Final response

We would like to thank the referees and the editor for their helpful comments. We made changes accordingly in a revised version of the manuscript. Here, we respond to the editor comments and provide a list of changes in the manuscript. In addition, we provided our responses to the referees again, but added more specific details about the changes. The editor and referee comments are written in italics in the following, line numbers in our responses refer to lines in the revised manuscript.

Response to Editor Comments

Your manuscript has now been seen by 2 referees, who both have provided very in-depth and constructive comments. While they both agree that the manuscript is suitable for a technical note, but also have a number of comments that prevent me from accepting the manuscript in its present form. In particular referee #2 comments on the length of the manuscript, and a lack of a more mathematically-based discussion. Given the nature of the comments, I believe revisions are needed, and I will check the revised version with the referees again before making a decision.

Thank you for your assessment. We added a more mathematically-based discussion in the revised version in Section 3.1. Please note that referee #2 only commented on the length of the abstract, which we shortened in the revised manuscript.

List of changes

- Abstract

We shortened the abstract by removing some technical details, as requested by referee #2.

- Introduction

We updated the historical perspective, as requested by referee #1. We also added more references to recent applications of the Budyko framework, as commented on by referee #2. At the same time, we tried to emphasize how the n-parameter is interpreted in the literature.

- Methods

We added Equation 1, to provide some more mathematical context, as asked for by referee #2.

We added also the number of arid and wet catchments in Section 2.4.

- Results and Discussion

In Section 3.1, we added a more mathematically based discussion.

In Section 3.2, we added more discussion about the interpretation of the distances in the different projections.

The axes of Figure 2 have been adjusted, as suggested by referee #2.

- Supplements

We added Supplement S2 based on a question of referee #2.

Response to Referee #1

We would like to thank Referee #1 for the constructive comments. The Referee mentions several important points that we will improve on in a revised version of the manuscript. Below, we address the comments of Referee #1, with the referee comments written in italics.

This is a good manuscript, appropriate for a technical note. Your historical description is unprecise: since you chose to start with an historical perspective, your history should not be approximate!

Thank you for this comment and the given references. We updated our historical description in the introduction (lines 25-33).

All the Budyko framework starts with Oldekop's work (1911), if you want to get convinced by yourself, the original publication of Oldekop is available as a supplementary material of one of our papers : Andréassian et al. (2016). This is not a secret, Budyko cites widely the work of Oldekop as his source of inspiration. Recently, Zhang and Brutsaert (2021) even suggested to rename the « Budyko framework » into the « Oldekop framework »;

Indeed, we agree that the work of Oldekop (1911) served as a base of Budyko's work. We also mention this in line 8, but will more specifically state this in our revised manuscript. However, we also believe the work of Schreiber (1904) is important to mention here, as Budyko took the arithmatic mean of both curves. Note also that Zhang and Brutsaert (2021) talk about the Schreiber-Oldekop hypothesis.

From the point of view of the graphical representation, there were originally two concurrent representations : that ot Turc (1954) and that of Budyko (1948). Nobody seems to have noticed this difference, and as far as we know the paper by Andréassian et al. (2016) is the first to mention it and to present both representations side by side. Other authors are now using this distinction (see e.g. Moussa & Lhomme, 2016 ;Porporato, 2022). Note that we failed to explain clearly this distinction in our 2012 paper (Andréassian & Perrin, 2012)... shame on us ;

Thank you for these references, we included them in our revised manuscript (line 53).

As far as I know, the most complete history of the Budyko-type formulas (Turc-Mezentsev and Tixeront-Fu) has been published as an appendix in Andréassian & Sari (2019) ;

Also here, thank you for these references, they are very helpful and we included them in our revised manuscript (line 33).

line 10: this is inexact. This formula was proposed independently by Turc (1954) in France and Mezentsev (1955) in the Soviet Union. This is why we most often name it « Turc-Mezentsev »

We corrected this in our revised manuscript (lines 29-30).

