

1 Supporting Information for

2 Effects of the dynamic effective porosity on watertable fluctuations and
3 seawater intrusion in coastal unconfined aquifers

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22 **Introduction**

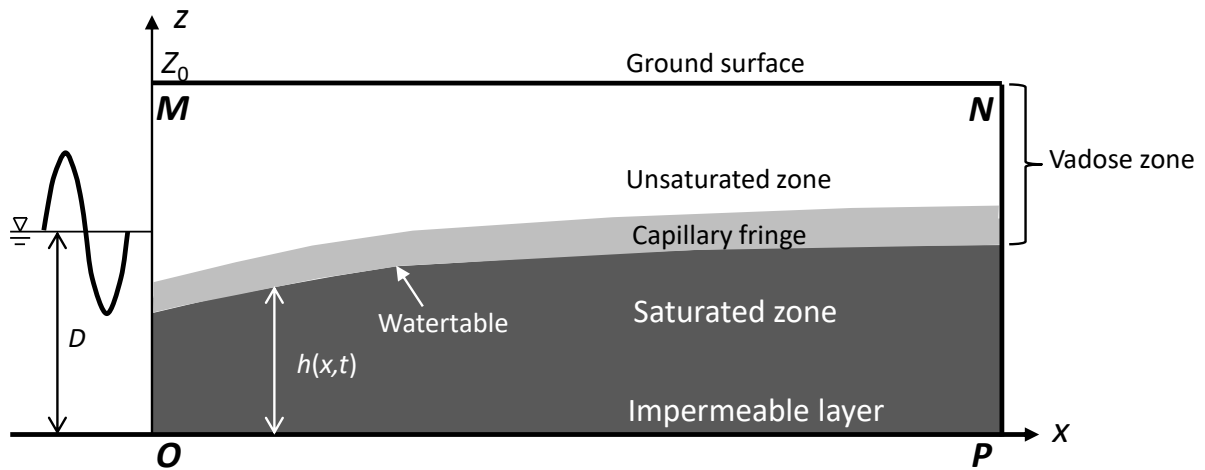
23 Figure S1 shows a schematic diagram of an unconfined aquifer.

24 Figure S2 shows the experimental apparatus used to quantify the dynamic effective porosity.

25 Figure S3 plots amplitude decay rate ($k_r D$) and phase lag increase rate ($k_i D$) versus non-
26 dimensional aquifer depth ($n_e \omega D / K_s$) for watertable waves obtained from experiments and
27 different existing theories.

