



1 **River-enhanced non-linear overtide variations in river**
2 **estuaries**

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20 **Abstract:** Tidal waves traveling into estuaries are modulated in amplitude and
21 shape due to bottom friction, funneling planform and river discharge. The role
22 of river discharge on damping incident tides has been well-documented,
23 whereas our understanding of the impact on overtide is incomplete. Inspired by
24 findings from tidal data analysis, in this study we use a schematized estuary
25 model to explore the variability of overtide under varying river discharge. Model
26 results reveal significant M₄ overtide generated inside the estuary. Its absolute
27 amplitude decreases and increases in the upper and lower parts of the estuary,
28 respectively, with increasing river discharge. The total energy of the M₄ tide
29 integrated throughout the estuary reaches a transitional maximum when the



30 river discharge to tidal mean discharge (R2T) ratio is close to unity. We further
31 identify that the quadratic bottom stress plays a dominant role in governing the
32 M_4 variations through strong river-tide interaction. River flow enhances the
33 effective bottom stress and dissipation of the principal tides, and reinforces
34 energy transfer from principal tide to overtide. The two-fold effects explain the
35 nonlinear M_4 variations and the intermediate maximum threshold. The model
36 results are consistent with data analysis in the Changjiang and Amazon River
37 estuaries and highlight distinctive tidal behaviors between upstream tidal rivers
38 and downstream tidal estuaries. The new findings inform study of compound
39 flooding risk, tidal asymmetry, and sediment transport in river estuaries.

40 **Key words:** River discharge; Overtide; Bottom stress; Estuary

41

42 1. Introduction

43 Tides are a primary force driving water motion and transport of sediment
44 and contaminant in estuarine and coastal environments. Examination of tidal
45 wave dynamics supports many aspects of coastal management, including
46 flooding risk mitigation, coastal erosion defense, and wetland conservation.
47 Tidal dynamics in oceanic and coastal waters have been extensively studied
48 for centuries (Green, 1837; Talke and Jay, 2020). It is already well established
49 that tidal waves traveling into estuaries are altered in amplitude and shape due
50 to water depth changes, channel convergence (Jay, 1991; Friedrichs and
51 Aubrey, 1994; Lanzoni and Seminara, 1998; Talke and Jay, 2020), and river
52 discharge (Godin, 1985; Horrevoets et al., 2004; Cai et al., 2014). Given tidal
53 wave celerity is a function of water depth in shallow environments, high water
54 travels faster than low water, leading to shorter rising tide and longer falling
55 tide, i.e., tidal wave deformation and tidal asymmetry. Tidal wave deformation
56 at the daily time scale is nicely represented by superimposition of M_2 and its
57 first overtide M_4 (Pugh, 1987). The amplitude of M_4 overtide is basically small
58 and insignificant in relatively deep and open coastal seas, but may become
59 profound inside tidal estuaries. The energy of M_4 overtide inside estuaries is



60 extracted from astronomical tides through the nonlinear processes (Parker,
61 1984; Talke and Jay, 2020). The behavior and dynamics of M_4 tide in
62 tide-dominant estuaries and lagoons have been extensively examined
63 because the resultant tidal asymmetry controls tide-averaged sediment
64 transport and morphological changes (Parker, 1984; Speer and Aubrey, 1985;
65 Friedrichs and Aubrey, 1988; Le Provost, 1991 etc.).

66 River flow enhances tidal energy dissipation and stimulates wave
67 deformation (Jay and Flinchem, 1997; Godin, 1999; Horrevoets et al., 2004;
68 Toffolon and Savenije, 2011). River discharge reinforces wave deformation by
69 prolonging falling tides and shortening rising tides, which is featured by larger
70 overtide amplitude under higher river discharge (Stronach and Murty, 1989;
71 Gallo and Vinzon, 2005). However, a small number of studies suggest that the
72 impact of river flow on tidal wave deformation and overtide generation exhibits
73 more spatial variability within river estuaries under varying river discharge. For
74 instance, Godin (1985, 1999) reported accelerated low water and retarded
75 high water in the upper Saint Lawrence Estuary under larger river discharge,
76 whereas the high water is hastened and the low water delayed in the lower part
77 of the estuary. In the Changjiang River estuary, the amplitude of the
78 quarter-diurnal tidal species (overtides and compound tides), resolved by
79 continuous wavelet transform method, becomes larger in the lower part of the
80 estuary, but smaller in the upper part of the estuary under high river flow
81 conditions (Guo et al., 2015). These findings imply that the M_4 overtide is
82 sensitive to river discharge magnitude and it displays different spatial
83 variations under different river discharge conditions, which is, however,
84 insufficiently understood.

85 The overtide generated locally within estuaries is inherently related to the
86 nonlinear dynamics in shallow waters. Non-linearity enters the mathematical
87 representation of a tidal system through the divergence of excess volume in
88 the continuity equation and the advection and bottom friction terms in the
89 momentum equation (Speer and Aubrey, 1985; Parker, 1984, 1991; Wang et al.



90 1999, 2002). Pioneering studies with scaling analysis suggested that the
91 advection term is insignificant when scaled with estuarine length or wavelength
92 in short and tide-dominated estuaries, thus was ignored in past analytical study
93 of tides (Speer and Aubrey, 1985; Friedrichs and Aubrey, 1994). However, in
94 the presence of a river flow, the advection term may play a role in slowing
95 down incident tidal waves and speeding up the reflected waves (Godin, 1985,
96 1991; van Rijn, 2011; Kästner et al., 2019). This is because river flow enlarges
97 the mean current, therefore the advection term becomes significant and cannot
98 be ignored in river estuaries (Talke and Jay, 2020). Parker (1984) provided a
99 thorough analysis of the importance of frictional effects on tidal interactions.
100 The quadratic bottom shear stress has the effect of reducing tidal amplitudes
101 and decreasing wave celerity (Proudman, 1953; Godin, 1985, 1991, 1999; Jay,
102 1991; Horrevoets et al., 2004), and stimulating the generation of new
103 harmonics (Proudman, 1953; Pingree and Maddock, 1978; Parker, 1984, 1991;
104 Wang et al., 1999). Given all the three nonlinear terms are attributed to
105 creating forced harmonics (Parker, 1984; Walters and Werner, 1991; Wang et
106 al., 1999), it would be helpful to determine their relative importance. Gallo and
107 Vinzon (2005) provided an evaluation of the relative importance of the
108 nonlinear terms on overtide for the Amazon River estuary. But the results were
109 only presented for a mean river discharge condition. It still remains an open
110 question as to which nonlinear term plays a more significant role under varying
111 river discharge conditions.

112 In this contribution we deploy a numerical model to explore river-tide
113 interaction and subsequent impact on overtide behavior in a schematized long
114 estuary. We aim to explore 1) how varying river discharge would modulate the
115 overtides, and 2) what is the controlling impact of the nonlinear terms on the
116 spatial variability of overtide under different river discharge.

