Theta_{cr}$ , calculated as

$$108 \quad \theta_{cr_c} = (0.056 - 0.033e^{-0.0115D_*} + 0.12e^{-0.25D_*} + 0.48e^{-3.8D_*}) \times \\ 109 \quad (e^{9.8 \times 10^{-4} \times \tau_r \times \exp(-0.4D_*)}). \quad (3)$$

110 The erodibility ( $K$ ) and critical shear stress ( $\tau_c$ ) were also used as resistance  
111 parameters for the cohesive sediments. Mahalder et al. (2018) tested different pressure  
112 levels on soils with clay contents of 15–25%, to obtain the maximum critical shear stress  
113 ( $\tau_c$ ), which varied from 12.43 to 26.80 Pa; maximum erodibility values ( $K$ ) of 3.84–24.2  
114  $\text{cm}^3 \text{N}^{-1} \text{S}^{-1}$  were also obtained.

115 Graf (1984) noted that the relation for shear stress in a cohesive material can be  
116 written as

$$117 \quad \frac{\tau_0}{(y_s - y)d} = A_1'' + C_0, \quad (4)$$

118 where  $\tau_0$  is the shear stress;  $y_s$  and  $y$  are the sediment- and soil-specific weights,  
119 respectively;  $d$  is the average diameter of the grain;  $A_1''$  is the sediment coefficient; and  $C_0$   
120 is the coefficient of cohesion for the material. The coefficient  $A_1''$  can be omitted for  
121 materials where the cohesive forces are much larger than the other forces. However, in  
122 soil mechanics, the shear stress corresponding to a failure can be approximated by

$$123 \quad \tau = \sigma \tan \phi + C, \quad (5)$$

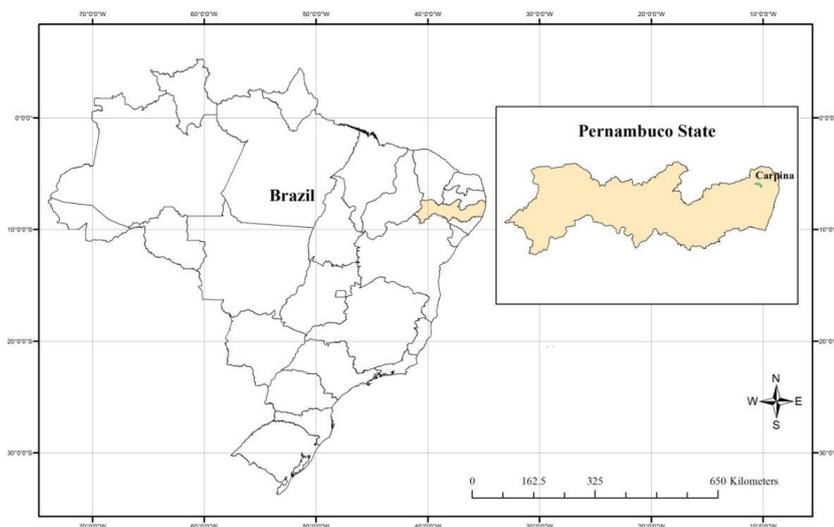


124 where  $\tau$  is the shearing strength or shearing resistance,  $\sigma$  is the effective pressure, and  $C$   
125 denotes the cohesion. This equation is known as *Coulomb's* equation and is similar to Eq.  
126 4. The cohesion at saturation water content, dispersed particle size, and soil aggregate  
127 stability (i.e., the soil's resistance to water) appear to be the most important elements of  
128 the extensive and complex physical and mechanical properties of cohesive soils.  
129 Consequently, Mirtskhoulava (1996a) reported that resistance to scouring increases under  
130 an increase in cohesive strength, owing to the moisture content.

131 This study aimed to evaluate the parameters of erosion resistance, including the  
132 cohesion, soil erodibility, critical shear stress, and yield shear stress of highly cohesive  
133 channels. In addition, to obtain a flow velocity equation for cohesive channels, we applied  
134 dimensional analysis to experimental data, considering the hydraulic and cohesive  
135 parameters governing the concentrated flow velocity under these conditions.

## 136 **2. Material and Methods**

137 This study was performed in the Experimental Station of Rural Federal  
138 Pernambuco University located in Carpina city, Pernambuco state, Brazil (7° 51' 13" S,  
139 35° 14' 10" W) (Figure 1), 180 m above sea level. The climate in the study area has a  
140 Köppen climate classification as "Ams,"; which is a rainy tropical climate with dry  
141 summers (less than 60 mm in the driest month) and the total precipitation is 1200  
142 mm.year<sup>-1</sup>. This area has been cultivated with Sugarcane crops for more than 400 years,  
143 and the native vegetation was the Atlantic Forest. The experiment was located on the  
144 landscape medium part with a slope between 0.13-0.16 m.m<sup>-1</sup>, under a Ultisol according  
145 to the Soil Taxonomy USDA (1998), with a sand surface horizon of 40 cm depth and an  
146 Argillic B horizon with a 60% clay proportion. The soil attributes are enclosed in tables  
147 1 and 2.



148

149 **Figure 1.** Sugarcane Experimental Station of the Rural Federal University of  
 150 Pernambuco, located in Carpina City, Pernambuco State, Brazil. Map entirely created by  
 151 authors.

152

153 **Table 1.** Some physical attributes of the horizon B of a Ultisol from Brazil. Soil density  
 154 ( $\rho$ ), particle density ( $\rho_s$ ), total porosity ( $\alpha$ ), and water content. Average values considering  
 155 four repetitions.

$\rho_d$	$\rho_p$	$\alpha$	$\theta$
----- $\text{kg m}^{-3}$ -----	----- $\text{m}^3 \text{m}^{-3}$ -----		
1381	2675	0.484	0.356

156

157 **Table 2.** Particle size distribution, organic carbon content, and soil texture of the horizon  
 158 B of a Ultisol from Brazil. Average values considering four repetitions.

159

Treat.	OC	Sand	Silt	Clay	Textural classification
	%	----- $\text{g kg}^{-1}$ -----			
1	0.54	187.75	247.56	564.70	Clay
2	0.45	122.72	211.78	665.50	Clay
3	0.51	167.33	201.59	631.08	Clay



4      0.69      173.55      217.31      609.15      Clay

160            Organic carbon analysis was performed using the dry combustion method, and the  
161 soil density ( $\rho$ ) was determined using the methodology of Grossman and Reinsch (2002).  
162 The particle density ( $\rho_s$ ) was obtained according to Blake and Hartge (1986), and the total  
163 porosity ( $\alpha$ ) was calculated following Flint and Flint (2002). Table 1 summarizes the  
164 results of the physical analyses.

### 165 **2.1 Pre-formed cohesive channels and experimental procedure**

166            The bed channels had a soil texture with a high clay content (Table 2), which was  
167 pre-formed under the B-horizon of the Ultisol. Initially, the Ultisol surface soil horizon  
168 was removed and deposited adjacent to the experimental area (Figure 2). Then, 16  
169 channels were pre-formed; these consisted of four larger channels of width 0.5 m, depth  
170 0.2 m, and length 4 m and 12 smaller channels of width 0.1 m, depth 0.05 m, and length  
171 4 m. A gutter was placed at the lower end of the channels to sample water and solid  
172 discharges. Water was added to the channels by pipes connected to a water reservoir  
173 maintained at a constant level. These cohesive channels were applied at the following  
174 flow levels:  $Q_1 = 70$ ,  $Q_2 = 132$ , and  $Q_3 = 210$  for smaller channels, and  $Q_4 = 545 \text{ L min}^{-1}$   
175 for larger channels. All tests lasted for 20 min.



