Reviewers #2 comments:

Review of “Model Comparisons Between Canonical Vine Copulas and Meta-Gaussian for Agricultural Drought Forecasting over China” by Haijiang Wu, Xiaoling Su, Vijay P. Singh, Te Zhang, and Jixia Qi.

This paper developed an agricultural drought forecasting model based on canonical vine copulas under three-dimensions (3C-vine model). With the meta-Gaussian (MG) model as a reference model, they found that the 3C-vine model showed better performances than the meta-Gaussian model for agricultural drought forecasting over China. Any such model aimed at improving the forecasting of drought should be encouraged. The topic falls into the scope of HESS.

Overall, the paper is well written and structured, and I support the publication of this work after major revision based on the comments below. Some works are needed to improve in the methodology, results, and discussion. I have some suggestions/recommendations to improve the manuscript, which are given below:

General concern:
The major concern is about why the authors compare the vine copula model with the Meta Gaussian model. the latter one is generally based on the Gaussian distribution, and the prediction function is expected to be not superior than other competitors. More justifications or involving some other statistical models are expected through the paper.

Other concerns:
1. In comparison with the MG model, what are the superiority of the 3C-vine model or C-vine copula? The authors need a further statement about this in the Introduction section or discuss more about this in the Discussion section. Also in Line 57, the authors made a list of exiting model for the drought prediction; yet those models are all statistical models, some physical-based hydrological models are also widely used in hydrological prediction, the droughts included as well. A elaborate introduction is expected herein.

2. Page 3 Line 62: I suggest the authors add the ‘aforementioned’ before the ‘conventional statistical methods’, to avoid the broad statement.
3. Page 5 Lines 90-91: “The propagation between meteorological drought and agricultural drought…” should be changed as “The propagation from meteorological drought to agricultural drought…”, as the meteorological drought is a source of the agricultural drought. Be careful with the wording.

4. Page 5 Lines 95-97: Authors mentioned that the 3C-vine and MG models are employed to forecast the agricultural drought in August. It is rather confusing. I strongly suggest the authors provide some compelling reasons for choosing this month. Of course, if the authors can display the agricultural drought forecast in any interested months (e.g., the forecasted of extreme agricultural drought in June), it can further strengthen the robust of 3C-vine model.

5. Page 6 Line 126: I think the ‘three’ should be changed to ‘top-three’. Please check it.

6. Page 8 Line 155: The $\mu y_3|(y_2,y_1)$ in Equation (3) should be removed. Be careful with the checking.

7. Page 9 Line 187-188: “Here, regarding the conditional distribution of $z$ given the conditions $w$…”, the terms ‘$z$’ is confusing here, maybe it should be revised as ‘$y$’ according to the Equation (5). Please check it.

8. Page 11 Line 213-220: A graphical representation or flowchart of this process would be helpful, maybe in the Methodology section. I am actually quite intrigued by it.

9. Page 11 Line 226: The numerator term in the Equation (11) may be have problematic. Be careful with the checking.

10. Figure 6: I suggest the authors should add the PDF curve for the MG model. Maybe the authors need to consider completing it via the simulations.

11 Page 17 Lines 342-344: I think the ‘at time $t-1$ (t denotes target month)’ should be removed. Please check it.
Responses to the comments from Reviewer #2

We would like to thank the anonymous reviewer for reviewing our manuscript. These constructive comments are very important for us to improve the present manuscript. The reviewer’s comments are italicized and our responses immediately follow.

Review of “Model Comparisons Between Canonical Vine Copulas and Meta-Gaussian for Agricultural Drought Forecasting over China” by Haijiang Wu, Xiaoling Su, Vijay P. Singh, Te Zhang, and Jixia Qi.

This paper developed an agricultural drought forecasting model based on canonical vine copulas under three-dimensions (3C-vine model). With the meta-Gaussian (MG) model as a reference model, they found that the 3C-vine model showed better performances than the meta-Gaussian model for agricultural drought forecasting over China. Any such model aimed at improving the forecasting of drought should be encouraged. The topic falls into the scope of HESS.