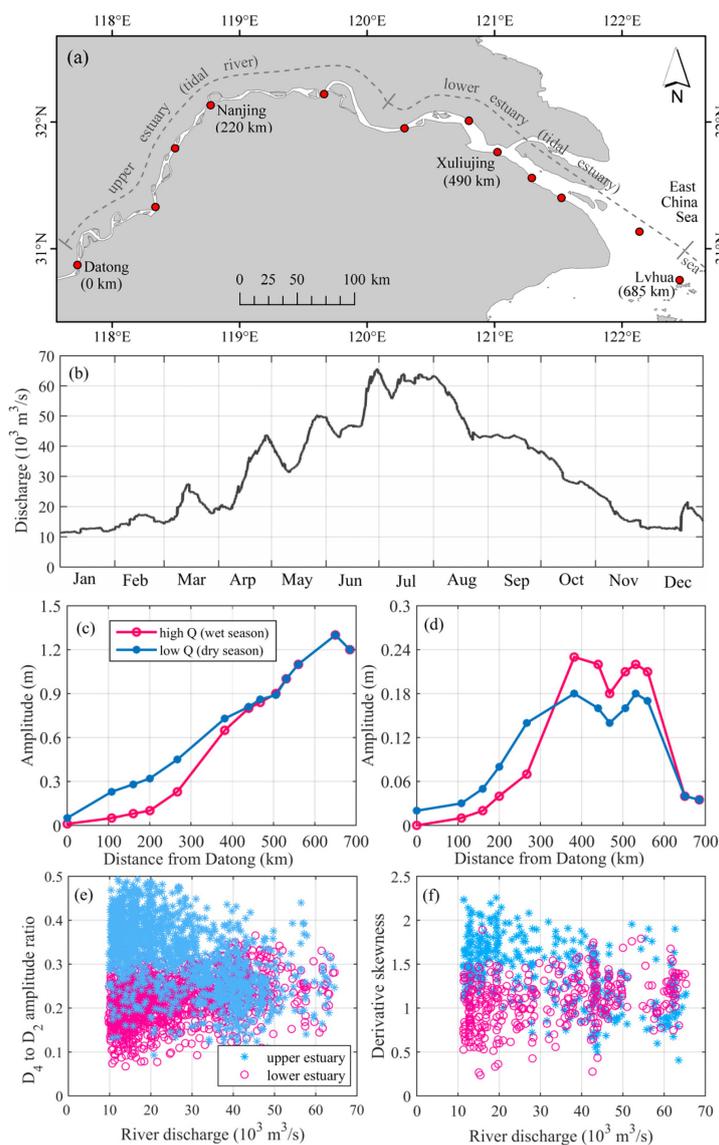
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118 **2. Model setup and data analysis**

119 **2.1 Inspirations from the Changjiang River estuary**



120 The rationale of this study comes from tidal analysis in the Changjiang
121 River estuary, which is a large tidal system with a tide-influenced river reach as
122 along as 650 km (Figure 1a). River discharge at the tidal wave limit, Datong,
123 varies seasonally in the range of 10,000-60,000 m³/s in the post-Three Gorges
124 Dam period (Figure 1b; Guo et al., 2018). The incoming tides are semi-diurnal
125 with a maximum tidal range of 5.9 m, and the M₂ is the most significant
126 constituent, followed by S₂, O₁, and K₁. Based on harmonic analysis
127 (Pawlowicz et al., 2002) of tidal height data in the time periods when river
128 discharge varies in a small range (close to a stationary situation, see Guo et al.
129 (2016) for further details), we see that the incoming tidal waves are firstly
130 amplified before they travel into the estuary, owing to a landward decrease in
131 water depth (Figure 1c). They are, however, predominantly dissipated inside
132 the estuary, despite the width convergence in the lower part of the estuary
133 seaward of Jiangyin, because of stronger influence of bottom friction and/or
134 river discharge. The river-enhanced tidal damping is more significant in the wet
135 season when the river discharge is higher, particularly in the upper part of the
136 estuary upstream of Jiangyin.



137

138 **Figure 1.** (a) The geometry and tidal gauges in the Changjiang River Estuary,
 139 (b) river discharge variations within a year course, along-river (c) M₂, and (d)
 140 M₄ amplitude variations in the dry and wet seasons, (e) amplitude ratios of the
 141 quarter-diurnal to semi-diurnal tides, and (f) skewness of the time derivative of
 142 tidal water levels in the upper (Nanjing) and lower (Xuliujing) estuaries. Details
 143 of the Changjiang River estuary and the tidal data are given in Guo et al.
 144 (2015). The numbers in the brackets in panel (a) indicate the seaward distance



145 from Datong. The data in panels (c)-(e) is from Guo et al. (2016) and that in
146 panel (f) is from Guo et al. (2019).

147

148 A significant M_4 overtide is detected inside the estuary while it is
149 insignificant to seaward of the estuary (Figure 1d). The smaller M_4 amplitude in
150 the region km-380 and km-520 in both dry and wet seasons is attributed to
151 interaction between the two main branches around Xuliujing. Apart from that,
152 the M_4 amplitude is larger in the lower part of the estuary in the wet season
153 when the river discharge is higher (Figure 1d). Moreover, the amplitude ratios
154 of the quarter- to semi-diurnal tidal species (derived by continuous wavelet
155 transform) decrease with increasing river discharge in the upper part of the
156 estuary but increase in the lower estuary (Figure 1e; Guo et al., 2015). Similar
157 analyses, using the skewness of the time derivative of tidal water levels, show
158 that the duration asymmetry between falling and rising tides exhibits similar
159 variations as the amplitude ratios (Figure 1f; Guo et al., 2019). These results
160 regarding the longitudinal M_4 amplitude variations by harmonic analysis
161 (Figure 1d), the amplitude ratios of the quarter- to semi-diurnal tidal species
162 (Figure 1e), and the derivative skewness variations (Figure 1f) consistently
163 demonstrate that the overtides display distinctive variations between the upper
164 and lower parts of the estuary in response to low and high river discharge
165 conditions. However, such changing behavior was insufficiently discussed in
166 previous studies. One challenge is that the river discharge varies continuously
167 in reality and induces non-stationary variations in tidal dynamics. Conventional
168 harmonic analysis is unable to accurately resolve the tidal changes when the
169 river discharge varies continuously in a big range (Jay and Flinchem, 1997;
170 Jay et al., 2014; Guo et al., 2015). Specifically, it is unknown how the overtides
171 will behave as the river discharge varies between the low and high limits other
172 than the results shown in Figure 1d.