176  
177 **Figure 2.** Cohesive channel preparation under a cohesive horizon of a Ultisol. Figure  
178 from authors.



179

180 The mean flow velocities for all cohesive channels were obtained using an  
181 electromagnetic current meter and the dye method. For the dye method, methylene blue  
182 was introduced at the start of the channels, and the time taken for the dye to reach the  
183 channel's outlet was measured using a chronometer. The channel length divided by the  
184 time taken by the dye yielded the superficial velocity ( $V_s$ ) of the flow and was expressed  
185 in  $\text{m s}^{-1}$ . In this procedure, the superficial velocity values were multiplied by  $\alpha = 2/3$ ; then,  
186 a correction factor was applied, and the mean velocity ( $V_m$ ) was ultimately recorded in  $\text{m}$   
187  $\text{s}^{-1}$  (Farenhorst and Bryan, 1995; Cassol et al., 2004; Cantalice et al., 2017).

188 The water discharge ( $Q$ ) was computed from the sampling runoff obtained in  
189 plastic pots placed on the channel outlets at 5-min intervals; this was obtained  
190 concomitantly with flow velocity measurements. A linear ruler was used to measure the  
191 flow depth ( $h$ ) (cm) for larger channels, and a profilometer was used to take the hydraulic  
192 radius ( $R_h$ ) (m) for the smaller channels, as well as the channel area ( $A$ ) ( $\text{m}^2$ ). The  
193 kinematic viscosity was determined using the equation proposed by Julien (1995), in  
194 which we used the water temperature ( $^{\circ}\text{C}$ ) measured by a thermometer in each test. The  
195 Froude number ( $Fr$ ) and Reynolds number ( $Re$ ) were obtained according to Simons and  
196 Senturk (1992).

197 The Darcy–Weisbach coefficient ( $f$ ) was used to express the hydraulic resistance:

$$198 \quad f = \frac{8gR_h S}{V^2}. \quad (6)$$

199 Here,  $f$  is the Darcy–Weisbach coefficient (dimensionless),  $R_h$  is the hydraulic radius (m),  
200  $S$  is the water surface slope ( $\text{m m}^{-1}$ ),  $V$  is the mean flow velocity, and  $g$  is the gravitational  
201 acceleration ( $\text{m s}^{-2}$ ).

## 202 2.2 Mirtskhoulava's permissible velocity

203 Mirtskhoulava's permissible (Mirtskhoulava, 1996a, Mirtskhoulava, 1991)  
204 velocity was used to verify the equation's performance on cohesive channels, mainly  
205 through a paired comparison to velocity values obtained by an electromagnetic current  
206 meter within the channels. Mirtskhoulava's equation (1991) is expressed as

$$207 \quad V = \log_{10} \frac{8.8R_h}{d} \sqrt{\frac{2gm}{2.6\gamma n'} [(\gamma_s - \gamma)d + 1.25C_f K]}, \quad (7)$$



208 where  $V$  is the permissible velocity ( $\text{m s}^{-1}$ ),  $R_h$  is the hydraulic radius (m),  $d$  is the median  
209 surface grain diameter ( $D_{50}$ , mm),  $g$  is the acceleration due to gravity ( $\text{m s}^{-2}$ ), and  $m$  is the  
210 working condition coefficient, which expresses the influences of different factors on the  
211 operating conditions and is usually considered equal to 1 for recently prepared soils.  $\gamma$  and  
212  $\gamma_p$  are the specific weights of water and sediment ( $\text{t m}^{-3}$ ), respectively; and  $n'$  is the  
213 overload coefficient considering the change in scouring flow capacity (which is  
214 influenced by pulsating velocities and other probable cases of loads exceeding their  
215 calculated values). The overload coefficient it is obtained from the following expression:

$$216 \quad n' = 1 + \frac{d}{0.00005 + 0.3d}. \quad (8)$$

217 Here,  $C_f$  is the soil fatigue strength needed to rupture (Pa); it is a function of soil cohesion,  
218 obtained via

$$219 \quad C_f = 0.035C, \quad (9)$$

220 where  $C$  is the soil cohesion (Pa) obtained by direct shear tests, and  $K$  is the clay soil  
221 homogeneity coefficient, which characterizes the probability that the cohesion indices  
222 deviating unfavorably from their mean values; this coefficient is obtained from

$$223 \quad K = 1 - \frac{\alpha\sigma}{C}, \quad (10)$$

224 where  $\alpha$  is a coefficient characterizing the minimum probability of soil resistance or the  
225 safety coefficient [usually taken as 3 (Mirskhoulava, 1966a)], and  $\sigma$  is the standard  
226 deviation of the data.

### 227 **2.3 Detachment rates for concentrated flow in cohesive channels**

228 The soil detachment rates under the concentrated flow conditions were calculated  
229 to the level needed to overcome the critical shear stress arising from the cohesive channel,  
230 based on the methods of Partheniades (1965), Elliot et al. (1989), Flanagan and Nearing  
231 (1995), and Thoman and Niezgodá (2008), as follows:

$$232 \quad D_c = K(\tau - \tau_{cr})^1. \quad (11)$$

233 Here,  $D_c$  is the detachment capacity ( $\text{kg m}^{-2} \text{s}^{-1}$ ),  $K$  is the erodibility of the soil ( $\text{kg N}^{-1} \text{s}^{-1}$ )  
234 <sup>1</sup> in response to shear stress  $\tau$  ( $\text{N m}^{-2}$  or Pa), and  $\tau_c$  is the critical shear stress of the soil  
235 ( $\text{N m}^{-2}$  or Pa). Therefore, the shear stress  $\tau$  was obtained as

$$236 \quad \tau = \gamma R_h S, \quad (12)$$



237 where  $\gamma$  is the specific weight of water ( $\text{N m}^{-3}$ ),  $R_h$  is the hydraulic radius (m), and  $S$  is  
238 the soil surface slope ( $\text{m m}^{-1}$ ). According to Partheniades (1965), the cohesive bed  
239 erodibility ( $K$ ) is considered as the angle coefficient  $b$  of a linear regression model  
240 between the soil detachment rate and shear stress  $\tau$ , and the critical shear stress ( $\tau_c$ )  
241 corresponds to the intercept value of  $\tau$ , where the detachment rate  $D = 0$ .

242 The soil detachment rates from the concentrated flow were obtained from  
243 sediment sampled every 5 min using (Flanagan and Nearing, 1995)

$$244 \quad D = \frac{QC}{LP_w}, \quad (13)$$

245 where  $D$  is the soil detachment rate in response to the concentrated flow ( $\text{kg m}^{-2} \text{s}^{-1}$ ),  $Q$  is  
246 the flow rate ( $\text{L s}^{-1}$ ),  $C$  is the sediment concentration ( $\text{kg L}^{-1}$ ),  $L$  is the length of the channel  
247 (m), and  $P_w$  is the cohesive channel width (m).

248

#### 249 **2.4 Sampling for mechanical soil analysis**

250 Disturbed soil samples were collected on the cohesive bed channels, air-dried, and  
251 sieved through a 2 mm mesh. Thirty-two undisturbed soil samples were collected and  
252 placed into a rectangular stainless-steel box ( $0.06 \times 0.06 \times 0.043$  m) encased in bubble  
253 plastic, to ensure proper readings of the cohesive channels' physical and mechanical  
254 parameters.