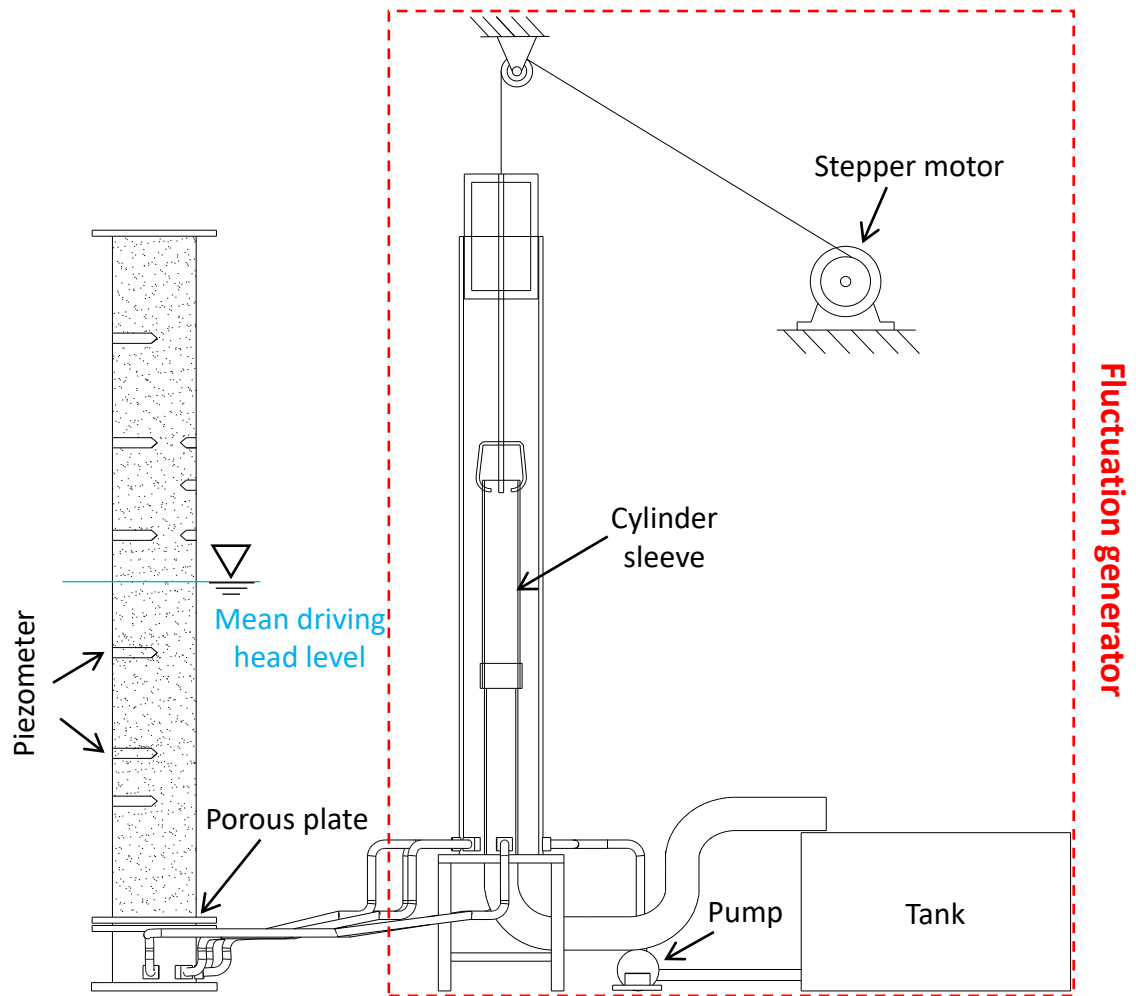
28 Figure S4 plots amplitude decay rate ($k_r D$) and phase lag increase rate ($k_i D$) versus non-
29 dimensional aquifer depth ($n_e \omega D / K_s$) for watertable waves obtained from experiments and
30 existing theory that combines equation (2) with the complex-valued expression for the
31 dynamic effective porosity proposed by Cartwright et al. (2005).

32 Figure S5 gives the numerical model setup for the base case.



33

34 **Figure S1.** Schematic diagram of an unconfined aquifer (modified from Kong et al. (2015)).
 35 *OPNM* represents the aquifer normal to the shoreline with a vertical sea boundary (*OM*). *OP*
 36 and *PN* are impermeable while *MN* is the ground surface, t [T] is time, x [L] is the horizontal
 37 distance from the vertical seaward boundary, D [L] is the aquifer depth (equal to the mean
 38 height of the boundary forcing), and h [L] is the watertable height above the impermeable
 39 layer.



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41 **Figure S2.** Experimental apparatus used to quantify the dynamic effective porosity, modified

42 from Luo et al. (2020).

