Supporting Information: Frequency domain water table fluctuations reveal recharge in fractured aquifers depends on both intense and seasonal rainfall and unsaturated zone thickness

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Exploring the parameter set using a radial groundwater model

Setting up the radial model

Here, the impact of model geometry on aquifer parameters estimates and recharge fluctuations is explored. This section and the next one are focused on the pumping case. In this case, the 1D analytical groundwater model need a "width" parameter. Another option is to consider a radial model instead of the 1D model.

As for 1D geometry, an analytical solution is obtained in radial coordinates involving Bessel functions (Matlab models are available on http://hplus.ore.fr/en/guillaumot-et-al-2020-wrr-data). In this case, pumping rate is applied at the center of the system, at \( r = 0 \). At \( r = L \), we keep an imposed constant head. Recharge rate is still uniformly distributed. As pumping is equally distributed in all directions, boreholes position is only defined by their distance from pumping wells as for the 1D model. Finally, such radial model contains only three parameters: transmissivity, storativity and length. As for the 1D model, characteristic time is defined by \( t_c = \frac{L^2}{D} \), \( D \) being the hydraulic diffusivity. For each parameter, a realistic range is set and regularly sampled: transmissivity \( T \in [10^{-4};1.10^{-1}] \ m^2.s^{-1} \) (40 values), storage coefficient \( S \in [10^{-3};2.10^{-1}] \) (35 values), length \( L \in [800;5000] \ m \) (43 values). For a better comparison with the 1D model, only the "Thornthwaite" recharge model is given as input. Thus, a total of 60200 models where run for the Ploemeur site using this radial model. Groundwater levels are simulated from 1996 to 2012.

Results of the parameter set exploration

In the same way as developed in the previous 1D model, storativity is the more constrained parameter. In addition, as for the 1D model, characteristic time is still well constrained and defined for each borehole (Fig. S1). However, estimated values are typically two times higher when using the radial model. Consequently, we can conclude that the radial model gives the same answer, regarding water table fluctuations, than the 1D model with a higher characteristic time. Such difference is explained by the fact that hydraulic head propagation (the propagation of recharge events mainly) is faster in radial model even if the characteristic time is the same. This difference is due to groundwater flow convergence in radial system.
Exploring the parameter set including the "width" parameter ($W$) in the 1D groundwater model

Setting up the 1D model including $W$ as parameter

In the study described in the main text, the width of the 1D model in the pumping case was set to 1000 $m$. Intuitively, for a given model, increasing (decreasing) $W$ decreases (increases) pumping influence on simulated hydraulic head. In order to investigate the impact of model width $W$ on aquifer parameters estimates, we run the parameter set exploration adding $W$ as a free parameter.

For each parameter, a realistic range is set and regularly sampled: transmissivity $T \in [10^{-4}; 1.10^{-1}]$ $m^2.s^{-1}$ (40 values), storage coefficient $S \in [10^{-3}; 2.10^{-1}]$ (35 values), length $L \in [800; 5000]$ $m$ (43 values) and $W \in [100; 1500]$ $m$ (12 values). For a better comparison with the radial model, only the "Thornthwaite" recharge model is given as input. Thus, a total of 722400 models were run for Ploemeur site using this 1D model. Groundwater levels are simulated from 1996 to 2012.
Results of the parameter set exploration

Parameter set exploration, including model width as parameter, indicates that characteristic time estimate is not sensitive to the model width as long as $W > 500 m$. Indeed, for each borehole the estimated characteristic time is very close from values found with $W$ imposed to 1000 m (Fig. S2 upper graph). Next, we found that model length ($L$) and width ($W$) are not completely constrained but both parameters show a lower limit. This information is summarized on Figure S2 (lower graph). The model surface (length times width) clearly has to be higher than $1.5 \, km^2$, unless for borehole F20 and F11 whose the threshold could be $1 \, km$. As a comparison, mean pumping rate divided by mean potential recharge rate (from Thornthwaite and SURFEX models) gives a recharge surface roughly equals to $3.8 \, km^2$.