Response to Referee #2

We would like to thank Referee #2 for the comments and thoughts. We will improve on the issues in a revised version of the manuscript and give thorough thought about the ideas of the referee. Below, we address the comments of Referee #2, with the referee comments written in italics.

(1) I appreciate the thorough assessment of differences occurring due to the use of different projections. However, from my point of view, there is a different understanding in the interpretation of the n-value between the two projections per se. The n-value has no apriori physical interpretation and is, technically, a mere mathematical entity. That means there is no necessity for the n-value to be invariant under different projections. Or is there? As much as I appreciate the illustration of your results using real-world data, I think a more mathematically-based discussion of the issue is also needed.

We agree with the referee that the n-value does not have a real physical meaning and can be considered a mathematical entity. However, many authors relate it to physical variables and attempt to give a physical explanation to the parameter. For example, several studies relate the parameter n to vegetation, such as Zhang et al. (2001), Yang et al. (2009) or Ning et al. (2017). Donohue et al. (2012) related this parameter even to a larger set of variables including local rooting depths, storm depths and soil water storage capacities. Moreover, Roderick and Farquhar (2011) argued that this parameter reflects all local catchment properties combined (except for climate). We discuss the meaning of n and the notion that n should be constant as catchment properties or vegetation stays constant more in our accompanying manuscript (Nijzink and Schymanski, 2022). Here, we now added more discussion in the introduction in lines 39-45. Nevertheless, if n is indeed related to physical variables (we do not assess this in this technical note and do not take a stand on this), these n-values should be the same for the different projections. At least, we would like to emphasize that if n-values are compared between studies, authors should keep in mind that their values may depend on the projections used.

We added a more mathematically-based discussion. Technically, the two projections can be re-written into another and we will mathematically illustrate that for any combination of E, Ep, and P, the n-value is the same regardless of the projection. In this way, we believe that the meaning of the n-parameter should not change. We added Equation 1 for this, and described how Equations 2 and 3 result from it. At the same time, we added more discussion in lines 160-164.

(2) A similar comment can be made concerning the deviations from the curve and the limits. The interpretation of deviations within the E/P and the E/Ep space is different per se. In other words, I do not expect similar deviations in terms of E/P and E/Ep. If this has been used interchangeably within the literature, I think it needs to be highlighted in your manuscript by providing more references and a more in-depth discussion of the issue.

We thank the referee for this important point and will clarify in the revised manuscript that the distance to E/Ep=1 indicates decreased energy use efficiency, whereas the distance to E/P=1 indicates decreased rain use efficiency by evapo-transpiration. This will also strengthen our argument that it is more meaningful to look at the non-contracted part of the Budyko-space only, as in the contracted part, a value of 1 cannot be achieved due to other constraints. We highlighted this more and added a more in-depth discussion about this in, especially, lines 175-181.

Abstract: Even though it is a Technical Note, the abstract is a bit too technical (and a bit too long) in my opinion. I would appreciate it if you try to condense some of the methodologies and put more emphasis on the interpretation of your results.

We shortened the abstract and condensed it in the revised version of the manuscript.

Introduction: You introduce the Budyko framework as being super popular in recent years (which is true). However, most of your references are at least 10 years old or older. I would appreciate a slightly more extensive introduction outlining more recent references, highlights, and applications of the Budyko framework and how this connects to the objective of your manuscript.

We elaborated more on recent applications of the Budyko framework and connect this to the issues raised in our manuscript (lines 34-38).

p.5, l. 8: Would be great to acutally provide the number of arid/humid catchments.

There are 247 arid catchments and 110 humid catchments, we added this to the text (lines 124-125).

Fig. 2: Maybe consider adjusting the y-axis of the plots to better highlight the differences. Given the current scale, the differences seem rather unremarkable.

Thank you for the suggestion, we adjusted the axes.