255 The direct shear test was performed according to the norm D-3080/98 of the  
256 American Society for Testing and Materials (ASTM D 3080-98, 2003); this was  
257 conducted using a direct shear press device with a shear velocity of  $0.125 \text{ mm min}^{-1}$ . The  
258 normal pressures used during the tests were 50, 100, 150, and 200 kPa. At the end of the  
259 test, the data required for the equations:

$$260 \quad \sigma_n = \frac{N}{a}, \quad (14)$$

261 (where  $\sigma$  is the normal stress,  $N$  is the normal force applied to the test body, and  $a$  is the  
262 transverse section area of the sample) and

$$263 \quad \tau_c = \frac{T}{a}, \quad (15)$$

264 (where  $\tau_c$  is the shear stress and  $T$  is the force applied to the test body) were obtained.



265 Soil cohesion (C) was determined using the value of direct shear stress under each  
266 normal stress at the end of the test, by plotting the relation between the two. The cohesion  
267 values were obtained from the intercept of the equation for the line formed in the graph.

## 268 2.5 Shields critical parameter for cohesive sediment ( $\Theta_{cr}$ )

269 In the Shields critical parameter for cohesive sediment ( $\Theta_{cr}$ ) determination, the  
270 dimensionless yield stress parameter ( $\tau_r$ ) was incorporated according to Zhang and Yu  
271 (2017) and defined by Eq. (2). The yield stress values ( $\tau_y$ ) were obtained from direct shear  
272 stress tests on bed cohesive samples under saturated conditions, which consisted of the  
273 shear stress observed when the bed cohesive samples were subjected to different normal  
274 stress levels. The numerical calculation of the Shields critical parameter for cohesive  
275 sediment values ( $\Theta_{cr}$ ) was obtained using Eq. (3), where the dimensionless particle  
276 diameter ( $D_*$ ) was obtained using

$$277 \quad D_* = d_{50} \left[ g \left( \frac{\rho_s - \rho}{\rho \nu^2} \right) \right]^{\frac{1}{3}}, \quad (16)$$

278 where  $d_{50}$  is the size diameter (m),  $\rho_s$  is the particle density ( $\text{kg m}^{-3}$ ),  $\rho$  is the water density,  
279  $g$  is the gravitational acceleration ( $\text{m s}^{-2}$ ), and  $\nu$  is the kinematic viscosity of flow ( $\text{m}^2 \text{s}^{-1}$ ).  
280

## 281 2.6 Dimensional analysis

282 The dimensional analysis was based on the Buckingham  $\pi$  theorem and the  
283 repeated variable method (Fox et al., 2015). This analysis is based on the difference  
284 between the number of dimensional variables that describe a process ( $k$ ) and the number  
285 of dimensions that reference these variables ( $r$ ); this results in the group's dimensionless  
286 number (denoted as  $\pi$ ). A set of fundamental dimensions is used as a reference, such as  
287 [mass] = M, [length] = L, and [time] = T (Dym et al., 2010). Initially, the dependent and  
288 independent variables were defined according to

$$289 \quad q_1 = f(q_2, q_3, q_4, \dots, q_n). \quad (17)$$

290 The theorem establishes that it is possible to adjust the relationship between  $n$   
291 variables, as follows:

$$292 \quad q(q_1, q_2, q_3, q_4, \dots, q_n) = 0. \quad (18)$$

293 These  $n$  variables can be grouped into  $k - n$  independent dimensionless ratios, or  
294  $\pi$  parameters, which are expressed in a functional form as



295  $G(\pi_1, \pi_2, \pi_3, \pi_4, \dots, \pi_{k-n}) = 0,$  (19)

296 and rewritten according to

297  $\pi_1 = G_1(\pi_2, \pi_3, \pi_4, \dots, \pi_{k-n}).$  (20)

298 After determining the number of  $\pi$  groups observable, a dimensional parameter set  
299 describing all primary dimensions was established based on the procedures of Fox et al.  
300 (2014); these parameters are referred to as repeating parameters ( $m$ ); typically,  $m = r$ .  
301 Thus, the repeated parameters were combined with the remaining ones.

302 Based on Díaz (2012), we investigated whether an observed variable belonged to  
303 the  $\pi$  group. The first task was to place dimensionless variables in a  $\pi$  group, and the  
304 second was to designate any two variables of identical dimensions as constituting a  $\pi$   
305 group. Finally, the dimensional groups were resolved and made dimensionless (Munson  
306 et al., 2004).

### 307 **2.7 Statistical analysis**

308 This study was conducted in a randomized block with four treatments (four flow  
309 levels) and four repetitions, totaling 16 cohesive experimental channels. The data were  
310 initially analyzed using descriptive statistics to identify outliers that could compromise  
311 the behavior of the studied parameters; then, the data were subjected to a two-way  
312 analysis of variance. Other tests were also applied, such as the Kolmogorov–Smirnov and  
313 Shapiro–Wilk tests, to verify data normality; the F test, for variance analysis; and the  
314 Tukey test, to obtain a mean comparison between treatments at a 5% probability.

## 315 **3. Results and Discussion**

### 316 **3.1 Hydraulic behavior of the larger cohesive channels under the applied flow**

317 Table 3 summarizes the hydraulic behavior obtained from the four larger cohesive  
318 channels, in which the mean flow velocity was obtained for the applied flow levels. In all  
319 tests, the Reynolds numbers were turbulent, and the Froude number values showed a slow  
320 or tranquil flow. The obtained hydraulic radius was between 0.055 and 0.59 m, which  
321 meant that all channels could reach similar areas. Descriptive statistics confirmed the  
322 homogeneity of the generated flows.

323 Figure 3 shows the mean velocity behavior, as measured using the electromagnetic  
324 current meter and dye method under the applied flows within the larger cohesive  
325 channels; a high correlation coefficient was obtained for the paired velocities when  
326 compared to the velocities obtained under these two methods.

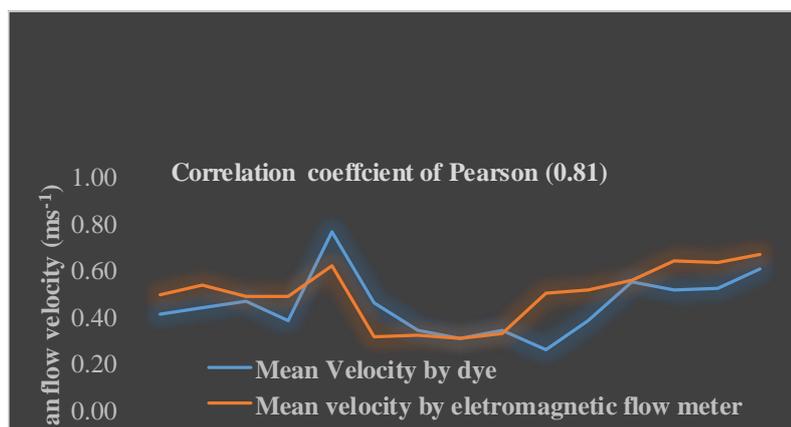


327 **Table 3.** Hydraulic variables under applied flows on larger cohesive channels under a  
 328 cohesive B horizon of the studied Ultisol in Brazil. Average values considering four  
 329 repetitions,  $n = 16$ .

Variables	Applied flows			
	Run 1	Run 2	Run 3	Run 4
$Q$ ( $\text{m}^3 \text{s}^{-1}$ )	0.00897	0.00713	0.00708	0.00593
$\tau$ (Pa)	93.89	88.37	76.20	55.74
$V_m$ ( $\text{m s}^{-1}$ )	0.4987	0.5044	0.3497	0.5988
Re (Adm.)	33060.85	31471.87	23771.38	40403.02
Fr (Adm.)	0.659	0.687	0.456	0.784
$f$	2.97	2.73	4.90	1.22
$P_m$ (m)	0.964	0.831	1.023	1.006
$R_h$ (m)	0.0584	0.0550	0.0599	0.0595
$A$ ( $\text{m}^2$ )	0.0555	0.0450	0.0604	0.0590
$S$ ( $\text{m m}^{-1}$ )	0.1611	0.1611	0.1275	0.0940

All variables were normal distribution by the Kolmogorov test at 5% probability.

