Supplement S1

March 2, 2022

1 Budyko Equations

$E = E_p P \left(E_p^n + P^n\right)^{-\frac{1}{n}}$

To express E/E_p as a function of P/E_p, we will divide both sides by E_p, then again divide the nominator and denominator of the right hand side by E_p and substitute P/E_p for P/E_p to obtain:

$E/E_p = P/E_p \left(P/E_p^n + 1\right)^{-\frac{1}{n}}$

Proof that multiplication of eq_EEp_Rod1 by E_p recovers eq_E_Rod1:

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True
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Similarly, to express E/P as a function of E_p/P, we will divide both sides of eq_E_Rod1 by P , then again divide the nominator and denominator of the right hand side by P and substitute E_p/P for E_p/P to obtain:

$E/P = E_p/P \left(E_p/P^n + 1\right)^{-\frac{1}{n}}$

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True
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$E = E_p + P - \left(E_p^\omega + P^\omega\right)^{\frac{1}{\omega}}$

$E/P = E_p/P - \left(E_p/P^\omega + 1\right)^{\frac{1}{\omega}} + 1$

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True
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$E/E_p = P/E_p - \left(P/E_p^\omega + 1\right)^{\frac{1}{\omega}} + 1$

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True
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