Sec. 3.2: Deviations from the curve and the limits can be larger for water-limited catchments by design. For energy-limited catchments, the total range is always smaller than 1. You highlight that in your discussion of the contracted vs. the uncontracted side. However, I was just wondering if you would obtain similar results if you just assess randomly distributed numbers within the Budyko space.

This is an interesting thought experiment that we tried out. We sampled the x-values of Ep/P from a uniform distribution (100,000 samples between a value of 0 and 2). In the next step, we sampled an n-value for each realization of Ep/P from a truncated normal distribution (similar to Greve et al., 2015), truncated at 0.0 to avoid negative n-values, with a mean of 1.9 and standard deviation of 0.5 . In this way, we determined the accompanying Ea/P and Ea/Ep values by using the Budyko equation with a certain realization of n. With this set of data, we repeated our analysis and fitted a curve (i.e this curve will be close to the curve obtained with the mean of the n-values of the distribution used to create the data).
The results in Figure 1 mainly confirm that energy-limited catchments always have a smaller distance to the curve in a projection based on a dryness index compared to a projection based on a wetness index. Moreover, it can be seen that the distances of water-limited catchments with an aridity close to 2.0, doubles when changing from a projection with a wetness index to a projection with a dryness index.
In addition, we sampled Ea, Ep and P also from a uniform distribution with values between 0 and 2000 mm/year. This was done for 100,000 samples, but unrealistic combinations of Ea, Ep and P, were removed (e.g. Ep/Ea > 1.0), as well as samples with an aridity higher than 2 or lower than 0.5. Eventually, 24937 samples remained. These results are shown in Figure 2. Interestingly, energy-limited catchments (Ep/P < 1) still have a smaller distance to the curve in a projection based on a dryness index compared to a projection based on a wetness index.

We added this analysis to Supplement S2.