330



331

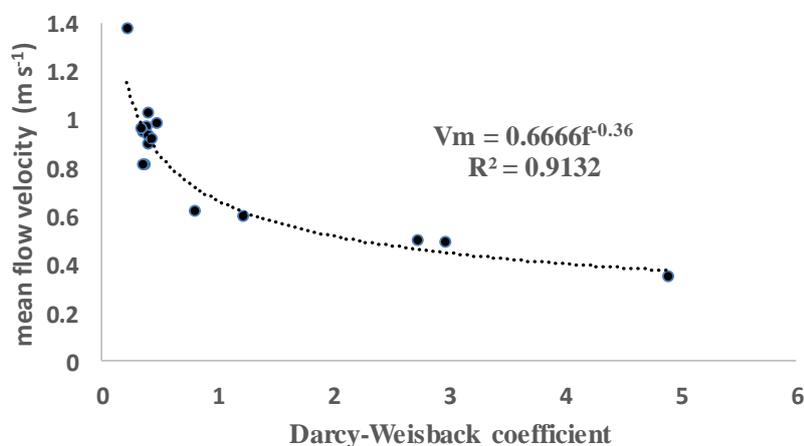
332 **Figure 3.** Relationship between mean velocities measured by the electronic current meter  
 333 and by dye method on cohesive channels under different flows applied. Average values  
 334 considering four repetitions and  $n$  (runs number) = 16.

335

336 In the cohesive channels under applied flows, the resistance hydraulic  
 337 (represented by the Darcy–Weisbach coefficient) showed an inverse response to the mean  
 338 flow velocity (Figure 4), , indicating that the shear stress response to the flow influenced



339 the mean velocity via the residual energy. The shear stress level obtained was comparable  
340 to that of a cohesive bed with a high clay content, as observed by Thoman and Niezgoda  
341 (2008) and Grabowski et al. (2010).  
342



343  
344 **Figure 4.** Relationships between mean flow velocity ( $V_m$ ) and resistance hydraulic  
345 represented by Darcy-Weisbach ( $f$ ) on cohesive channels under different flows applied.  
346 Average values considering four repetitions and  $n$  (runs number) = 16.  
347

### 348 3.2 Hydraulic behavior of all cohesive channels for all levels of applied flow

349

350 Table 4 summarizes the hydraulic behavior observed in the smaller cohesive  
351 channels in response to the different applied flows. Because of the significant increase in  
352 the applied flow, the channel geometry was altered by increasing the wetted perimeter  
353 and area. This allowed for the differentiation of the hydraulic radii and, thereby, an  
354 ultimately significant increase in the obtained shear stress ( $\tau$ ).

355

356

357

358



**Table 4.** Hydraulic variables channels of cohesive channels for different levels of applied flows on cohesive channels. Average values considering four repetitions.

Hydraulic Variables <sup>1</sup>	Different levels of applied flows.			
	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>
Q (L min <sup>-1</sup> )	49.97a	66.01 ab	72.13 b	436.52 c
$\tau$ (Pa)	35.64 b	46.62 b	49.08 b	78.55 a
Re (Adm.)	23073.71 b	31335.26 ab	39040.01 a	32176.78 ab
Fr (Adm.)	1.58 a	1.84 a	1.88 a	0.64 b
$f$	0.482 a	0.388 a	0.363 a	2.956 b
V <sub>m</sub> (m s <sup>-1</sup> )	0.7895 ab	0.9703 a	1.0617 a	0.4879 b
P <sub>m</sub> (m)	0.1319 a	0.1682 ab	0.2282 b	0.9558 c
R <sub>h</sub> (m)	0.0256 a	0.0284 ab	0.0322 b	0.0582 c
A (m <sup>2</sup> )	0.0034 a	0.0047 ab	0.0073 b	0.055 c
S (m m <sup>-1</sup> )	0.1396 a	0.1644 a	0.1528 a	0.1359 a

359 <sup>1</sup>Means followed by the same small letter did not differ in column (Tukey,  $p < 0.05$ ).  
 360

361 The flow regimes for all channels were turbulent and slow according to the  
 362 Reynolds and Froude numbers, respectively. This is in accordance with the findings of  
 363 Simons and Senturk (1992), who reported that this dynamic frequently occurs in natural  
 364 alluvial channels. However, the Froude and Reynolds numbers showed a significant  
 365 increase under the applied flow. Following Slattery and Bryan (1992) and Bezerra et al.  
 366 (2010), we noted whether any of the cohesive channels achieved a Froude number of 2.8.  
 367 The hydraulic resistance obtained for the Darcy–Weisbach coefficients was only  
 368 significant for higher applied flows, most likely attributable to the high clay content of  
 369 the cohesive channels.

### 370 **3.3 Resistance and rheological parameters of cohesive channels: critical shear stress, 371 yield stress, and channel erodibility**

372 Table 5 summarizes the average values for the soil detachment rate (D), sediment  
 373 concentration (C<sub>s</sub>), dimensionless particle diameter (D\*), yield stress ( $\tau_y$ ), dimensionless  
 374 yield stress ( $\tau_r$ ), and Shields critical parameter, as obtained from cohesive agricultural  
 375 channels. As observed in Table 4, significant differences occurred for higher applied  
 376 flows when the yield stress and dimensionless yield stress parameter ( $\tau_r$ ) were different.  
 377 However, these values were obtained from the saturated direct shear stress, thereby  
 378 demonstrating the exact behavior of the cohesive channels under the flow-generated shear



379 stress. Therefore, the yield shear showed an exact difference when the flow shear stress,  
 380 channel geometry, and hydraulic resistance were altered at the highest flow level applied.  
 381 Zang and Yu (2017) indicated that yield stress is primarily related to incipient movement  
 382 under cohesive sediment conditions.

383

384 **Table 5.** Resistance parameters of the cohesive agricultural channels: Cohesion (C),  
 385 failure cohesion ( $C_f$ ), critical shear stress ( $\tau_c$ ), soil erodibility (K), yield shear stress ( $\tau_y$ ),  
 386 dimensionless yield shear stress parameter ( $\tau_r$ ), and Shields's parameter to cohesive  
 387 sediment ( $\Theta_{cr}$ ) for the applied concentrated flows. Average values considering four  
 388 repetitions.

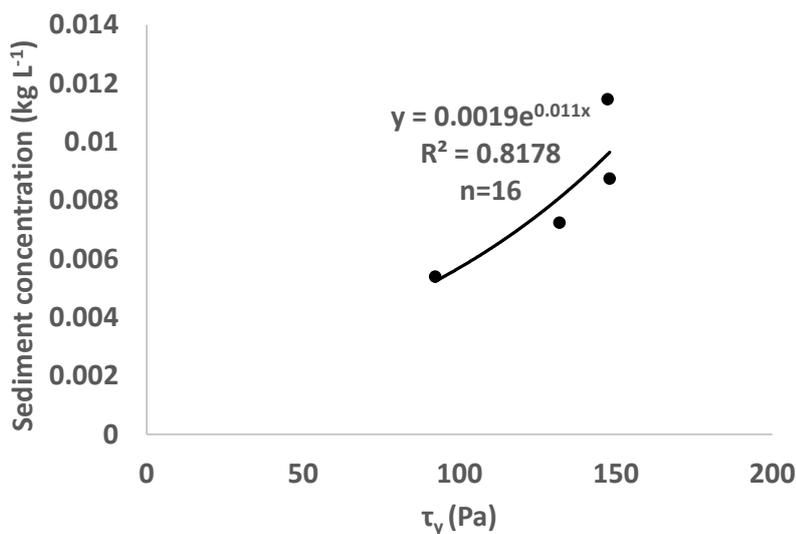
$D_r$ ( $\text{kg m}^{-2} \text{s}^{-1}$ )	$C_s$ ( $\text{kg m}^{-3}$ )	$D^*$	$\tau_y$ (Pa)	$\tau_r$	$\theta_{cr}$
0.003a	1.986a	0.005158a	147.33a	13.36a	0.639a
0.002a	1.741a	0.005415a	148.015a	13.78a	0.639a
0.001a	1.292a	0.006077a	131.98a	11.70a	0.635a
0.001a	0.771a	0.007495a	92.24b	8.30b	0.630a