Figure S2. Evolution of the minimal normalized $RMS$ error for Ploemeur wells as a function of parameters "characteristic time" (upper graph) and "surface" (lower graph), defined by length times width, when using the 1D groundwater model.
3 Impact of the model geometry on estimated recharge fluctuations

To compare recharge fluctuations sensitivity to model geometry, recharge fluctuations are inverted analytically using the best aquifer parameters for the radial model and two 1D models. Recharge fluctuations are computed from water table fluctuations observed at boreholes F9 and F19. These boreholes are chosen because of their different situation. Indeed, F9 is located to the West, far from the pumping site, while F19 is located to the South and is more sensitive to pumping rates fluctuations.

Figure S3 shows recharge fluctuations estimated from the radial model and from two 1D models with $W = 917 \text{ m}$ and $W = 1500 \text{ m}$. The figure highlights that results are very similar whatever the model geometry. Whatever the width parameter value, recharge fluctuations estimates overlap (blue and green lines). Recharge fluctuations estimated from the radial model (red line) are also almost similar. Main differences occur during summer season when pumping is higher and recharge close to zero. This difference is more important on borehole F19 which is closer from pumping wells.
Figure S3. Comparison between potential recharge fluctuations \([\text{mm.month}^{-1}]\) estimated from Thornthwaite model and recharge fluctuations estimated from observed water table at boreholes F9 (upper graph) and F19 (lower graph) on Ploemeur site. Recharge fluctuations from observed water table are estimated using the radial model or two 1D groundwater models with \(W = 917\,\text{m}\) and \(W = 1500\,\text{m}\). "Fluctuations" means that time variations are expressed as anomalies compared to the mean value along the study period.

### 4 Comparing estimated recharge fluctuations at two boreholes on Ploemeur

Recharge fluctuations estimated from WTF on the pumping site are quite similar. However, recharge noise increases when the borehole is well connected to the pumped fractured zone, as suggested by difference in RF between F9 and F19 ((Fig. S4)), respectively 519 m and 268 m far from the pumping station.
Figure S4. Comparison between recharge fluctuations (anomalies compared to the mean value) from three potential recharge models (Thornthwaite, GR4J, SURFEX) and from GW analytical model (in mm.month\(^{-1}\)) at boreholes F9 (upper graph) and F19 (lower graph) on Ploemeur site.

5 Impact of the size of the study period on storage coefficient estimates

While the main study was focusing on time series of several years, we proposed here to study the impact of the length of time series. Therefore, we used the same 1D groundwater model to calibrate water table fluctuations series of different lengths. The same methodology was used. We simply compared the informative content of observed water table fluctuations in function of the length of the time series.

First, water table variations recorded in well F19 at Ploemeur site was decomposed in five time series. Then, each time series was converted into fluctuations by removing the mean value of each time series. The obtained water table fluctuations are presented on Figure S5 (black lines) as well as the original data set. Note the two first times series (top-left on Figure S5) focus on smaller periods of 1 and 4 months. Each time series has been filtered to keep only a range of frequencies contained in the initial water table fluctuations:

1. Frequencies between 1/30 and 1/8 days\(^{-1}\)
2. Frequencies between 1/120 and 1/30 days\(^{-1}\)
3. Frequencies between 1/8 and 1/4 month\(^{-1}\)
4. Frequencies between 1/18 and 1/8 months\(^{-1}\)
5. Frequencies < 1/2 years\(^{-1}\)

Thus, each time series focuses on a specific time scale. Figure S5 shows the best models (red lines) obtained from the parameter set exploration. Criteria (normalized \(RMS\) error) highlight a good fit between observed and modeled water table fluctuations.
whatever the studied frequency range. However, the model seems better to reproduce low frequencies variations and specifically frequencies between 1/18 and 1/8 months$^{-1}$.

Regarding parameters estimation, we focused here on storage coefficient. Interestingly, this parameter is less constrain (larger uncertainty) at high frequencies (< 1/Amonths$^{-1}$). Moreover, estimated values are lower for the two highest frequencies as illustrated on Figure S6. It means that storage coefficient is lower at short time scale than at long-term scale (typically > 200days). Finally, Figure S6 also shows that at low frequency variations, we obtained the same storage coefficient than the value (around 0.04, dotted red lines) obtained from the initial time series of water table fluctuations.

Figure S5. Water table fluctuations observed at borehole F19 (black lines). The graph in the middle represents the initial data. From initial data, five data sets of water table fluctuations are generated. For each time series, different period lengths are considered and only a range of frequencies are studied.
Figure S6. Estimated storage coefficient in function of the studied time scale ("Period"). Note water table fluctuations are all well reproduced by the model whatever the frequency range.