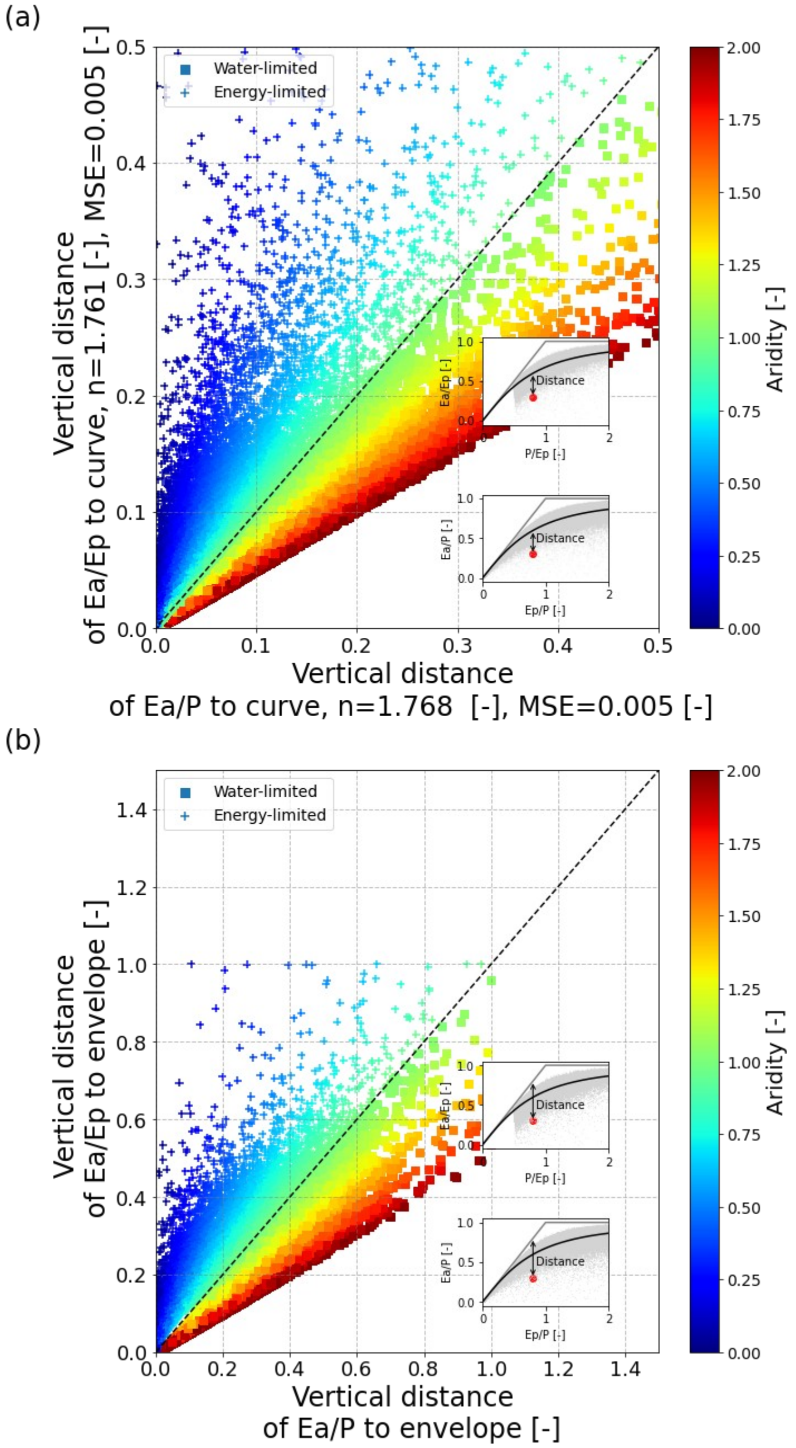

Figure 1. Vertical absolute distances to a) vertical distances to the fitted Budyko curve and b) the envelope of the physical limits of the Budyko framework, both for projections normalized by precipitation (x-axes) and potential evaporation (y-axes). Ep/P-data is sampled from a uniform distribution with values between 0.0 and 2.0, n-values are sampled from a truncated normal distribution with a mean of 1.9 and standard deviation of 0.5, the distribution was truncated at 0. 100,000 data points were sampled. Water-limited catchments (Ep/P >1) are shown with stars, whereas energy-limited catchments are shown with crosses. The colorscale indicates the aridity of the catchments.