389 Values followed by the same letter in the column do not differ (Tukey,  $P < 0,05$ ).

390

391 The sediment concentration exhibited an exponential increase in yield stress ( $\tau_y$ )  
 392 (Figure 5), indicating an increase in sediment concentration during the transition from the  
 393 solid to the liquid phases; this was attributable to yield stress. Therefore, when the  
 394 cohesive particles of the saturated channel bed were detached by the yield stress, the  
 395 sediment concentration increased. This finding is in accordance with Yang et al. (2014),  
 396 who stated that the sediment concentration is affected by the rheological properties of the  
 397 cohesive sediment, such as the yield stress.

398



399

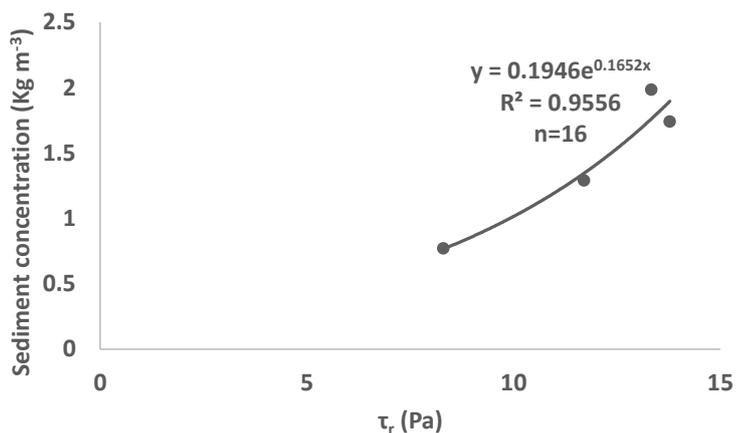
400 **Figure 5.** The exponential relationship between yield stress ( $\tau_y$ ) and sediment  
401 concentration ( $C_s$ ) from cohesive channels under different flows was applied. Average  
402 values considering four repetitions and n (runs number) = 16.

403

404 Similarly, Figure 6 shows a sediment concentration that increases exponentially  
405 with respect to the dimensionless yield stress parameter ( $\tau_r$ ) for a determination  
406 coefficient ( $R^2$ ) of 0.9537; this further demonstrates that the sediment concentration was  
407 affected by the yield stress. Figures 5 and 6 show that the cohesive sediment reacted  
408 similarly to the shear stresses generated by flow in the channels and by direct shear testing  
409 in the laboratory, respectively.

410 A strong exponential relationship ( $R^2 = 0.9791$ ) between the Shields critical  
411 parameter ( $\theta_{\text{cre}}$ ) and the dimensionless particle diameter ( $D^*$ ) was obtained using the  
412 methodology proposed by Zhang and Yu (2017) (Figure 7). In the studies by Van Rijn  
413 (1984) and Yu and Lim (2003),  $\theta_{\text{cre}}$  was negatively correlated with the dimensionless  
414 particle diameter of the alluvial channels. However, the Shields critical parameter ( $\theta_{\text{cre}}$ )  
415 is a positive exponential function of the dimensionless particle diameter of the clay  
416 sediment  $D^*$ , as verified by Zhang and Yu (2017).

417

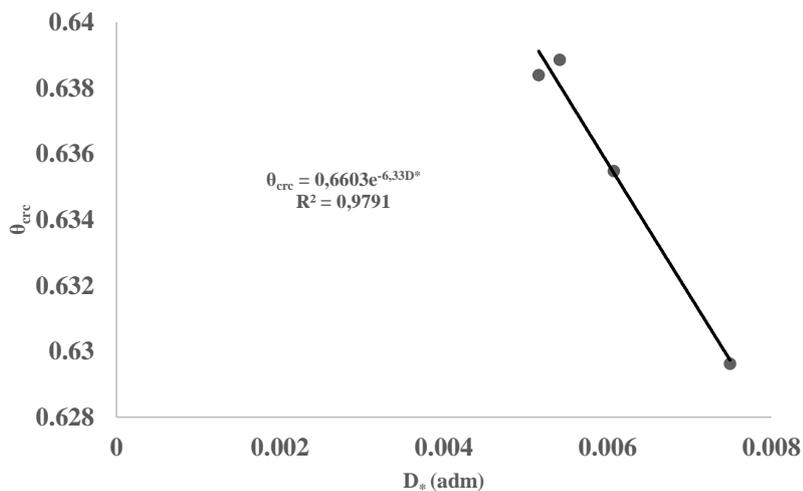


418

419 **Figure 6.** The exponential relationship was applied between dimensionless yield stress  
 420 parameter ( $\tau_r$ ) and sediment concentration ( $C_s$ ) from cohesive channels under different  
 421 flows. Average values considering four repetitions and  $n$  (runs number) = 16.

422

423



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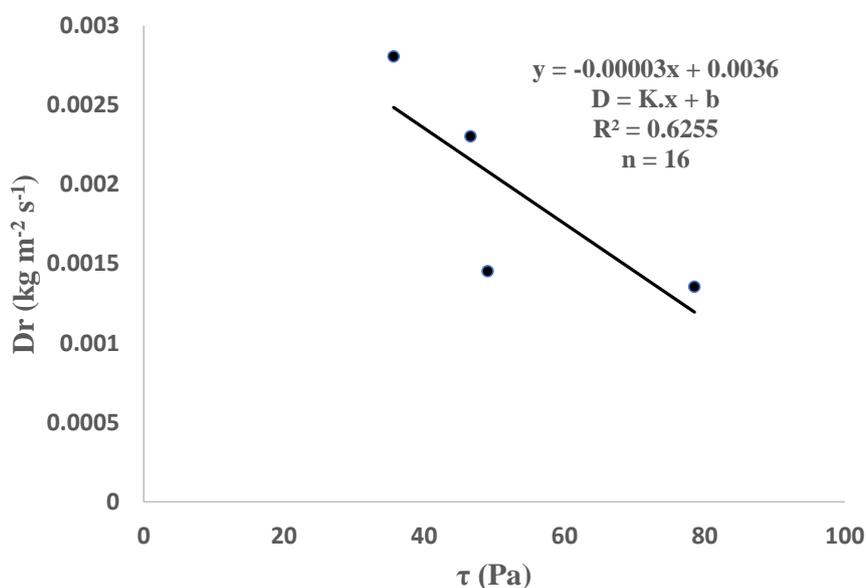
425 **Figure 7.** Cohesive Shields parameter behavior, according to the equation proposed by  
 426 Zhang and Yu (2017), about the dimensionless cohesive sediment diameter.

