hess-2022-16 Responses to comments by referee #2

Dear Editor, dear Referee,

We thank the second referee for the review of our manuscript. We will in the following reply to the comments point by point. The Referee comments are in blue.

Comment 1: The technical note is on the topic of applying information theory to the analysis of observed hydrometeorological time series (precipitation and flow). The method contributed here (as far as I understand) consists of computing the entropy over many slices (windows) of the original time series varying the width of the slices. These entropies are seen as measures of system uncertainty. A separate measure of system complexity is defined by the "entropy of entropies". The "system uncertainty" is graphed versus "system complexity" and this curve is given the term "c-u curve". This analysis is applied to several time series including from general dynamic systems (Lorenz attractor) and from hydrological systems (basin streamflow). I believe the proposed ideas hold merit and can potentially make a valuable contribution to data analysis in hydrology.

Reply 1: Thank you for the concise summary of our approach and the overall positive evaluation.

Comment 2: However the current presentation and its format make it difficult to ascertain the contributions. The paper is submitted in the format of a "technical note", but it gives the impression of a full paper with the introduction / literature review essentially missing, …

Reply 2: Presenting the approach as a technical note or research paper: This question was also raised by referee #1 (RC1 comment 2), so obviously this is an important point to consider. Initially we indeed thought about presenting the c-u-curve method in a full scientific paper, with the method description and a range of applications, including:

- Hydrological classification: Use data from hydrologically distinctly different catchments such as groundwater dominated, interflow dominated, dominated by reservoir operation, arid, humid and snow-influenced catchments from large data sets (e.g. Addor et al. 2017, Kuentz et al., 2017) and see if and how these differences are reflected by c-u-curve properties).
- Comparison to existing hydrological classifiers and signatures (such as those discussed in Jehn et al., 2020; Addor et al. 2018; Kuentz et al., 2017) at the example of large data sets (e.g. Addor et al., 2017). This includes evaluating the classification power of c-u-curve and the evaluation how similar or dissimilar its classifications are to those from existing classifiers.
- Use for model improvement: Test if c-u-curve characteristics can be used for targeted model improvement, either as an objective function for parameter identification during model calibration, or as a signature which may point to model structural deficiencies
- System analysis: Compare c-u-curve characteristics for input, internal states and output of hydrological systems to analyse system behaviour: Are uncertainty and complexity increasing or decreasing on the way through the system?

Looking at this list we decided it will be too much material for a single paper to introduce the method and provide all these use-cases. So instead we decided to introduce the c-u-curve method in a compact manner in a technical note, along with a few examples illustrating its properties and behaviour, and presenting the other aspects in a follow-up scientific paper. We suggest that this is the best compromise for rapid yet thorough presentation of the method. To clarify this to the reader, we suggest adding in a revised version of the manuscript a sentence to the last paragraph of section 4 ('summary and conclusions'), explaining possible further avenues of research along the above bullet-point list.

Comment 3: … and with important method descriptions referenced to a Zenodo archive (line 64) which is not a peer-reviewed publication.

Reply 3: Ehret (2022), the Zenodo archive to which the referee refers, is not an extra publication but a part of this manuscript. It provides all the code required to reproduce the results of the manuscript, and as such it is required by the HESS publication rules. Through the review process of this manuscript, it will become part of a peer-reviewed publication. Please note that the step from uni- to multivariate, and from deterministic to ensemble application is very straightforward (as described in 'test_c_u_curve.m' on Zenodo): It simply means to expand the input data set into the second dimension (for multivariate) and into the third dimension (for ensemble). Therefore, as mentioned in the manuscript in lines 63-65, for clarity we use the 1-d deterministic case for demonstration of the method in the manuscript, and point the reader to the demo examples on Zenodo for the other cases.

Please note that since the first submission of the manuscript, we have updated the code archive. The DOI for the new archive is https://doi.org/10.5281/zenodo.7124382. We will update the DOI in a revised version of the manuscript.

Comment 3: As such the entire literature review seems limited to 4-5 lines (lines 48-52). This would seem insufficient for a publication claiming to provide a general method for analysis of dynamic systems.

Reply 3: We agree that the literature review should be more comprehensive. In a revised version of the manuscript, we suggest to include and briefly discuss the work on complex system analysis in general by Ladyman et al. (2013), Lloyd (2001), LopezRuiz et al. (1995), Feldman and Crutchfield (1998), and hydrological complexity in particular by Jakeman and Hornberger (1993), Vapnik (2006), Vapnik and Chervonenkis (2015), Pande and Moayeri (2018), Omabdi et al. (2021).

Please also see our reply to comment 4, where we suggest further literature to be included in a revised version of the manuscript.

Comment 3: The intended scope of the contributed method is also not clearly defined. If the claimed contribution is for general systems, then the literature review should be far more general than the hydrological literature. Otherwise if the contribution is made in the context of hydrological data analysis then the title and abstract should be toned down and made more specific.