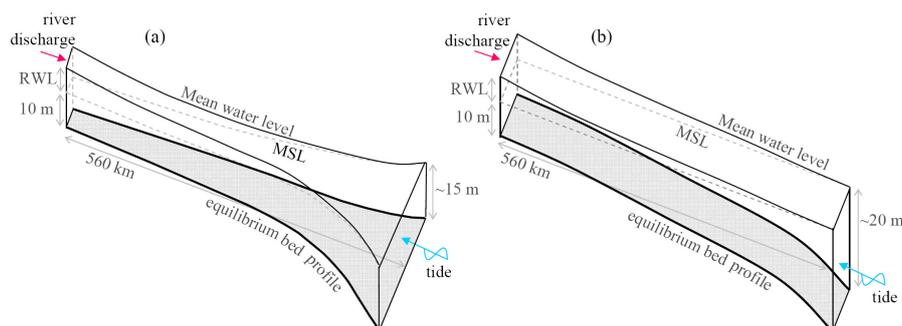
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174 **2.2 Model setup**



175 Examination of tidal data has provided a basic framework for our
176 understanding of tidal dynamics (Dronkers, 1964; Godin, 1985). However,
177 conventional harmonic analysis may not accurately resolve tidal constituents
178 owing to the non-stationary river discharge variations (Jay and Flinchem,
179 1997). In addition, analytical solutions of the tidal dynamic equations have
180 facilitated examination of leading-order wave propagation such as landward
181 damping or amplification of astronomical tides, given its advantages in terms of
182 fast setup and transparency in unraveling physical processes (Jay, 1991;
183 Friedrichs and Aubrey, 1994; Lanzoni and Seminara, 1998; Savenije, 2005).
184 However, analytical models usually assume tidal propagation as a single wave
185 component, based on simplified tidal dynamic equations after scaling analyses,
186 e.g., adopting a linear assumption or a nonlinear expansion of the friction term
187 (Green, 1837; Kreiss, 1957; Jay, 1991; Parker, 1991; Friedrichs and Aubrey,
188 1994; van Rijn, 2011). Analytical models may not fully capture the nonlinearity
189 imbedded in tidal dynamics, considering that the importance of different
190 nonlinear terms is likely not the same in different parts of long systems,
191 particularly under strong river flow conditions in long estuaries.

192 In this study we seek to capture the nonlinear dynamics by using a
193 numerical model, i.e., the open-source Delft3D codes, which has been widely
194 validated and used in varying estuarine and coastal environments (Lesser et
195 al., 2004). We construct a schematized 1D estuary model with a convergent
196 planform mimicking the Changjiang River estuary. The model domain
197 describes a 650 km long estuary that is composed of a weakly convergent
198 upstream segment (km-0 to km-400, width varying from 2 to 5 km) and a
199 strongly convergent downstream segment (km-400 to km-650, width varying
200 from 5 to 32 km (Figure 2a). Another situation with a uniform prismatic channel
201 (i.e., 2 km width and similar length) is adopted as part of the sensitivity analysis
202 to see the influence of basin geometry (Figure 2b).



203
204 **Figure 2.** Sketches of the schematized estuary model outline and settings
205 considering (a) a convergent and (b) a prismatic planform. The shade face
206 indicates the equilibrium bed profile. The RWL and MSL indicate residual water
207 level and mean sea level, respectively.

208
209 The model is forced by river discharge and tides. A combination of different
210 tidal constituents is imposed, and for simplicity we mainly consider a
211 semi-diurnal M_2 constituent with an amplitude of 1.0 m. Extra simulations
212 considering both M_2 and S_2 constituents (an amplitude of 0.5 m) are included
213 to facilitate more tidal interactions and generation of representative compound
214 tide such as MS_4 . Other astronomical constituents like O_1 and K_1 are excluded
215 because they would not affect the M_2 propagation very much. River discharge
216 is prescribed by constant values of 0, 10,000, 30,000, 60,000 m^3/s , symbolized
217 as Q_0 , Q_1 , Q_3 , and Q_6 scenarios, respectively, to facilitate harmonic analysis
218 with a stationary assumption. A dimensionless parameter, defined as the ratio
219 of river discharge to tide-averaged mean discharge (i.e., tidal prism divided by
220 tidal period) at the mouth section (R2T ratio), is estimated to be 0, 0.5, 2.6, and
221 42, which can be classified into tide-dominant, low, medium, and very high
222 river discharge circumstances, respectively (see section 3.3). The size of the
223 schematized estuary and the forcing conditions are characterized for a large
224 river estuary and in this case key dimensions from the Changjiang River
225 Estuary are used.

226 To obtain a suitable bottom profile for the tidal model, we first run a



227 morphodynamic simulation based on the above-mentioned model outline, with
228 an M_2 tide and a river discharge seasonally varying between 10,000 and
229 60,000 m^3/s as the boundary forcing conditions, as that in Guo et al. (2016).
230 The long-term morphodynamic simulation starts from an initial sloping bed with
231 depth varying from 5 m to 15 m seaward, considers sediment transport and
232 bed level changes, which leads to a morphodynamic equilibrium when bed
233 level changes become small at the time scales greater than decades (Guo et
234 al., 2016). The eventual equilibrium bed profile is then used as the bottom level
235 condition in the tidal simulations. The purpose of using this equilibrium bed
236 profile is to maintain consistency between the forcing and morphological
237 conditions. Based on this equilibrium bed profile and given high river discharge
238 imposed, the incoming tides are largely dissipated in the landward region of
239 the estuary, thus the influence of wave reflection is minimized. Details of the
240 morphodynamic model can be found in Guo et al. (2016), thus are not
241 repeated here.

242 Past studies using similar 1D representation of tidal estuaries confirm the
243 capture of leading-order dynamic processes (Friedrichs and Aubrey, 1994;
244 Lanzoni and Seminara, 1998). But it is noteworthy that the 1D model excludes
245 tidal flats and assumes uniform water density. These excluded processes may
246 have additional impact on tidal dynamics, e.g., additional momentum loss,
247 reduction in bottom drag, and extra tidal asymmetry (Friedrichs and Aubrey,
248 1988; Talke and Jay, 2020). Although simplified, the model provides a virtual
249 lab where tidal wave propagation, deformation and associated overtide
250 dynamics under varying river discharge can be isolated from the influences of
251 basin geometry and irregular shoreline, which enables straightforward
252 exploration of river-tide interactions.

253

254 **2.3 Data analysis**

255 The 1D model solves the width-averaged shallow water equations, i.e., the
256 continuity and momentum conservation equations, when the effect of Coriolis



257 force and density variations are neglected (Dronkers, 1964), as follows,

$$258 \quad \frac{\partial \eta}{\partial t} + \frac{\partial u(h + \eta)}{\partial x} = 0 \quad (1)$$

$$259 \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \eta}{\partial x} + \frac{gu|u|}{C^2(h + \eta)} = 0 \quad (2)$$

260 where u is velocity, η is water height above mean sea level, h is water depth
 261 below mean sea level, g is gravitational acceleration (9.8 m²/s), and C is a
 262 Chezy friction coefficient prescribed as 65 m^{1/2}/s uniformly.

263 As the bed level is prescribed as an equilibrium profile, the water depth h is
 264 constant. In the presence of a river discharge, the water level height is
 265 composed of two parts, namely a mean water height related to river flow η_0 ,
 266 and a tide-induced water level oscillation,

$$267 \quad \eta(x, t) = \eta_0(x) + \eta_{M_2}(x) \cos(\omega t - kx) \quad (3)$$

268 in case of the presence of M₂ tide only, in which η_{M_2} is the surface amplitude of
 269 M₂, and ω is the frequency of M₂, and k is tidal wave number. Similarly, the
 270 current is composed of a mean current and a tidal component,

$$271 \quad u(x, t) = u_0(x) + u_{M_2}(x) \cos(\omega t - kx - \theta) \quad (4)$$

272 in which u_0 is the mean current velocity, u_{M_2} is the velocity amplitude of M₂, θ
 273 is the phase difference between tidal surface wave and tidal currents.