427

428 The observed critical shear stress ( $\tau_c$ ) and bed erodibility ( $K$ ) were obtained via a  
 429 linear regression between the detachment rates and applied shear stresses, as shown in  
 430 Figure 8. The bed resistance parameters, expressed by the obtained critical shear stress



431 ( $\tau_c$ ) and erodibility, were inversely proportional to the shear stress tension. The critical  
432 shear stress and cohesive bed erodibility were 120 Pa and  $0.00003 \text{ kg N}^{-1} \text{ s}^{-1}$ , respectively,  
433 when the critical shear stress exceeded the applied shear stress. This suggests that the  
434 shear stresses could have been higher; consequently, the straight line in Figure 8 could  
435 have been steeper.  
436



437  
438 **Figure 8.** Relationship between soil detachment rates ( $D$ ) and shear stress ( $\tau$ ) obtained on  
439 cohesive channels under different flows applied. Average values considering four  
440 repetitions and  $n$  (runs number) = 16.

441  
442 Hanson and Simon (2001) obtained critical shear stress values ranging from 1.31  
443 to 256 Pa for a cohesive streambed; these values are similar to the critical shear stress  
444 values obtained in this study. Simon and Thomas (2002) found similar results in the  
445 Yalobusha River Basin in Mississippi, with jet testing results indicating a range of critical  
446 shear stress values ( $\tau_c$ ) from 0 to 400 Pa and a mean value of 105 Pa. These values are  
447 close to the value of 110.8 Pa obtained in the current study. Additionally, Thoman and  
448 Niezgodá (2008), when studying the stability of cohesive channels, obtained high  $\tau_c$   
449 values between 0.11 to 15.35 Pa, and erodibility values between  $0.27$  to  $2.38 \text{ cm}^3 \text{ N}^{-1} \cdot \text{s}^{-1}$ .



450           These findings are similar to those of Geng et al. (2017), who studied the spatial  
451 variations in soil resistance ( $K$  and  $\tau_c$ ) under concentrated flows in 36 different soils, and  
452 who obtained  $K$  values of  $0.000456\text{--}0.826 \text{ kg N}^{-1} \text{ s}^{-1}$ . These authors suggested that clay  
453 exhibited the highest resistance during wetting, owing to the greater cohesion produced  
454 by its number of bonds between clay particles, which decreases its erodibility. Garde and  
455 Raju (2000) showed that for a flat limestone material (10–20 mm thick and 40–60 mm  
456 long), the critical tractive stress is  $\sim 56 \text{ N.m}^2$ . This suggests that in the present study (in  
457 which the predominant particles were clay, with a clay content of 60%), the critical shear  
458 stresses may take high values.

459           Grabowski et al. (2010) and Grabowski et al. (2011) reported that the  
460 hydrodynamic aspects of erosion and sediment transport are thoroughly understood.  
461 However, cohesive sediment erodibility has proven to be more challenging to address and  
462 predict, because interparticle attraction is influenced by many sediment properties that  
463 interact in complex ways. Wuddivira et al. (2013) reported that the detachment forces  
464 acting upon strength and erodibility under tropical conditions arise frequently in cohesive  
465 soils, and that soil erodibility is determined by the complex interactions between the clay  
466 and organic matter involved in the shear strength and erodibility, rather than by a single  
467 factor.

468           The applied shear stresses ( $\tau$ ) followed the increases in the applied flow level,  
469 reaching values between 35 and 73 Pa (Table 4). Statistical analysis indicates a  
470 statistically significant difference at the highest applied flow ( $Q_4 = 545 \text{ L min}^{-1}$ ). This  
471 result indicates that for cohesive channels under a cohesive Ultisol, higher flow rates are  
472 required to increase the soil detachment.

473           Soil cohesion is a crucial parameter for understanding the resistance behavior of  
474 cohesive channels. Thus, the shear stress and soil fatigue strength needed to produce  
475 cohesion failure ( $C_f$ ), according to Mirskhoulava (1966a), were 56135 Pa and 1964.72  
476 Pa, respectively. The cohesion value was high, and the soil fatigue strength to rupture  
477 value reflected the high clay content ( $617.60 \text{ g.Kg}^{-1}$ ) of the cohesive channels at a depth  
478 of 40 cm. This high cohesion value is related to the applied shear stress; therefore, in this  
479 study, higher detachment rate values were not achieved. These results agree with those of  
480 Kothyari and Jain (2008), who observed that the threshold condition can vary with respect  
481 to the clay content, shear stress, and soil moisture.



482 **3.4 Mean velocity obtained and permissible velocity estimated by Mirtskhoulava's**  
483 **equation for the cohesive channels**

484 Table 6 summarizes the observed mean velocities obtained under the applied  
485 flows, as well as the permissible velocities estimated by Mirtskhoulava's equation for  
486 cohesive channels. To estimate the permissible velocities, we used the following  
487 parameters:  $D_{50} = 0.959$  mm,  $g = 9.81$  m s<sup>-2</sup>,  $m = 0.8$  (considering the recently revolved  
488 channel surface),  $\gamma_w = 9771$  t m<sup>-3</sup>,  $n' = 3.84003$ ,  $\gamma_s = 26241.8$  t m<sup>-3</sup>, cohesion = 56135 Pa,  
489 failure cohesion = 1964.72 Pa,  $\sigma = 17.41$ , and  $K = 0.9991$ ; the hydraulic radius ( $R_h$ ) was  
490 as stated in Table 3.

491

492 **Table 6.** Observed mean velocities generated by applied flows and permissible velocities  
493 estimated by Mirtskhoulava's equation on cohesive channels. Average values considering  
494 four repetitions.

Applied flows	$V_m$	$V_{Mirtskhoulava}$
	-----	m s <sup>-1</sup> -----
Q <sub>1</sub>	0.789 ab	1.494 c
Q <sub>2</sub>	0.970 a	1.523 b
Q <sub>3</sub>	1.062 a	1.557 ab
Q <sub>4</sub>	0.488 b	1.719 a

495

496 The observed mean velocity values only differed at higher applied flows, owing  
497 to the large, pre-formed, wetted perimeter and hydraulic radius. Thus, under increased  
498 applied flow, the shear stresses did not generate sufficient detachment to cause significant  
499 erosion.

500 The permissible velocities estimated by Mirtskhoulava's equation exhibited  
501 values that followed the increases in applied flow. Graf (1996) observed water flow  
502 velocities for clay materials, ranging from 1.3 m s<sup>-1</sup> in clay aggregates to 2.87 m s<sup>-1</sup> in  
503 dispersed clay materials. However, according to Mirtskhoulava (1991) and Wuddivira et  
504 al. (2013), clay soils with high cohesion values can support velocities ranging from 1.56  
505 to 2.76 m s<sup>-1</sup>.

506 The shear stresses ( $\tau$ ) obtained from the cohesive channels were not high enough  
507 to produce sufficient detachment, and all observed velocity values measured via the



508 electromagnetic current meter were lower than the permissible velocities estimated by  
509 Mirtskhoulava's equation. These experimental field results indicate that Mirtskulava's  
510 equation adequately estimates the permissible velocities in cohesive channels.

511

### 512 **3.5 Dimensional analysis and permissible velocities in cohesive channels**

513 Dimensional analysis was performed by considering the experimental parameters  
514 involved in measuring the flow velocity in cohesive channels; these constituted the  
515 hydraulic and geometric characteristics of the cohesive channel under a concentrated  
516 flow. Twelve parameters were arranged empirically using the following mathematical  
517 relationship:

$$518 V_m = f(Q, R_h, C_s, D_r, \rho, S, f, C, C_f, \tau_f, \tau_{cr}). \quad (21)$$

519 Here,  $V_m$  = mean flow velocity ( $\text{m s}^{-1}$ ),  $Q$  = liquid discharge ( $\text{m}^3 \text{s}^{-1}$ ),  $R_h$  = hydraulic radius  
520 (m),  $C_s$  = sediment concentration ( $\text{kg L}^{-1}$ ),  $D$  = detachment rate from the concentrated  
521 flow ( $\text{kg m}^{-2} \text{s}^{-1}$ ),  $\rho$  = water density ( $\text{kg m}^{-3}$ ),  $S$  = channel slope ( $\text{m m}^{-1}$ ),  $f$  = the Darcy–  
522 Weisbach hydraulic resistance coefficient (dimensionless),  $C$  = cohesion coefficient (Pa),  
523  $C_f$  = failure cohesion (Pa),  $\tau$  = flow shear stress (Pa), and  $\tau_{cr}$  = critical shear stress (Pa).