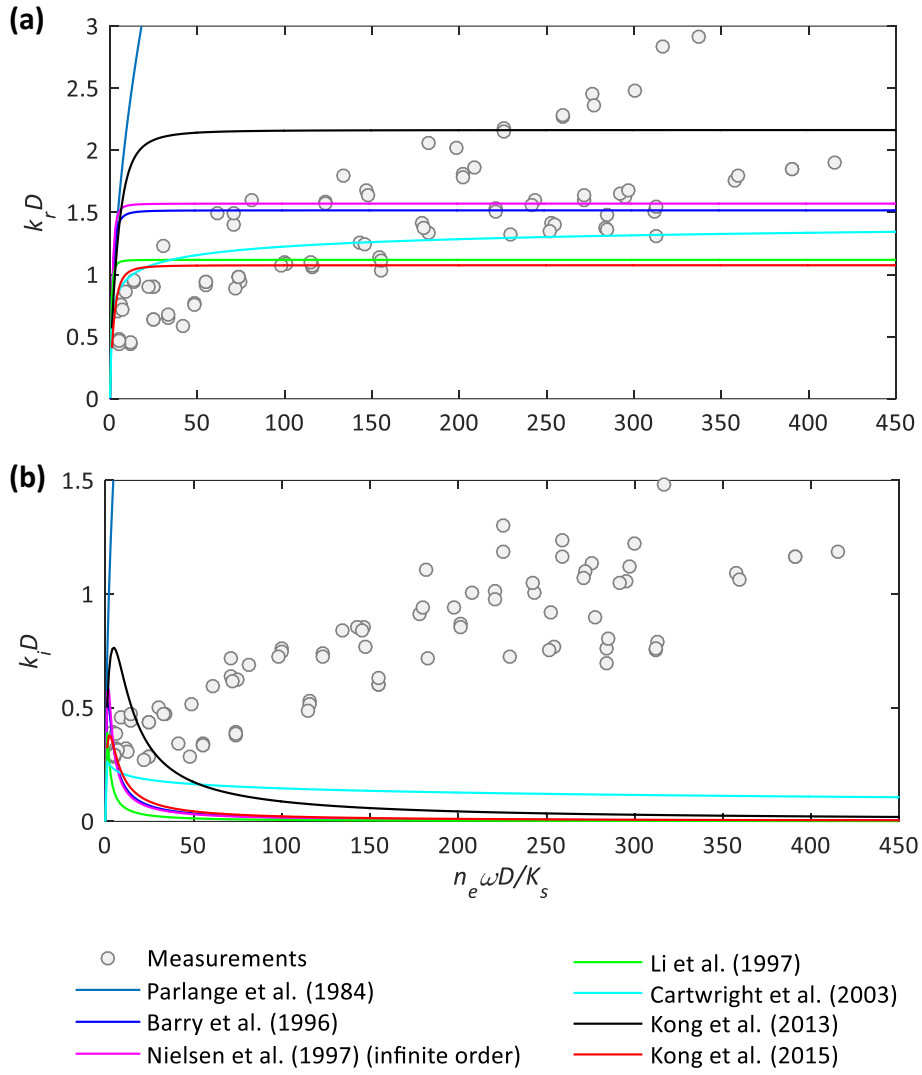
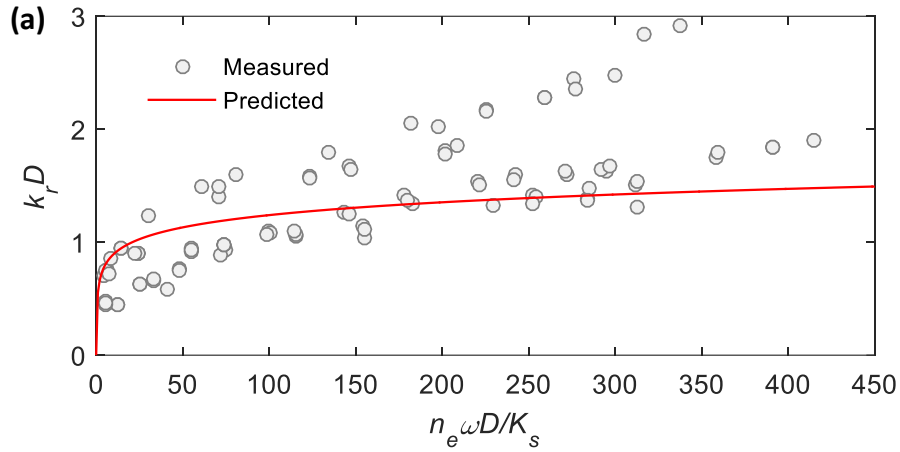
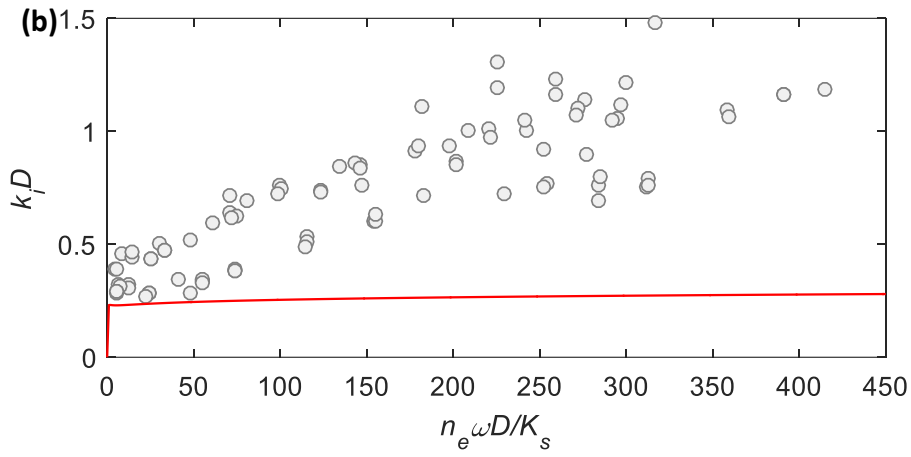