274 Three nonlinear terms are identified in the tidal wave equations, namely
 275 the discharge gradient term in the continuity equation, and the advection and
 276 quadratic friction terms in the momentum equation:

$$277 \quad \bullet \text{ discharge gradient: } \frac{\partial u(h + \eta)}{\partial x} = \frac{\partial(uh)}{\partial x} + \frac{\partial(u\eta)}{\partial x} \quad (5)$$

$$278 \quad \bullet \text{ advection: } u \frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left(\frac{u^2}{2} \right) \quad (6)$$

$$279 \quad \bullet \text{ bottom friction: } \frac{gu|u|}{C^2(h + \eta)} \approx \frac{g}{C^2} \left(\frac{u|u|}{h} - \frac{\eta u|u|}{h^2} \right) \quad (7)$$

280 The bottom friction term is approximately expanded into a bottom shear
 281 stress term and a term considering depth variations, as the two terms on the
 282 right hand of Eq. (7), respectively, according to Godin and Martinez (1994),
 283 given the tidal amplitude to water depth ratio ($|\eta|/h$) is generally smaller than



284 one. Note that the bottom friction term can be calculated accurately with
 285 resolved water depths and velocities in the numerical model, while the
 286 approximation of Eq. (7) is just used to analytically demonstrate how the
 287 friction would lead to local generation of compound tides and overtides. Firstly
 288 considering a situation when river discharge is small and the associated mean
 289 current (u_0) is insignificant, the quadratic bottom stress can be further
 290 expressed by Fourier decomposition according to Le Provost (1991) and Wang
 291 et al. (1999):

$$292 \quad \frac{u|u|}{h} \approx \frac{u_{M_2}^2}{h} \sum_{n=0,1,2,\dots} (-1)^{n+1} \frac{8}{(2n-1)(2n+1)(2n+3)\pi} \cos(2n+1)\omega t \cos(nkx) \quad (8)$$

293 Equation (8) suggests that the self-interaction of M_2 tide through the
 294 quadratic bottom stress produces a series of overtide harmonics with
 295 odd-multiple frequencies, e.g., M_6 and M_{10} (Parker, 1984). In addition, Eq. 8
 296 also yields a contribution to the same frequency as M_2 (when $n=0$), which
 297 suggests tidal energy dissipation via the quadratic shear stress term (Wang et
 298 al., 1999). Similarly, the depth variation term in Eq. 7 can be expressed as:

$$299 \quad \frac{\eta u |u|}{h^2} \approx \frac{\eta_{M_2} u_{M_2}^2}{h^2} \sum_{n=0,1,2,\dots} (\dots) \cos(\omega t) \cos[(2n+1)\omega t]$$

$$300 \quad = \frac{\eta_{M_2} u_{M_2}^2}{h^2} \sum_{n=0,1,2,\dots} (\dots) \left[\frac{1}{2} \cos(2n\omega t) + \frac{1}{2} \cos(2n+2)\omega t \right] \quad (9)$$

301 Equation (9) suggests that the self-interaction of M_2 tide through the depth
 302 variation term generates even-multiple frequency harmonics, e.g., M_4 and M_8 .
 303 Similar decomposition analysis for the advection and discharge gradient term
 304 suggests the generation of even-frequency overtide as well (Parker, 1984;
 305 Wang et al., 1999). Following similar logic, when two components such as M_2
 306 and S_2 tides are prescribed, compound tides with frequencies the sums (e.g.,
 307 MS_4) or differences (e.g., MS_f) of the prescribed constituents are generated.
 308 The main focus of this study is devoted to M_4 overtide, given it is the first
 309 overtide of M_2 and of profound importance for tidal asymmetry.

310 Following the above analyses that qualitatively indicates the possibility of



311 local overtide generation, we attempt to further quantify the relative
 312 contribution of the nonlinear terms. For this purpose, we employ another
 313 approximation of the quadratic shear stress term, as follows, according to
 314 Godin and Martinez (1994) and Godin (1999),

$$315 \quad u |u| \approx 0.35u + 0.71u^3 \quad (10)$$

316 Replacing the Eq. (7) with Eq. (10) and using the expansion of Eqs. (3) and
 317 (4), a harmonic decomposition using the sine and cosine summation rules is
 318 used to identify the contribution of the nonlinear discharge gradient, advection,
 319 and bottom friction terms based on Eqs. (5) to (7). It follows the method in
 320 Gallo and Vinzon (2005) and Leberthal et al. (2019), but considering both
 321 quadratic bed shear and depth variation terms.

$$322 \quad \bullet \text{ discharge gradient: } 0.5u_{M_2} \frac{d\eta_{M_2}}{dx} + 0.5\eta_{M_2} \frac{du_{M_2}}{dx} \quad (11)$$

$$323 \quad \bullet \text{ advection: } 0.5u_{M_2} \frac{du_{M_2}}{dx} \quad (12)$$

$$324 \quad \bullet \text{ friction: } \frac{1.065g}{C^2 h} u_0 u_{M_2}^2 + \quad (13)$$

$$\frac{g}{C^2 h^2} [1.065\eta_0 u_0 u_{M_2}^2 + 0.525u_0^2 \eta_{M_2} u_{M_2} + 0.355\eta_{M_2} u_{M_2}^3]$$

325 The first term in Eq. 13 is ascribed to the quadratic bottom shear while the
 326 other terms are attributed to the depth variations. Again, the Eqs. (11) to (13)
 327 suggest that the interaction between the mean current and M_2 velocity would
 328 generate even-frequency harmonics like M_4 via both the quadratic bed shear
 329 stress and depth variation terms, implying river influence through river-tide
 330 interaction. Harmonic analyses of the model-output time series of water levels
 331 and currents provide mean water height, mean current, and the amplitudes
 332 and phases of surface wave and velocity of M_2 and M_4 tides for Eqs. (11) to
 333 (13). To indicate their relative importance, the advection and friction terms are
 334 then normalized by squared maximum velocity, and the discharge gradient
 335 term is normalized by the product of maximum velocity and maximum water
 336 level range. Comparison of the four scenarios forced by different river
 337 discharge thus helps to demonstrate the variability (see section 3.3).