524 According to Fox et al. (2014), when applying the  $\pi$  value or Buckingham theorem  
525 for dimensional analysis, the dimensions L, M, and T are considered fundamental for  
526 specifying the dimensions of each of the parameters involved, as follows:  $[V_m] = \text{L T}^{-1}$ ,  
527  $[Q] = \text{L}^3 \text{T}^{-1}$ ,  $[R_h] = \text{L}$ ,  $[C_s] = \text{M L}^{-3}$ ,  $[D] = \text{M L}^{-2} \text{T}^{-1}$ ,  $[\rho] = \text{M L}^{-3}$ ,  $[C] = \text{M L}^{-1} \text{T}^{-2}$ ,  $[C_f] =$   
528  $\text{M L}^{-1} \text{T}^{-2}$ ,  $[\tau_f] = \text{M L}^{-1} \text{T}^{-2}$ ,  $[\tau_{cr}] = \text{M L}^{-1} \text{T}^{-2}$ ; the remaining  $S$  and  $f$  are dimensionless.

529

530 By analyzing the chosen parameters and  $\pi$  properties, we observed that the  
531 channel slope ( $S$ ) and Darcy–Weisbach hydraulic resistance coefficient ( $f$ ) are already  $\pi$   
532 terms, because they are dimensionless and correspond to  $\pi_4$  and  $\pi_5$ , respectively. Because  
533 the variables had the same reference dimensions, other  $\pi$  terms were observed, such as  
534 the water density ( $\rho$ ), sediment concentration ( $C_s$ ), cohesion coefficient ( $C$ ), failure  
535 cohesion ( $C_f$ ), shear stress of the flow ( $\tau$ ), and critical shear stress ( $\tau_c$ ). Accordingly, to  
536 determine the remaining two groups, the parameters  $V_m$ ,  $R_h$ , and  $C_s$  were considered  
537 repetitive, and  $\pi_1$  and  $\pi_2$  were thus determined.

538



539 Because we utilized 10 dimensional variables ( $k = 10$ ) and three dimensions (M,  
540 L, and T) to describe the physical process, the difference between the number of  
541 dimensional variables ( $k$ ) describing a process and the number of reference dimensions  
542 ( $r$ ) resulted in seven dimensionless groups, as follows:

$$543 \quad \pi_1 = \frac{Q}{VR_h^2}, \quad (22)$$

$$544 \quad \pi_2 = \frac{D_r}{VC_s}, \quad (23)$$

$$545 \quad \pi_3 = \frac{\rho}{c_s}, \quad (24)$$

$$546 \quad \pi_4 = S, \quad (25)$$

$$547 \quad \pi_5 = f, \quad (26)$$

$$548 \quad \pi_6 = \frac{c}{c_f}, \quad (27)$$

$$549 \quad \pi_7 = \frac{\tau_f}{\tau_c}. \quad (28)$$

550 Thus, the results were arranged into  $\pi$  groups as

$$551 \quad \pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7), \quad (29)$$

552 and in a dimensionless group arrangement, as

$$553 \quad \frac{Q}{VR_h^2} = f\left(\frac{D_r}{VC_s}, \frac{\rho}{c_s}, S, f, \frac{c}{c_f}, \frac{\tau_f}{\tau_c}\right). \quad (30)$$

554 Finally, the dimensionless terms made it possible to calculate the  $\pi$  numerical  
555 values, as summarizes in Table 7, as well as to proceed with regression analyses and  
556 obtain a new model. All terms obtained (except  $\pi_6$ ) varied, which corresponded to the  
557 relationship between two constant parameters throughout the experiment. Considering all  
558 this information, several relationships pertaining between the dimensionless parameters  
559 were tested, as shown in Figure 9.

560 Figure 9 shows the linearity of the relationships between  $\pi_1$  and  $\pi_2$  and  $\pi_1$  and  $\pi_5$   
561 ( $R^2 = 0.9978$  and  $R^2 = 0.9221$ , respectively) and the exponential relationship between  $\pi_1$   
562 and  $\pi_3$  and  $\pi_1$  and  $\pi_7$ . There was no correlation between  $\pi_1$  and  $\pi_6$ ; consequently,  $\pi_6$  was  
563 not considered in the model development; however, we included 16 runs for each  
564 variable.

565

566

567

568

569



570 Table 7. Numerical values corresponding to the  $\pi$  terms obtained from cohesive channels  
571 represent the flow velocities.

Runs	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$\pi_7$
1	6.75E-07	0.020	325.65	0.161	0.800	33.192	3.006
2	1.16E-06	0.006	505.63	0.141	0.400	33.192	2.906
3	4.76E-07	0.002	748.40	0.128	0.370	33.192	3.811
4	5.40E-07	0.005	640.98	0.128	0.355	33.192	4.019
5	8.72E-07	0.012	317.29	0.188	0.405	33.192	2.206
6	9.96E-07	0.002	762.90	0.161	0.394	33.192	2.537
7	1.05E-06	0.005	534.25	0.141	0.356	33.192	2.950
8	7.64E-07	0.001	1588.59	0.168	0.398	33.192	2.720
9	1.08E-06	0.002	689.24	0.154	0.217	33.192	2.302
10	1.43E-06	0.002	710.18	0.179	0.472	33.192	2.060
11	1.42E-06	0.002	774.47	0.121	0.335	33.192	3.029
12	8.70E-07	0.001	983.16	0.158	0.429	33.192	2.592
13	6.14E-05	0.001	1889.61	0.161	2.970	33.192	1.278
14	4.27E-05	0.002	1476.99	0.161	2.732	33.192	1.358
15	7.26E-05	0.004	1088.94	0.128	4.900	33.192	1.575
16	3.50E-05	0.002	1041.49	0.094	1.223	33.192	2.153

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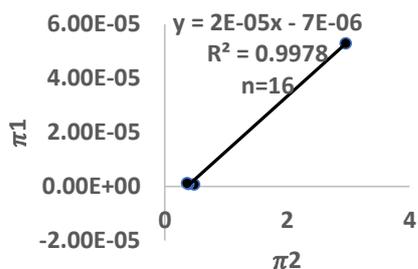
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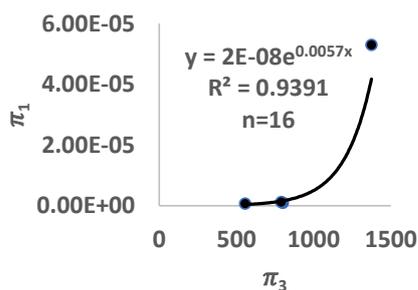


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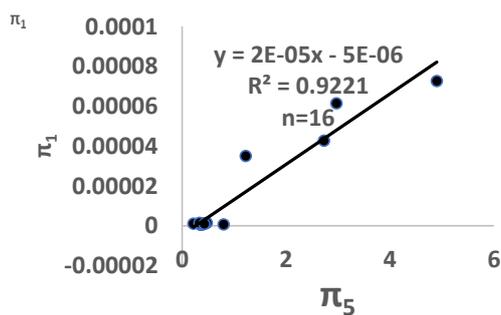
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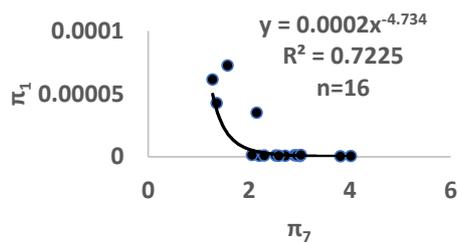
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583



584 **Figure 9.** Relationships between the terms  $\pi$  dimensionless, dependent, and independents  
585 represent the mean velocities obtained from cohesive channels in 16 runs.