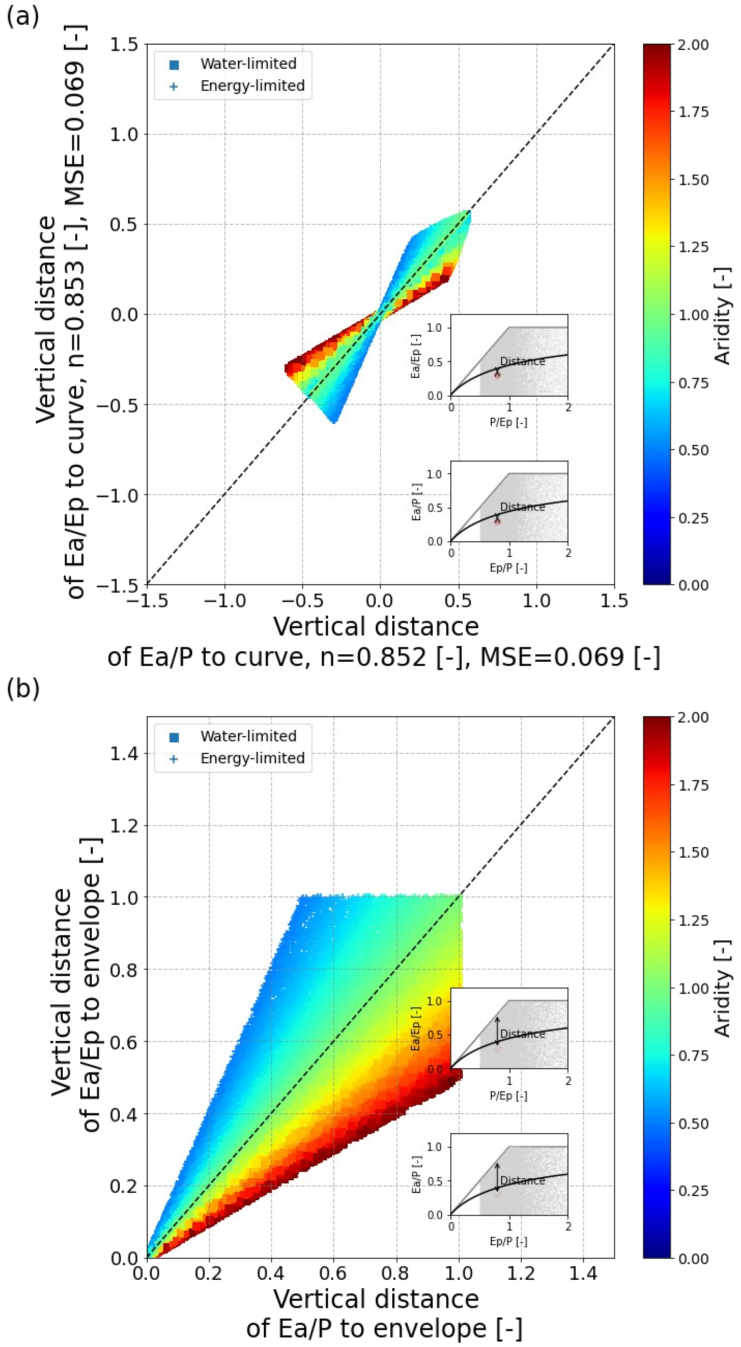

Figure 2. Vertical distances to a) the fitted Budyko curve and b) vertical distances to the envelope of the physical limits of the Budyko framework in absolute terms, both for projections normalized by precipitation (x-axes) and potential evaporation (y-axes). Ep, P and Ea data are sampled from a uniform distribution with values between 0.0 and 2000.0 mm/year. Originally, 100,000 samples were drawn, but unrealistic combinations of Ep, P and Ea were removed leading to 24937 samples. Water-limited catchments (Ep/P >1) are shown with stars, whereas energy-limited catchments are shown with crosses. The colorscale indicates the aridity of the catchments.

References

Donohue, R. J., Roderick, M. L., and McVicar, T. R.: Roots, storms and soil pores: Incorporating key ecohydrological processes into Budyko's hydrological model, Journal of Hydrology, 436–437, 35–50, https://doi.org/10.1016/j.jhydrol.2012.02.033, 2012.

Greve, P., Gudmundsson, L., Orlowsky, B., and Seneviratne, S. I.: Introducing a probabilistic Budyko framework, 42, 2261–2269, https://doi.org/10.1002/2015GL063449, 2015.

Nijzink, R. C. and Schymanski, S. J.: Vegetation optimality explains the convergence of catchments on the Budyko curve, Hydrol. Earth Syst. Sci. Discuss. [preprint], https://doi.org/10.5194/hess-2022-97, in review, 2022.

Ning, T., Li, Z., and Liu, W.: Vegetation dynamics and climate seasonality jointly control the interannual catchment water balance in the Loess Plateau under the Budyko framework, Hydrol. Earth Syst. Sci., 21, 1515–1526, https://doi.org/10.5194/hess-21-1515-2017, 2017.

Roderick, M. L. and Farquhar, G. D.: A simple framework for relating variations in runoff to variations in climatic conditions and catchment properties, 47, W00G07, https://doi.org/10.1029/2010WR009826, 2011.

Yang, D., Shao, W., Yeh, P. J.-F., Yang, H., Kanae, S., and Oki, T.: Impact of vegetation coverage on regional water balance in the nonhumid regions of China, 45, W00A14, https://doi.org/10.1029/2008WR006948, 2009.

Zhang, L., Dawes, W. R., and Walker, G. R.: Response of mean annual evapotranspiration to vegetation changes at catchment scale, 37, 701–708, https://doi.org/10.1029/2000WR900325, 2001.