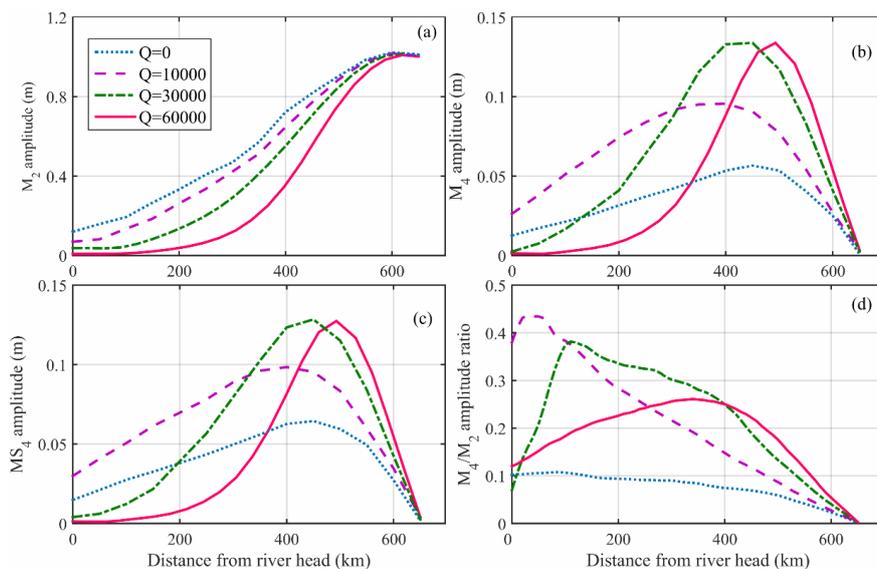


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339 **3. Model results**

340 **3.1 Tidal variations under varying river discharge**

341 The longitudinal amplitude variations of both the principal and forced
342 constituents are shown in Figure 3. The M_2 tide is firstly slightly amplified in the
343 utmost seaward regions close to the mouth, owing to channel convergence,
344 (Figure 3a). Landward of that, the incoming M_2 tide is predominantly dissipated,
345 and river discharge enhances the damping. In addition, a considerable M_4 tide
346 is detected in the Q0 scenario (no river discharge) with a local amplitude
347 maximum around km-450. The M_4 amplitude becomes larger throughout the
348 estuary in the Q1 scenario compared with that that in Q0 (Figure 3b). However,
349 under higher river discharge, the M_4 amplitude reduces in the upper part of the
350 estuary, e.g., landward km-300, but continues to increase in the lower reaches,
351 e.g., seaward km-500 (Figure 3b). The location with maximal M_4 amplitude
352 moves slightly landward as the river discharge increases from zero (i.e., from
353 km-450 in the Q0 scenario to km-400 in the Q1 scenario), but seaward as the
354 river discharge further increases (i.e., from km-420 in the Q3 scenario to
355 km-500 in the Q6 scenario). The M_4 to M_2 amplitude ratio exhibits similar
356 variations as the absolute M_4 amplitude, but the ratio is overall larger in the
357 upper parts of the estuary in which the absolute amplitudes of both M_2 and M_4
358 tides are small (Figure 3d). The increasingly damped and distorted tidal waves
359 further illustrate the river impact on the incoming tides (see Figure S1 in the SI).
360 When both M_2 and S_2 are imposed in the seaward boundary, a compound
361 overtide MS_4 is detected inside the estuary, which exhibits similar spatial
362 variations as the M_4 tide (Figure 3c). We then focus on the M_4 overtide in the
363 following discussions.



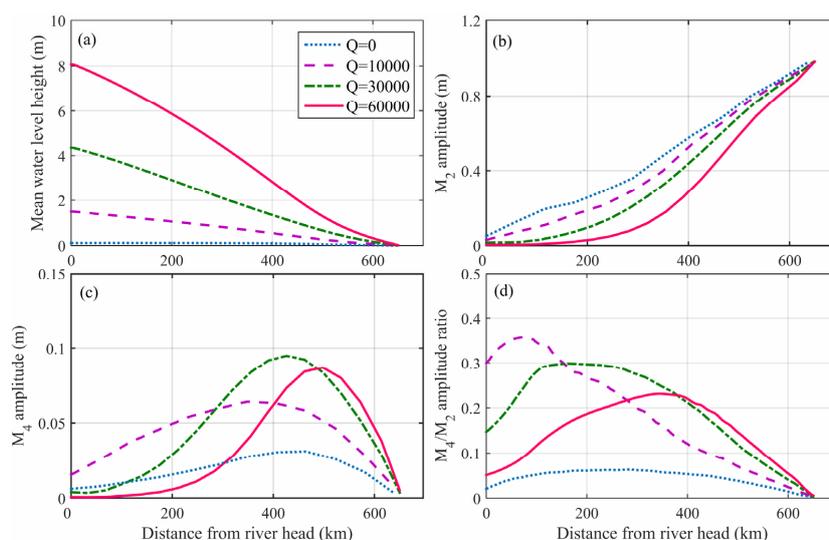
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365 **Figure 3.** The model-reproduced longitudinal variations of (a) M_2 tidal
366 amplitude, (b) M_4 tidal amplitude, (c) MS_4 amplitude (in the extra scenario
367 when both M_2 and S_2 are imposed at the boundary), and (d) the M_4 to M_2
368 amplitude ratios in the convergent estuary.

369 370 **3.2 Sensitivity to channel convergence**

371 Channel convergence is expected to affect tidal wave propagation and
372 wave deformation (Jay, 1991; van Rijn, 2011; Talke and Jay, 2020). To
373 demonstrate the sensitivity of the model results to width variations, we setup a
374 prismatic channel model with similar settings as the convergent estuary. A
375 close-to-equilibrium bed profile is again obtained via morphodynamic
376 simulation for the prismatic estuary. River discharge elevates the mean water
377 level and mean current (Figure 4a). The incoming M_2 tide is overall damped
378 inside the estuary, without any amplification (Figure 4b). Similar M_4 overtide
379 is generated as well, but its amplitude is approximately 30% smaller than that in
380 the convergent estuary (Figure 4c). A smaller tidal prism owing to a smaller
381 mouth width and surface area explain the smaller tidal amplitude in the
382 rectangular estuary. Apart from the differences in the absolute amplitudes, the



383 longitudinal variations of both the principal and forced tides and their spatial
384 dependence on river discharge exhibit similar patterns as the convergent
385 estuary (Figures 3 and 4). These consistent results imply that channel
386 convergence does not fundamentally changes the spatial dependence of
387 overtide behavior on river discharge, thus the findings from the prismatic
388 estuary is taken to have implications for estuaries in general and will be the
389 focus of further more detailed examination in order to highlight the controlling
390 impact of river discharge.



391
392 **Figure 4.** Model-reproduced longitudinal variations of (a) mean water level
393 height, (b) M_2 tidal amplitude, (c) M_4 tidal amplitude, and (d) the M_4 to M_2
394 amplitude ratio under different river discharge in the prismatic estuary.