Reply 3: Like the multiscale entropy method the referee mentions in comment 4, the c-u-curve method is applicable to any dynamical system, which we point out in the abstract (lines 7-10 and 18-19) and in the summary and conclusions (lines 256-257), and which we illustrate by using three typical synthetical time series (constant, random noise, Lorenz attractor). As our domain expertise is in hydrology, and as hydrological systems - and signals coming from them - often qualify as complex (see the related explanation in lines 46 – 48), we additionally use hydrological time series to demonstrate that the related c-u-curve characteristics are in accordance with domain (here: hydrological) system understanding. We therefore suggest that we made the general scope of the proposed method sufficiently clear in the manuscript. Nevertheless, we agree with the referee that we should give a more detailed overview on dynamical system analysis in general. For this, please see our reply on comment 4.

Comment 4: In terms of previous work, the c-u method looks similar to the "Multiscale Entropy" methods which also look at entropies for a range of time resolutions, and also define terms of uncertainty, complexity, etc. For example a quick search indicates:

- Hu, M., Liang, H. (2017). Multiscale Entropy: Recent Advances. https://sapienlabs.org/lab-talk/understanding-multiscale-entropy/

- Wu et al (2013) Entropy, 15(3), 1069-1084; "Time Series Analysis Using Composite Multiscale Entropy; https://doi.org/10.3390/e15031069

I am not implying the methods are the same, however as a reviewer I believe if a journal paper (regardless of its format as full paper or technical note) claims as its contribution a new way of using entropy to quantify uncertainty-complexity of general dynamical systems then the onus is on the authors to conduct a thorough literature review, define the scope of the innovation, discuss advantages and disadvantages with respect to existing approaches, etc.

Reply 4: Thank you for pointing us at methods related to the concept of multiscale entropy (MSE). Indeed the c-u-curve shares with MSE the idea that the entropy values are calculated for various aggregations of the original data, and that from the joint display and comparison of these entropy values much can be learned about the underlying dynamical system. The difference is that in MSE, the aggregation is typically done by adding consecutive values, and aggregated entropies are plotted versus the time-scale of aggregation, while in the c-u-curve method, entropies are always computed from the original data, but in blocks of various sizes, and that for a given block-size, complexity is calculated as the entropy of all block-entropies. In a revised version of the manuscript, we suggest to briefly introduce multiscale entropy methods in section 1 (Introduction), and discuss their main similarities and differences with the c-u-curve method in section 2.1 (Method description), including the following references: Costa et al. (2002), Li and Zhang (2008), Guzmán-Vargas et al. (2008), Brunsell (2010), Wu et al. (2013).

Also, when revisiting the manuscript, we found another interesting property of the c-u-curve, which links to work by Conrad (2022). We suggest including a short description of the property, and the reference into a revised version of the manuscript. The following explanation of the property is copied from the reply to referee #1 (last comment).

Pondering over many versions of Fig. 3 in the manuscript, it occurred to us that an upper limit of complexity as a function of uncertainty exists, which in parts (for very low and high uncertainty) is lower than the general upper limit indicated by the "max complexity" line. To explore this, we calculated c-u-pairs for all time series we had used, for many block averaging schemes, and for many slice widths, and plotted the c-u-pairs in Fig. R1 (see below). Please note that the axes in Fig. R1 are scaled to [0,1], just to give the editor and referee a flavour of a normalized version of the c-u-plot. Obviously, an upper complexity limit emerges. It arises from the fact that for very small and very high uncertainty values - which is the mean of all time slice entropies – the variability of these values – which is complexity – is limited. So an upper hull curve for complexity should arise from solving the question "what is the highest-entropy discrete distribution for which the mean is known?". This limit indeed exists, as shown by Conrad (2022), Theorem 5.12, and Example 5.13. The solution is semi-analytical, i.e. the unknown value of parameter β_0 in Eq. (5.10) is determined numerically, but the overall shape of the distribution function is analytically determined, and an exponential function of the expected values of each bin and β_0 (Eq. 5.5). In Fig. R1, this new limit is plotted as red line. Obviously, the limit serves as a useful reference, against which complexities of time series of interest can be compared. E.g. the overall area under the theoretical limit could be seen as an upper limit for total integral complexity achievable by a time series, which it would reach only if it is maximally complex for each slice width. Taking the ratio between the reference area and the area under a particular c-u-curve could then serve as a single-numbered measure of the overall complexity of a time series. We therefore suggest introducing and discussing this limit in a revised version of the manuscript in section 3.2, where we also explain the other bounds (lines 160 pp). We also suggest plotting this limit in Figs 2 and 3, and in the Appendix provide the relevant equations and procedure to calculate it.