Figure S3. Comparison of (a) amplitude decay rate ($k_r D$) versus non-dimensional aquifer depth ($n_e \omega D / K_s$) and (b) phase lag increase rate ($k_i D$) versus non-dimensional aquifer depth ($n_e \omega D / K_s$) for watertable waves obtained from experiments and different existing theories. Parameters used are consistent with Shoushtari et al. (2016): $D = 0.92$ m, $H_\psi = 0.61$ m and $K_s = 4.7 \times 10^{-4}$ m/s. Parameter definitions are given in the main text.

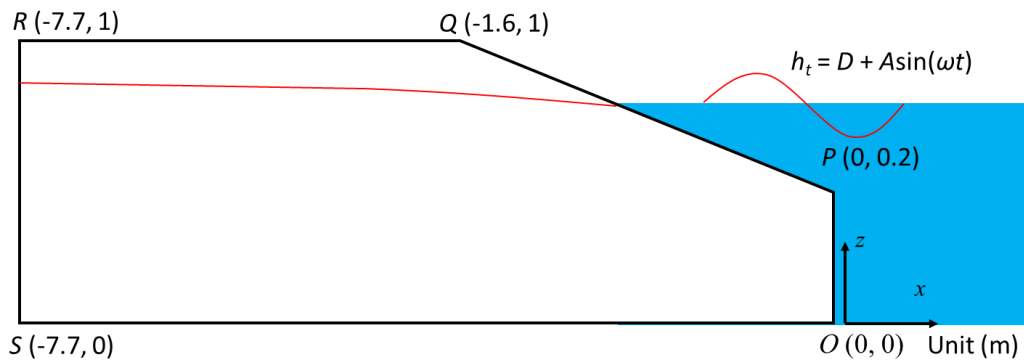


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52 **Figure S4.** Comparison of (a) amplitude decay rate ($k_r D$) versus non-dimensional aquifer
 53 depth ($n_e \omega D / K_s$) and (b) phase lag increase rate ($k_i D$) versus non-dimensional aquifer depth
 54 ($n_e \omega D / K_s$) for watertable waves obtained from experiments and existing theory that
 55 combines equation (2) with the complex-valued expression for the dynamic effective
 56 porosity proposed by Cartwright et al. (2005). Parameters used are consistent with
 57 Shoushtari et al. (2016): $D = 0.92$ m, $H_\psi = 0.61$ m and $K_s = 4.7 \times 10^{-4}$ m/s.