Overall, the paper is well written and structured, and I support the publication of this work after major revision based on the comments below. Some works are needed to improve in the methodology, results, and discussion. I have some suggestions/recommendations to improve the manuscript, which are given below:

Response: Thank you for your help and encouragement. Modifications will be done according to the reviewers’ comments to improve the quality of the current manuscript.

General concern:

The major concern is about why the authors compare the vine copula model with the Meta Gaussian model. The latter one is generally based on the Gaussian distribution, and the prediction function is expected to be not superior than other competitors. More justifications or involving some other statistical models are expected through the paper.

Response: We agree with the reviewer’s suggestion. Modifications will be done according to the reviewers’ comments in the next revised manuscript.

Meta-Gaussian (MG) model, as a usually statistical method, has been extensively employed in drought forecast and risk assessment. The forecast skills of MG model for drought or compound dry-hot events, for example, are outperformed persistence-based or random forecast models (Hao et al., 2016; Hao et al., 2019; Wu et al., 2021b).
Therefore, we selected MG model as a reference drought model to evaluate the forecast skills of vine copula model.

**Other concerns:**

1. *In comparison with the MG model, what are the superiority of the 3C-vine model or C-vine copula? The authors need a further statement about this in the Introduction section or discuss more about this in the Discussion section. Also in Line 57, the authors made a list of existing model for the drought prediction; yet those models are all statistical models, some physical-based hydrological models are also widely used in hydrological prediction, the droughts included as well. A elaborate introduction is expected herein.*

**Response:** We would like to thank the reviewer for the constructive comments. More information will be added to our next revised manuscript (in Introduction section) according to your valuable suggestions.

Meta-Gaussian (MG) model, as a usually statistical method, has been extensively employed in drought forecast and risk assessment. However, the MG model only depicts the linear relationship among explanatory variables (predictors) and predictand variable via covariate matrix, it cannot characterize the nonlinear or tail dependence existed in variables (Hao et al., 2016). Fortunately, vine copulas (also known as pair copula constructions; here, canonical vine (C-vine) copulas, a sub-classes of vine copulas, is of primary interest) are capable of bridging this gap. C-vine copulas can flexibly combine multiple variables via bivariate copula to characterize numerous or complex dependencies. However, only limited relevant studies have applied vine copulas for drought forecast (Wu et al., 2021a). Therefore, investigations on drought forecasting skills between C-vine copulas and MG models are an implication to obtain reliable drought forecasts.

In hydrology, some physical-based hydrological models (e.g., Distributed Time-Variant Gain Hydrological Model (DTVGM; Ma et al, 2021) and Soil and Water Assessment Tool (SWAT; Wu et al., 2019)) are widely used in hydrological simulation and prediction, the droughts included as well. However, the physical-based hydrological models typically apply to a catchment or sub-regional scale, and generally require numerous hydrometeorological variables to achieve more accurate real-time predictions (Liu et al., 2021; Xu et al, 2021). The traditional methods have been extensively employed to forecast drought, such as regression models, machine learning
models, and hybrid models (by considering both statistical and dynamical predictions) (Hao et al., 2016). Yet, these models tend to be limited in considering the complex nonlinear (e.g., regression models), explicit physical mechanisms and over-fitting (e.g., machine learning models), as well as the demand of massive hydroclimatic data input (e.g., hybrid models).

2. Page 3 Line 62: I suggest the authors add the ‘aforementioned’ before the ‘conventional statistical methods’, to avoid the broad statement.

Response: It will be done accordingly. A new sentence will add in our next revised manuscript as follows:

“The copula functions overcome the limitations of the aforementioned conventional statistical methods. Since copulas can flexible joining arbitrary marginal distributions of variables, …”

3. Page 5 Lines 90-91: “The propagation between meteorological drought and agricultural drought…” should be changes as “The propagation from meteorological drought to agricultural drought…”, as the meteorological drought is a source of the agricultural drought. Be careful with the wording.