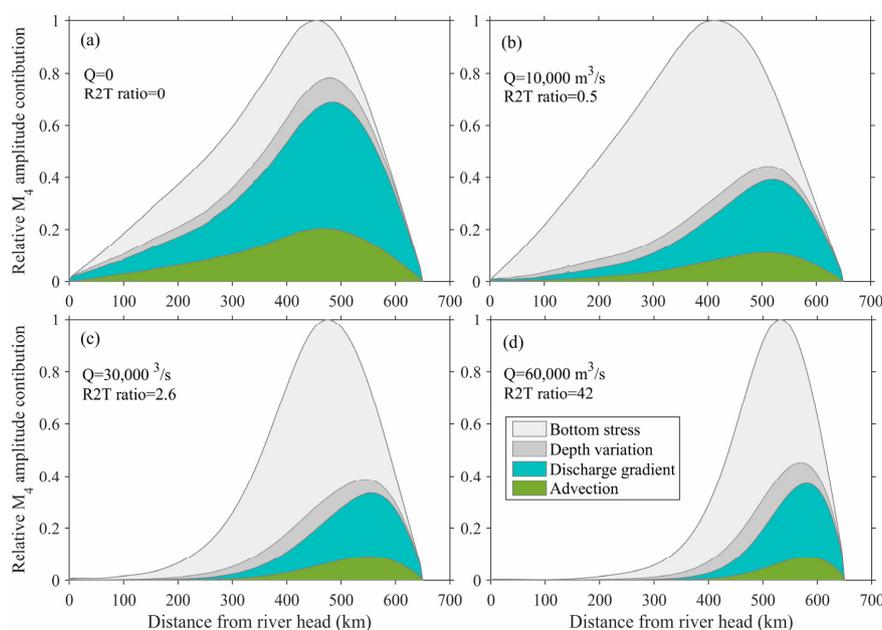
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396 3.3 Contribution of the nonlinear terms

397 We then use the harmonic decomposition method proposed in section 2.3
398 to quantify the contribution of the different nonlinear terms on M_4 variations. In
399 the absence of river discharge (Q_0 scenario), the discharge gradient term is
400 the largest contribution owing to strong landward damping of M_2 and
401 subsequent longitudinal flux gradients, followed by bottom friction and
402 advection (Figure 5a). The bottom friction term becomes more significant in the



403 presence of river discharge (Figures 5b-d). The impact of the quadratic bottom
404 shear stress is much more important than that of depth variations. The
405 influence of the advection term is relatively small compared to the other two
406 nonlinear terms. Spatially, the impact of bottom friction is more profound in the
407 upper parts of the estuary, whereas the impact of discharge gradient and
408 advection is more apparent in the regions close the estuary mouth. The
409 location of maximal M_4 amplitude coincides with the peak in the combined
410 contribution of discharge gradient and advection in the Q0 scenario, but with
411 the peak in bottom friction in the other three scenarios. Overall, these results
412 suggest that the three nonlinear terms are equally important in the
413 tide-dominated long estuary, whereas the importance of the bottom friction, or
414 more precisely the quadratic bottom shear stress, stand out when there is
415 significant river discharge.



416
417 **Figure 5.** Quantification of the relative importance of three nonlinear terms on
418 M_4 overtide amplitude in the (a) $Q=0$ (R2T ratio=0), (b) $Q=10,000$ (R2T
419 ratio=0.5), (c) $Q=30,000$ (R2T ratio=2.6), and (d) $Q=60,000$ m^3/s (R2T ratio=42)
420 scenarios. The contribution of bottom friction is divided into the components of



421 bottom shear stress and depth variation. The relative M_4 amplitude is
422 normalized by the maximal value in each scenario.

423

424 The importance of the quadratic bottom shear stress can be furthermore
425 inferred when comparing the model results under quadratic and linear bottom
426 shear stress. The quadratic bottom shear stress can be linearized using the
427 first order of the energy dissipation condition (Lorentz, 1926), as that applied
428 by Zimmerman (1992) and Hibma et al. (2003) (see SI for more details). When
429 similar simulations are run using linear bottom stress, the landward damping
430 rates of the principal tides become smaller (see Figure S4). Measurable M_4
431 overtide is still detected, which is ascribed to the effects of other nonlinear
432 effects (e.g., the advection and depth variations), but its amplitude is
433 comparably smaller than that under a quadratic bottom stress (Figure S4).
434 Moreover, increasing river discharge neither induces more tidal wave damping,
435 nor more overtide generation under a linear bottom stress.

436

437 **3.4 Quantification of the river discharge threshold**

438 The abovementioned model results imply that the M_4 amplitude tends to
439 first increase and then decrease as the principle M_2 is increasingly dissipated
440 by larger river discharge. To better reveal the nonlinear variations, we run extra
441 simulations under constant river discharge in the range of 0 to 60,000 m^3/s at
442 an increment of 5,000 m^3/s . Since the tidal amplitudes vary along the estuary,
443 we then integrate the total (tide-averaged) energy of M_2 and M_4 tides ($\text{kg}\cdot\text{m}^2/\text{s}^2$)
444 throughout the estuary to represent overall tidal strength (van Rijn, 2011) by:

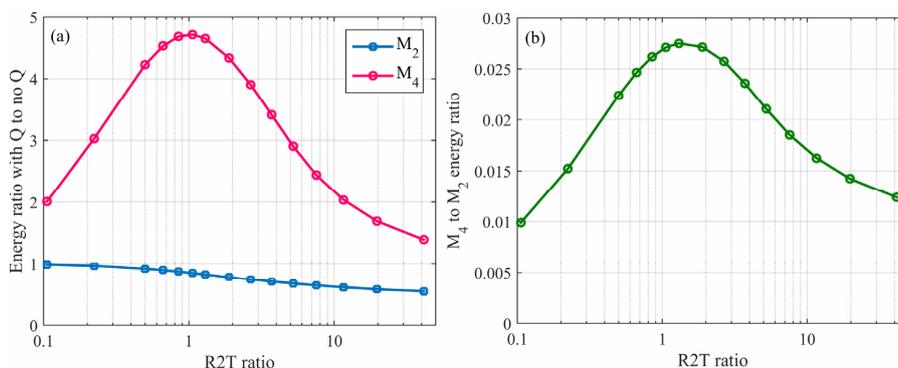
$$445 \int_0^L 0.5 \rho g b A(x)^2 dx / L \quad (14)$$

446 where: L is the channel length, ρ is the water density, b is channel width which
447 is uniform in the rectangular channel, and A is the surface amplitude of the M_2
448 or M_4 tide which varies along the estuary.

449 We see that the total energy of the M_2 tide decreases approximately



450 exponentially with increasing R2T ratios (Figure 6a). The decrease is more
451 significant for $R2T < 5$ (see Figure S2a). The total energy of M_4 overtide,
452 however, first increases with increasing R2T ratio from zero and reaches a
453 peak when the R2T ratio is approximately unity, followed by a decrease as the
454 R2T ratio further increases (Figure 6a). The maximum total energy of M_4 is 4.7
455 times the case with no river discharge, under the model framework in this study.
456 Similarly, the energy ratio of M_4 to M_2 displays similar variations as the total
457 energy variation of the M_4 tide, with a peak reached when the R2T ratio is
458 around 1-2 (Figure 6b). These results confirm that an intermediate river
459 discharge with R2T ratio close to unity, benefits maximal M_4 overtide
460 generation. Below this threshold, increased river discharge favors more M_4 tide
461 generation, whereas a larger river discharge above the threshold constrains
462 M_4 generation.