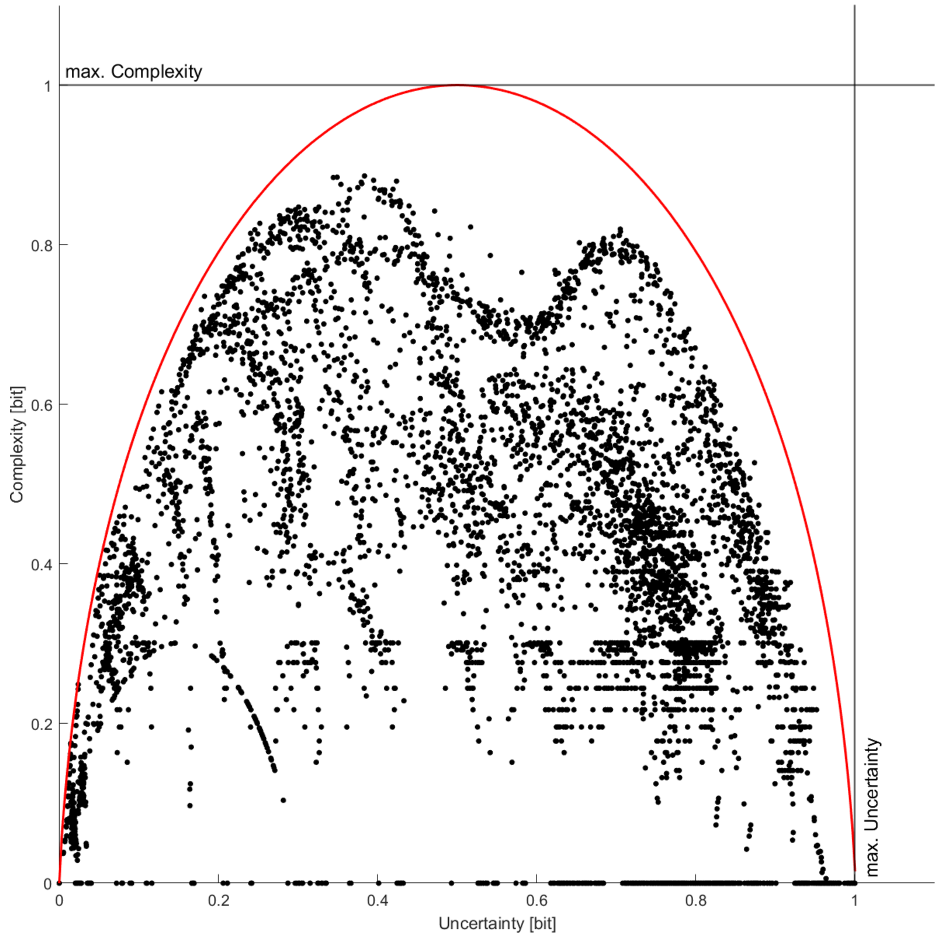

Fig. R1

Comment 5: The results section where the method is applied to basin precipitation and flow provides interesting ideas regarding the controls on streamflow complexity, for example lines 220-245 where the complexity is contrasted for two basins. For the benefits of the HESS audience, I would suggest more emphasis on this type of understanding would be useful.

Reply 5: We are glad the referee finds the hydrological application examples we provide in the manuscript useful. As explained in our reply to comment 2, we intend to provide an in-depth application to hydrological examples in a follow-up scientific paper. For the technical note, we believe that the mixture of general (line, random noise, Lorentz attractor) and hydrological applications (precipitation and streamflow STR, streamflow GR) is small enough to keep the technical note sufficiently lean, but suitable to demonstrate that the method is both generally applicable, and that it provides results which are in line with domain expertise when applied to data of a particular domain (here: hydrology).

Comment 6: Readers without a strong background in information theory would also benefit from some help on how to interpret information-theory concepts, for ex the axis scale in "bits" (eg Fig 3) and how to relate it to more common hydrological units.

Reply 6: In the description of the method in Sect. 2.1 (lines 79-82), we provide an intuitive interpretation of entropy measured in bits, but we agree with the referee that this may be too short for readers not yet familiar with the concept. In a revised version of the manuscript, we suggest adding to Sect. 2.1 a short discussion of how entropy compares to variance, a measure of spread of a data-distribution most readers are familiar with. We also suggest adding to the captions of Fig. 2 and Fig. 3 a reference to Sect. 2.1 for more information on how to interpret the axis units.

Comment 7: I would also suggest helping the reader through the results and discussion sections. Perhaps identify in advance some aims for this analysis and then follow them thru. Otherwise these sections are quite monolythic and a bit hard to follow.

Reply 7: We agree and suggest adding to a revised version of the manuscript, at the beginning of section 3.2, a short overview on the structure and content of the section.

Comment 8: Overall I recommend a major revision to address these issues and produce a clearer and stronger contribution.

Reply 8: We hope that the changes to our manuscript as proposed in the replies to comments 1-7 meet the referee's request to make our manuscript both clearer and stronger.

Yours sincerely,

Uwe Ehret and Pankaj Dey

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