Response: We will be modified it based on the reviewer’s suggestions in the next revised manuscript.

The original sentence “The propagation between meteorological drought and agricultural drought was characterized via the MG model (Xu et al., 2021)” will be changed to “The propagation from meteorological drought to agricultural drought was characterized via the MG model (Xu et al., 2021)”.

4. Page 5 Lines 95-97: Authors mentioned that the 3C-vine and MG models are employed to forecast the agricultural drought in August. It is rather confusing. I strongly suggest the authors provide some compelling reasons for choosing this month. Of course, if the authors can display the agricultural drought forecast in any interested months (e.g., the forecasted of extreme agricultural drought in June), it can further strengthen the robust of 3C-vine model.

Response: We agree with your comments. We will add more details about choosing August as the interested month and provide the forecast of extreme agricultural droughts in July in the next revised manuscript.
We used the 6-month timescale SPI and SSI to depict meteorological drought and agricultural drought, respectively. Therefore, the SPI (SSI) in August, which is calculated by the cumulative precipitation (soil moisture) from March to August, can indirectly reflect the surplus and deficit situations of water in the spring (March-April-May) and summer (June-July-August) seasons. This is a key growth period for crops (e.g., anthesis, fruiting, and seed filling) and vegetation and is also the period with frequent droughts. As such, the agricultural drought forecast in August is of primary interest in this study. Undoubtedly, the agricultural drought forecast can be implemented in any interested month based on the 3C-vine model and MG model.

To display the robustness of 3C-vine model for forecasting agricultural drought in any interested months, we further forecasted the extreme agricultural droughts in July for these selected typical regions (D1S–D7S; black rectangle boxes in Figure 5b) (Figure R1c–R1d). Compared with the MG model under different lead times, the agricultural drought forecasts made by the 3C-vine model are more accurate across different typical regions, in terms of the predictive uncertainty (i.e., width of PDF curve) as well as the difference between observed and forecasted extreme SSIs (Figure R1).
Figure R1 Probability density function (PDF) curve of (a) minimum and (b) maximum SSI under 1–3-month lead times for July and August during the 1961–2018 period over seven selected typical regions in climate regions D1–D7 (i.e., these black rectangle boxes in Figure 5b correspond to signify D1S–D7S, respectively). Black dash line and text indicate the (a and c) minimum and (b and d) maximum observations of SSI in August and July over D1S–D7S. These texts with red (light-blue), blue (yellow), and cyan (coral) colors of left (right) in each sub-figure are SSI forecasts under 1–3-month lead times of August (July), which correspond to the abscissa projected by the peak point of each PDF.
5. Page 6 Line 126: I think the ‘three’ should be changed to ‘top-three’. Please check it.

Response: It will be checked and modified in the next revised manuscript as follows:

“We employed the Standardized Precipitation Index (SPI, based on monthly precipitation) and Standardized Soil moisture Index (SSI, based on monthly cumulative soil moisture at top-three soil depths), respectively, to characterize meteorological drought and agricultural drought at a 6-month timescale.”

6. Page 8 Line 155: The $\mu_{3}(y_2,y_1)$ in Equation (3) should be removed. Be careful with the checking.

Response: It will be removed in the next revised manuscript.

7. Page 9 Line 187-188: “Here, regarding the conditional distribution of $z$ given the conditions $w$...”, the terms ‘z’ is confusing here, maybe it should be revised as ‘y’ according to the Equation (5). Please check it.

Response: We apologize for our carelessness. It will be modified in the next revised manuscript as follows:

“Here, regarding the conditional distribution of $y$ given the conditions $w$, …”.

8. Page 11 Line 213-220: A graphical representation or flowchart of this process would be helpful, maybe in the Methodology section. I am actually quite intrigued by it.