463
464 **Figure 6.** (a) The ratio of the total energy of M_2 and M_4 tides integrated
465 throughout estuary in the scenarios with river discharge to the case without
466 river discharge, and (b) the total energy ratio of M_4 tide to M_2 tide, as a function
467 of the ratio of river discharge to tide-mean discharge at the estuary mouth
468 (R2T ratio). Also see Figure S2 in the SI.

469

470 4. Discussion

471 4.1 Comparison with actual estuaries

472 The findings regarding overtide variability in the model are overall

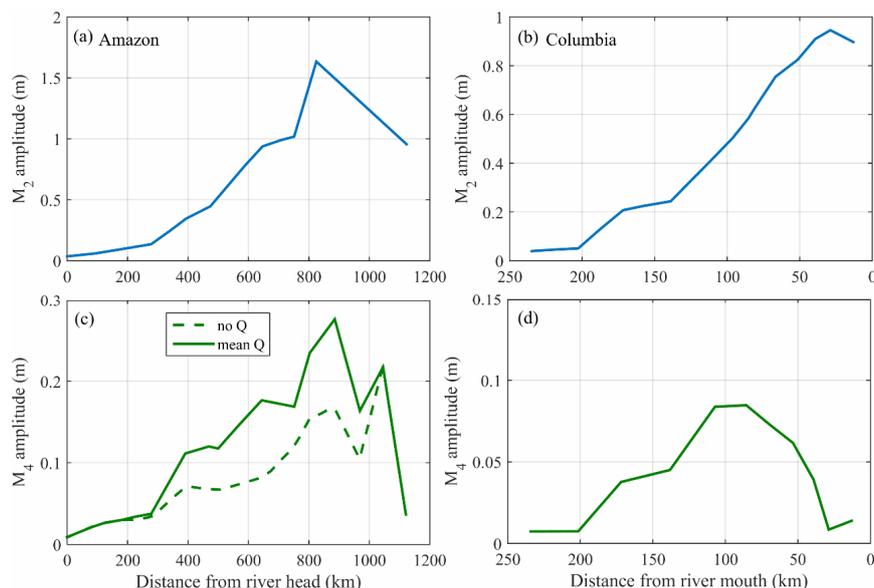


473 consistent with data analyses in actual estuaries like the Changjiang and
474 Amazon River estuaries. The modeled results between the Q1 and Q3
475 scenarios are consistent with the along-channel variations of the principle tide
476 and overtide under low and high river discharge in the Changjiang River
477 Estuary. The changes in the M_4 to M_2 amplitude ratios with varying river
478 discharge between the upper and lower parts of the Changjiang River Estuary
479 also agree well with model results in the schematized convergent estuary (see
480 Figure S3). In the Amazon River estuary where the river discharge is similarly
481 high and varies in a large range, the M_4 amplitude was overall larger under a
482 mean river discharge throughout the estuary compared to an idealized
483 situation with zero discharge (Figure 7c; Gallo and Vinzon, 2005). This result is
484 qualitatively consistent with the modeled differences between the Q0 and Q1
485 scenarios in this study. In the Columbia River estuary, a maximum in M_4
486 amplitude is approached in the lower part of the estuary, followed by a
487 subsequent decrease upriver under a year-mean river discharge (Figure 7d;
488 Jay et al., 2014). The model results can also explain why a higher river
489 discharge hastened the high water and delayed the low water in the lower part
490 of the Saint Lawrence Estuary (Godin, 1985, 1999). These field data and
491 model results confirm that the findings regarding the spatial dependence of
492 overtide on river discharge are likely to be ubiquitous for river estuaries.

493 Similar reports of the overtide behavior were, however, not widely found in
494 many other estuaries, given that the tidal dynamics have been intensively
495 studied worldwide. We think that it maybe because the majority of estuaries
496 worldwide are tide-dominated, such that river discharge is overall small and
497 rarely reaches a magnitude that exceeds $R2T=1$. Therefore, the role of river
498 discharge in stimulating wave deformation and associated overtide generation
499 has been widely observed and confirmed (when $R2T<1$), whereas the further
500 changes when $R2T>1$ are far less prevalent and hence less well documented.
501 Another possible explanation is that most tidal estuaries are small in physical
502 length (compared to wavelength), hence the spatial variations are less



503 apparent compared to long estuaries such as the Amazon, Changjiang, and
504 Columbia systems.



505
506 **Figure 7.** Amplitude variations of M₂ and M₄ overtide in the (a, c) Amazon
507 River estuary and (b, d) Columbia River estuary, from Gallo and Vinzon (2005)
508 and Jay et al. (2014), respectively.

509

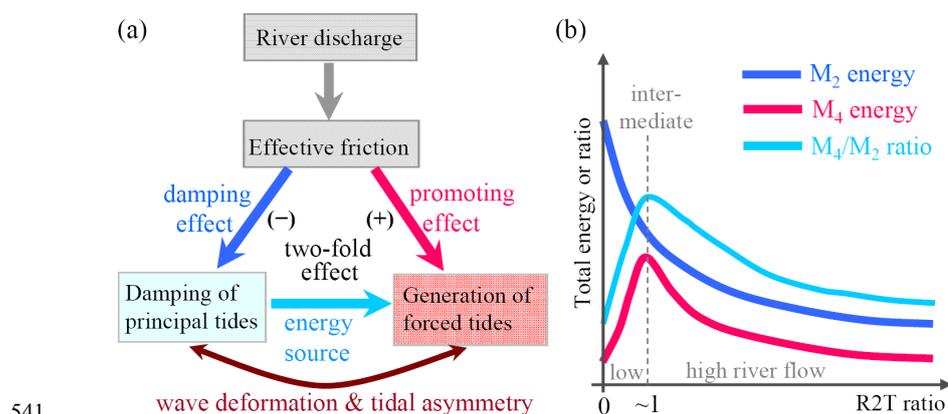
510 4.2 Role of river discharge

511 The majority of past studies have focused on either the damping effect of
512 river flow on principal tides or the enhancing effect on overtide. When linking
513 them together, we see that the two-fold effects of river flow nicely explain the
514 nonlinear overtide changes in river estuaries (Figure 8). River discharge
515 enlarges the currents and the effective friction on the moving flow. It induces
516 more damping of the principal tides, i.e., more energy dissipation of incident
517 tides. In addition, river-enhanced bottom friction reinforces the energy transfer
518 from the principal tide to overtide, i.e., stimulated overtide generation. As more
519 principal tidal energy is dissipated, particularly in the landward part of estuaries,
520 the energy available for transfer to overtide is also constrained. As a result, an
521 intermediate river discharge (when the R2T ratio is close to unity) provides an



522 effective bottom stress that will not dissipate the principal tides too much, and
 523 at the same time stimulates considerable energy transfer to overtones, leading
 524 to the occurrence of a maximum in overtide energy. Note that other
 525 high-frequency overtones display similar spatial variations as M_4 , e.g., MS_4 (see
 526 Figure 3c) and MN_4 tide when M_2 and N_2 constituents are prescribed (not
 527 shown).