Model setup

Boundary conditions:

RS: Constant head boundary with $h = 0.74$ m and $c = 0$ ppt

OPQ: Time-dependent head boundary with $D = 0.7$ m, $A = 0.075$ m and $T = 240$ s ($\omega = 2\pi/T$), and $c = 35$ ppt

OS: No flow boundary

QR: Atmospheric boundary

Initial conditions:

$h = 0.70$ m, $c = 0$ ppt

Soil properties:

VG parameters: $\alpha = 11 \text{ m}^{-1}$, $n = 6$,

Hydraulic conductivity: $K_s = 3 \times 10^{-3} \text{ m/s}$

Static effective porosity: 0.4

Diffusion coefficient: $1 \times 10^{-9} \text{ m}^2/\text{s}$

Longitudinal dispersivity: 0.006 m

Transversal dispersivity: 0.0006 m

Mesh sizes and time step:

$\Delta x = 0.02$ m, $\Delta z = 0.1$ m and $\Delta t = 4$ s

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59 **Figure S5.** Numerical model setup for the base case (Shen et al., 2020), which uses a van
60 Genuchten (VG) soil.

References

- Barry, D. A., Barry, S. J., & Parlange, J.-Y. 1996. Capillarity correction to periodic solutions of the shallow flow approximation, In *Mixing Processes in Estuaries and Coastal Seas*, C. B. Pattiaratchi (editor), *Coastal and Estuarine Studies*, Volume 50, American Geophysical Union, Washington, DC, pp. 496–510. <https://doi.org/10.1029/CE050>
- Cartwright, N., Nielsen, P., & Dunn, S. (2003). Water table waves in an unconfined aquifer: Experiments and modeling. *Water Resources Research*, 39(12), 1330. <https://doi.org/10.1029/2003WR002185>
- Li, L., Barry, D. A., Parlange, J.-Y., & Pattiaratchi, C. B. (1997). Beach water table fluctuations due to wave run-up: Capillarity effects. *Water Resources Research*, 33(5), 935-945. <https://doi.org/10.1029/96WR03946>
- Luo, Z., Kong, J., Ji, Z., Shen, C., Lu, C., Xin, P., Zhao, Z., Li, L., & Barry, D. A. (2020). Watertable fluctuation-induced variability in the water retention curve: Sand column experiments. *Journal of Hydrology*, 589, 125125. <https://doi.org/10.1016/j.jhydrol.2020.125125>
- Nielsen, P., Aseervatham, R., Fenton, J. D., & Perrochet, P. (1997). Groundwater waves in aquifers of intermediate depths. *Advances in Water Resources*, 20(1), 37–43. [https://doi.org/10.1016/S0309-1708\(96\)00015-2](https://doi.org/10.1016/S0309-1708(96)00015-2)
- Kong, J., Shen, C.-J., Xin, P., Song, Z., Li, L., Barry, D. A., Jeng, D.-S., Stagnitti, F., Lockington, D. A., & Parlange, J.-Y. (2013). Capillary effect on water table fluctuations in unconfined aquifers. *Water Resources Research*, 49(5), 3064–3069. <https://doi.org/10.1002/wrcr.20237>
- Kong, J., Xin, P., Hua, G.-F., Luo, Z.-Y., Shen, C.-J., Chen, D., & Li, L. (2015). Effects of vadose zone on groundwater table fluctuations in unconfined aquifers. *Journal of Hydrology*, 528, 397–407. <https://doi.org/10.1016/j.jhydrol.2015.06.045>
- Parlange, J.-Y., Stagnitti, F., Starr, J. L., & Braddock, R. D. (1984). Free-surface flow in porous media and periodic solution of the shallow-flow approximation. *Journal of Hydrology*, 70(1-4), 251-263. [https://doi.org/10.1016/0022-1694\(84\)90125-2](https://doi.org/10.1016/0022-1694(84)90125-2)
- Shen, Y., Xin, P., & Yu, X. (2020). Combined effect of cutoff wall and tides on groundwater flow and salinity distribution in coastal unconfined aquifers. *Journal of Hydrology*, 581, 124444. <https://doi.org/10.1016/j.jhydrol.2019.124444>