Response: We appreciate the reviewer’s comment. As the reviewer points out, we will provide a flow chart (Figure R2) in the Methodology section for the proposed method as follows.
Precipitation in CN05.1

6 month timescale SPI

Predictors: SPI$_t(y_1)$+SSI$_t(y_2)$

Meta-Gaussian (MG) model

LOOCV

\[ y_1 | (y_1, y_2) \sim N(\mu_{y_1|y_2}, \Sigma_{y_1|y_2}) \]
\[ y_2^{\text{new}} = \mu_{y_2} + \Sigma_{y_2}^{-1} \left( y_2 - \mu_{y_2} \right) \]

Forecast performance comparison between C-vine and MG models based on NSE, R$^2$, and RMSE

Typical agricultural drought forecast and sub-climate regions assessment

Cumulative soil moisture at top-three soil depths in ERA5

6 month timescale SSI

Predictand: SSI$_t(y_3)$

 Canonical vine copulas (C-vine) model

LOOCV

Determined tree structures and variables ordering via AIC and estimated parameters of each bivariate copulas

Monte Carlo simulation

Figure R2 Flowchart of agricultural drought forecasting based on canonical vine copulas (3C-vine) and meta-Gaussian (MG) model under three-dimensional scenarios. Here, \( t \) denotes the target month (e.g., August); \( i \) signifies the lead times (1–3-months); LOOCV is the abbreviate of leave-one-out cross validation; \( y_1^{\text{new}}(y_2^{\text{new}}) \) indicates the series after removing a sample \((y_1^{\text{new}}(y_2^{\text{new}}))\) for a specific year; and \( y_3^{\text{new}} \) is the agricultural drought forecast value for the target month of a specific year. Note that the optimal tree structure (i or ii on the right-hand side of this figure) is selected based on AIC to forecast agricultural drought.

9. Page 11 Line 226: The numerator term in the Equation (11) may be have problematic. Be careful with the checking.

Response: Thanks for your reminder. We apologize for our carelessness. It will be revised as follows:
\[ R^2 = \frac{\sum_{i=1}^{n} (AO_i - \overline{AO})(AP_i - \overline{AP})^2}{\sum_{i=1}^{n} (AO_i - \overline{AO})^2 \cdot \sum_{i=1}^{n} (AP_i - \overline{AP})^2} \quad R^2 \in [0,1] \] (11)

10. *Figure 6:* I suggest the authors should add the PDF curve for the MG model. Maybe the authors need to consider completing it via the simulations.

**Response:** Thank you. The PDF curve for the MG model will be added in Figure 6 (here Figure R1) in the revised manuscript. It will be modified as follows:

As shown in Figure R1, in comparison with the 3C-vine model, we found that the width of PDF curves in the MG model are broadened, indicating that the MG model performed more pronounced uncertainty for agricultural drought forecast. Furthermore, the skills of MG model are tended to deteriorate over many selected typical regions, especially for 2–3-month lead times of July and August. The difference between forecasted and observed extreme SSIs for MG model is larger than that of 3C-vine model in distinct typical regions, e.g., the forecasted maximum SSI in July on D4S (Figure R1d). As such, compared with the MG model, the 3C-vine model can provide a more reliable agricultural drought forecast under 1–3-month lead times.
Figure R1: Probability density function (PDF) curve of (a) minimum and (b) maximum SSI under 1–3-month lead times for July and August during the 1961–2018 period over seven selected typical regions in climate regions D1–D7 (i.e., these black rectangle boxes in Figure 5b correspond to signify D1S–D7S, respectively). Black dash line and text indicate the (a and c) minimum and (b and d) maximum observations of SSI in August and July over D1S–D7S. These texts with red (light-blue), blue (yellow), and cyan (coral) colors of left (right) in each sub-figure are SSI forecasts under 1–3-month lead times of August (July), which correspond to the abscissa projected by the peak point of each PDF.
Response: We will remove it in the next revised manuscript.

References