528 River impact on tidal wave propagation is space-dependent. River
 529 discharge substantially elevates the mean water level in the upper part of
 530 estuaries; the consequent larger water level gradient restricts landward wave
 531 propagation (Godin, 1985; Cai et al., 2019). In the lower part of estuaries
 532 where the incident tides are less dissipated, river flow plays a more important
 533 role in reinforcing the effective bottom friction. As a result, dissipation of the
 534 principal tide is more prominent in the upper part of estuaries, while tidal
 535 energy transfer and overtide generation is more substantial in the lower part of
 536 estuaries (Figures 8). These space-dependent dynamics explain the
 537 contrasting behavior of overtide in response to increasing river discharge, and
 538 also confirm that tidal wave deformation maybe one of the degrees-of-freedom
 539 of estuaries to maintain a state of minimum work by adjusting tidal wave
 540 shapes in response to different river discharge (Zhang et al., 2016).



541
 542 **Figure 8.** Sketches (a) showing the two-fold effects of river discharge on tides
 543 mainly through the bottom friction, and (b) showing the intermediate river



544 discharge threshold that benefits maximum overtide generation.

545

546 The two-fold river impact on tidal propagation is coherently related to the
547 bottom friction. Past studies have indicated that the effects of river flow on tidal
548 damping are exerted by a mechanism identical to bottom stress (Horrevoets et
549 al., 2004; Cai et al., 2014). River-tide interaction enhances the bottom stress,
550 which subsequently induces larger tidal damping (Alembregtse and de Swart,
551 2016). Past studies have also suggested that the nonlinear advection term is
552 the main cause of M_4 generation in tide-dominant estuaries, whilst the
553 nonlinear bottom stress term leads to generation of M_6 (Pingree and Maddock,
554 1978; Parker, 1984, 1991; Wang et al., 1999). In this work we see that the
555 quadratic bottom stress term also leads to significant M_4 , through river-tide
556 interaction, i.e., between a river-enhanced mean current and M_2 current. This
557 explains why the M_4 amplitude is larger in the presence of a river discharge
558 and a quadratic bottom stress, compared to the situation with no river
559 discharge and/or a linear bottom stress.

560 Given the comprehensive past studies of tidal dynamics in estuaries, the
561 contribution of this work lies in revealing the nonlinear overtide changes and
562 identification of a river discharge threshold that benefit maximum overtide
563 generation. A river flow above or below the threshold induce contrasting
564 overtide behavior along estuaries. Although the model results are obtained
565 under constant river discharge, the findings still hold true when considering
566 time-varying river discharge (see SI). One slight difference is that the tidal
567 damping rate would be slightly different during the rising and falling limb of a
568 river discharge hydrograph (Sassi and Hoitink, 2013), which may be due to a
569 time lag in the influence of river discharge along the length of the estuary.

570

571 **4.3 Implications and limitations**

572 Better understanding of the overtide behavior has implication for studies of
573 tidal bores, interpretation of extreme high water and associated flood risk, and



574 tide-averaged sediment transport. Tidal wave deformation changes the height
575 of high water and low water, which may then influence flooding risk
576 management and the water depth of navigational channels. Tidal bores are an
577 extreme phenomenon of tidal wave deformation when tides are concurrently
578 amplified and distorted to some degree (Bonneton et al., 2015). Tidal bores are
579 less likely to occur in river estuaries because of river-enhanced damping,
580 although deformation is enhanced. The interaction between tidally-averaged
581 mean current and quarter-diurnal overtide current may contribute to net water
582 transport (Alebrechtse and de Swart, 2016). Tide-averaged sediment transport
583 induced by tidal asymmetry related to M_2 - M_4 interaction plays a profound role
584 in controlling sediment import or export and resultant infilling or empty of
585 estuaries (Postma, 1961; Guo et al., 2014). It is noteworthy that the horizontal
586 velocity of the quarter-diurnal tide may exhibit more spatial variations than its
587 surface amplitude, owing to interaction with estuarine morphology and
588 inter-tidal interactions of eddy viscosity etc. (Dijkstra et al., 2017b; Lieberthal et
589 al., 2019).

590 Although we have argued that channel convergence will not fundamentally
591 change the model results and main findings, the potential impact of the
592 simplified model setting in this study still mandates careful evaluation when
593 applying them to actual estuaries. For instance, regional narrowing and
594 shallowness in geometry and morphology is expected to induce variations in
595 tidal damping rates and distribution of amplitudes. River-influenced estuaries
596 can be partially or highly stratified, and a density difference and associated
597 stratification affect tides by reducing the effective drag coefficient and changing
598 the pressure-gradient term (Talke and Jay, 2020). This impact maybe further
599 manifested in surface amplitude of overtide given the role of river-tide current
600 interaction in the nonlinear terms (Dijkstra et al. 2017a). Inter-tidal flats are
601 known as a sink of momentum and would exert additional impact on tidal wave
602 propagation (Hepkema et al., 2018). Exclusion of inter-tidal flats in this work
603 thus may lead to overestimation of the overtide amplitude. Furthermore, the



604 intermediate river discharge threshold that satisfies $R2T=1$ is expected to vary
605 with estuarine size and shape, given that the tidal mean discharge is strongly
606 affected by estuarine morphology. These dynamic complexities merit further
607 study for site-specific cases.

608

609 **5. Conclusions**

610 Based on past intensive studies of tidal dynamics in estuaries, this work is
611 devoted to examining the forced overtide behavior under varying river
612 discharge and the controlling nonlinear mechanism. We use a numerical
613 model for a schematized long estuary to capture the nonlinear dynamics as
614 much as possible. Model results reveal local overtide generation whose
615 amplitude however exhibits strong spatial dependence. While the principal M_2
616 tide is increasingly dissipated as the $R2T$ ratio increases from zero, the M_4
617 overtide amplitude decreases in the upper part of estuaries but increases in
618 the lower part of estuaries. With increasing $R2T$ ratio, the total energy of M_4
619 overtide integrated throughout the estuary first increases and reaches a peak
620 when the $R2T$ ratio approaches unit. Further larger river discharge induces a
621 decline in total energy of both M_2 and M_4 . The modeled nonlinear changes in
622 overtide are quantitatively validated by data in actual estuaries like the
623 Changjiang and Amazon River estuaries.

624 Further sensitivity simulations confirm the significant role of bottom friction
625 that is enhanced by river-tide interaction in controlling the overtide behavior.
626 The effective bottom friction is enhanced by the river discharge, and this has
627 two-fold impact: (1) dissipation of principal tidal energy and (2) stimulation of
628 energy transfer to overtides. The two-fold effect explains the occurrence of an
629 intermediate river discharge threshold that benefits maximal overtide
630 amplitude. This study demonstrates the need to look at both tidal wave
631 propagation and deformation at the same time in tidal wave dynamics, as well
632 as their nonlinear spatial variations in large river estuaries. The findings have
633 implications for study of tidal bores and tidal asymmetry and associated



634 morphological changes in river estuaries.

635

636 **Author contribution**

637 LG designed the experiments and carried them out. LG and IT prepared the
638 manuscript with contributions from all co-authors.

639

640 **Declare of interest conflict**

641 The authors declare that they have no conflict of interest.

642

643 **Data availability**

644 The model data are available on request at the corresponding author.

645

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658

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