586 Table 8 summarizes the permissible velocity values obtained via the dimensional  
 587 analyses that yielded the  $\pi_1$  term and the permissible velocity values calculated using the  
 588 equation proposed by Mirtskhoulava (1991) [Eq. (7)]. The average difference between  
 589 the permissible velocities calculated from the Mirtshloulava equation and  $\pi_1$  term  
 590 equation was only 17.24% of the 16 cohesive channels under increasing applied flows.  
 591 These obtained velocity values are in accordance with Mirtskhoulava (1991); however,  
 592 the clay soils with cohesion values varying between  $5 \times 10^4$  Pa had assumed permissible  
 593 velocities of 1.56–2.72 m s<sup>-1</sup> in the channels exhibiting cohesion values of  $5.6 \times 10^4$  Pa.  
 594

595 Table 8. Permissible velocities (m s<sup>-1</sup>) were calculated according to Mirtskhoulava  
 596 (1966b), and the  $\pi_1$  term was produced from hydraulic variables obtained on cohesive  
 597 channels with 60% clay content.

Permissible velocity Mirtskhoulava (1966b)	Permissible velocity $\pi_1 \left( \frac{Q}{VR_h^2} \right)$	Difference (%)
1.38	1.11	19.57
1.42	1.41	0.70
1.38	1.04	24.64
1.37	1.45	5.84
1.42	1.26	11.27
1.42	1.29	9.15
1.42	1.42	0
1.39	1.48	6.47
1.46	1.14	21.92
1.45	1.23	15.17
1.45	1.17	19.31
1.42	1.07	24.65
1.60	2.63	64.38
1.58	2.36	49.37
1.60	1.97	23.13
1.60	1.68	1.88

598 Permissible velocity equations, such as the Mirtshloulava equation [Eq. (7)] and  
 599 the  $\pi_1$  equation [Eq. (20)], were developed to predict the highest velocity that a flow can  
 600 reach without detaching the cohesive surface channel; however, the permissible velocity  
 601 values obtained (Table 8), based on the shear stress values (Tables 3 and 4) applied in this  
 602 study, were not sufficient to increase the detachment rate values.

603

### 604 3.6 Multiple regression

605 A multiple regression of the  $\pi$  terms was obtained without  $\pi_6$ , which was constant.  
 606 Thus, the dependent variable Y corresponded to X1, X2, X3, X4, and X6. Table 9  
 607 summarizes the correlation matrix for  $\pi$  terms.

608



609 **Table 9.** Correlation coefficient matrix for  $\pi$  terms in dimensionless terms.

Variables	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_7$	$\pi_1$
	$\left(\frac{D_r}{VC_s}\right)$	$\left(\frac{\rho}{c_s}\right)$	(S)	(f)	$\left(\frac{\tau_f}{\tau_c}\right)$	$\left(\frac{Q}{VR_h^2}\right)$
$\pi_2 \left(\frac{D_r}{VC_s}\right)$	<b>1</b>	-0.614	0.217	-0.114	0.221	-0.225
$\pi_3 \left(\frac{\rho}{c_s}\right)$	-0.614	<b>1</b>	0.006	0.549	-0.556	0.647
$\pi_4$ (S)	0.217	0.006	<b>1</b>	-0.132	-0.304	-0.224
$\pi_5$ (f)	-0.114	0.549	-0.132	<b>1</b>	-0.680	0.960
$\pi_7 \left(\frac{\tau_f}{\tau_c}\right)$	0.221	-0.556	-0.304	-0.680	<b>1</b>	-0.727
$\pi_1 \left(\frac{Q}{VR_h^2}\right)$	-0.225	0.647	-0.224	0.960	-0.727	<b>1</b>

610 Significance level:  $p < 0,05$ .

611

612 In the regression method, a high coefficient determination was obtained,  
 613 suggesting a 98% dependent variable variability, which was explained using the five  
 614 independent variables, as shown below:

615

$$616 \pi_1 = (4.75)10^{-5} + (3.71\pi_2)10^{-4} + (1.02\pi_3)10^{-5} - (2.40\pi_4)10^{-4} +$$

$$617 (1.20\pi_5)10^{-5} - (9.73\pi_7)10^{-6}. \quad (31)$$

618 Substituting the  $\pi$  terms via the variables applied in the dimensional analyses  
 619 produced

$$620 \frac{Q}{VR_h^2} = (4.75)10^{-5} + \left(3.71 \frac{D_r}{VC_s}\right) 10^{-4} + \left(1.02 \frac{\rho}{c_s}\right) 10^{-5} - (2.4 S)10^{-4} +$$

$$621 (1.20 f)10^{-5} - \left(9.73 \frac{\tau_f}{\tau_c}\right) 10^{-6}, \quad (32)$$

622 which predicts the permissible velocity in cohesive channels.

623 The regression model performed well under field conditions in the cohesive  
 624 channels for the applied flows and observed hydraulic variables; however, the latter  
 625 variables should be tested under higher-level flows.

626

#### 627 4. Conclusions

628 **Based on the flows applied to cohesive channels in the field experiment, the**  
 629 **following conclusions can be drawn:**

630 The soil resistance expressed by the critical shear stress ( $\tau_c$ ) and erodibility was  
 631 inversely proportional to the applied shear stress tension. In addition, a critical shear stress



632 and channel erodibility of 120 Pa and 0.00003 kg N<sup>-1</sup> s<sup>-1</sup>, respectively, were observed for  
633 the cohesive channel under the B-horizon of Ultisol, higher than the shear stress generated  
634 by the applied flows. This suggests that the applied shear stress flows could have been  
635 higher, and that higher soil detachment rates could have been realized.

636 The high cohesion value obtained and the soil fatigue strength to rupture value  
637 reflected the high clay content (617.60 g/Kg) of the cohesive channels. These cohesion  
638 values were related to the applied shear stress.

639 The yield stress was significant for the geometric alteration, flow velocity, and  
640 sediment concentration of the cohesive channels under the highest applied flow.

641 These results indicate that Mirstkulava's equation performed adequately in this  
642 field experiment when estimating the permissible velocities on cohesive channels.

643 The dimensional analysis applied to the obtained hydraulics variables produced  
644 the following equation, which predicts the permissible velocity in cohesive channels:

$$\begin{aligned} 645 \frac{Q}{VR_h^2} = & (4.75)10^{-5} + \left(3.71 \frac{D_r}{VC_s}\right) 10^{-4} + \left(1.02 \frac{\rho}{C_s}\right) 10^{-5} - (2.4 S)10^{-4} + \\ 646 & (1.20 f)10^{-5} - \left(9.73 \frac{\tau_f}{\tau_c}\right) 10^{-6}. \end{aligned} \quad (32)$$

647 This regression model performed well under field conditions in the cohesive channels for  
648 level-applied flows and observed hydraulic variables; however, the latter variables need  
649 to be tested under higher-level flows. We hope that this study will serve as a technical  
650 reference for engineers seeking to build effective, appropriate river structures in cohesive  
651 channels.

652

### 653 **Statements and Declarations**

654 There is no interest conflict and no disclosed or non-financial interests related to  
655 submitting this manuscript and future publication between the authors.

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659

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