



1 Experimental study of non-Darcian flow characteristics in permeable stones

2

3

4

5 Zhongxia Li¹, Junwei Wan¹, Tao Xiong¹, Hongbin Zhan^{2*}, Linqing He³, Kun Huang^{1*}

6 ¹School of Environmental Studies, China University of Geosciences, 430074 Wuhan,

7 China.

8 ²Department of Geology and Geophysics, Texas A & M University, College Station, TX

9 77843-3115, USA.

10 ³Changjiang Institute of Survey Technical Research MWR, Wuhan, China.

11 * Corresponding authors:

12 Dr. Hongbin Zhan (zhan@geos.tamu.edu);

13 Dr. Kun Huang (cugdr_huang@cug.edu.cn).

14

15

16

17

18

19

20



21 **Abstract**

22 This study provides experimental evidence of Forchheimer flow and transition between
23 different flow regimes from the perspective of pore size of permeable stone. We have firstly
24 carried out the seepage experiments of permeable stones with four different mesh sizes,
25 including 24 mesh size, 46 mesh size, 60 mesh size, and 80 mesh size, which corresponding
26 to mean particle sizes (50% by weight) of 0.71 mm, 0.36 mm, 0.25 mm, and 0.18 mm. The
27 seepage experiments show that obvious deviation from Darcian flow regime is visible. In
28 addition, the critical specific discharge corresponding to the transition of flow regimes (from
29 pre-Darcian to post-Darcian) increases with the increase of particle sizes. When the “pseudo”
30 hydraulic conductivity (K) (which is computed by the ratio of specific discharge and the
31 hydraulic gradient) increases with the increase of specific discharge (q), the flow regime is
32 denoted as the pre-Darcian flow. After the specific discharge increases to a certain value, the
33 “pseudo” hydraulic conductivity begins to decrease, and this regime is called the post-
34 Darcian flow. In addition, we use the mercury injection experiment to measure the pore size
35 distribution of four permeable stones with different particle sizes, and the mercury injection
36 curve is divided into three stages. The beginning and end segments of the mercury injection
37 curve are very gentle with relatively small slopes, while the intermediate mercury injection
38 curve is steep, indicating that the pore size in permeable stones is relatively uniform. The
39 porosity decreases as the mean particle sizes increases, and the mean pore size can faithfully
40 reflect the influence of particle diameter, sorting degree and arrangement mode of porous
41 medium on seepage parameters. This study shows that the size of pores is an essential factor
42 for determining the flow regimes. In addition, the Forchheimer coefficients are also discussed
43 in which the coefficient A (which is related to the linear term of the Forchheimer equation) is
44 linearly related to $1/d^2$ as $A = 0.0025(1/d^2) + 0.003$; while the coefficient B (which is related
45 to the quadratic term of the Forchheimer equation) is a quadratic function of $1/d$ as



46 $B = 1.14E-06(1/d)^2 - 1.26E-06(1/d)$. The porosity (n) can be used to reveal the effect of
47 sorting degree and arrangement on seepage coefficient. The larger porosity leads to smaller
48 coefficients A and B under the condition of the same particle size.

49 **Keywords:** permeable stone, mercury injection experiment, pore size, flow regime, non-
50 Darcian flow.

51 1. Introduction

52 [Darcy \(1857\)](#) conducted a steady-state flow experiment in porous media and concluded
53 that the specific discharge was proportional to the hydraulic gradient, which is the Darcy's
54 law described as follow:

$$q = KJ \quad (1-1)$$

55 where q is the specific discharge, J is the hydraulic gradient, and K is the hydraulic
56 conductivity. However, when the specific discharge increases above a certain threshold,
57 deviation from Darcy's law is evident and the flow regime changes from Darcian flow regime
58 to the so called non-Darcian flow regime ([Bear, 1972](#)), which was first observed by
59 [Forchheimer \(1901\)](#), who proposed a widely used non-Darcian flow equation (the
60 Forchheimer equation) as follow:

$$J = Aq + Bq^2 \quad (1-2)$$

61 where A and B are constants related to fluid properties and pore structure. The first and
62 second terms on the right side of Eq. (1-2) roughly reflect the contributions of viscous and
63 inertial forces (or resistance to flow), respectively.

64 From the Forchheimer equation, we can see that when the specific discharge is
65 sufficiently small, the inertial force can be ignored, and the equation is transformed to the



66 form of Darcy's law. On the other hand, when the specific discharge is sufficiently large, the
67 viscous force can be ignored, and the equation is transformed to the fully developed turbulent
68 flow.

69 In addition to the polynomial function such as the Forchheimer equation, there are also
70 several power-law functions proposed to describe the non-Darcian flow, and one of the most
71 commonly used power-law equations is the Izbash equation ([Izbash, 1931](#)), which is written
72 as:

$$J = aq^b \quad (1-3)$$

73 where a and b are the empirical parameters that depend on flow and materials properties, and
74 the coefficient b is usually between 1 and 2.

75 Because of its applicability for a wide range of velocity spectrum and its sound physics,
76 many scholars have adopted the Forchheimer equation (among many different types of
77 equations) to explore the non-Darcian flow. Besides, the theoretical background of the
78 Forchheimer equation has been discussed in details ([Panfilov and Fourar, 2006](#)). Numerous
79 experimental data have confirmed the validity of the Forchheimer equation for a variety of
80 nonlinear flow phenomena ([Geertsma, 1974](#); [Scheidegger, 1957](#); [Wright, 1968](#)). The
81 quadratic Forchheimer law has also been revealed as a result of numerical modelling by
82 simulating the Navier–Stokes flow in corrugated channels ([Koch and Ladd, 1996](#); [Skjetne et](#)
83 [al., 1999](#); [Souto and Moyne, 1997](#)). To sum up, the Forchheimer equation will be selected as
84 a representative to describe non-Darcy flow in this study.

85 Since the transition between Darcian flow and non-Darcian flow is important and
86 difficult to quantify, different scholars have carried out experiments using a wide range of
87 porous media, including homogeneous and heterogeneous porous media. Most of the
88 experimental studies have focused on the influence of mean particle size on flow state



89 transition using homogeneous porous media. In fact, it was believed that the nonlinear (or
90 non-Darcian) flow behavior in porous media was due to turbulent effect of flow in earlier
91 studies and the Reynold number (Re) was widely used to quantify the initiation of non-
92 Darcian flow. [Bear \(1972\)](#) concluded that the critical Re (denoted as Re_c) of flow states (or
93 the Re value at which flow starts to change from Darcian flow regime to non-Darcian flow
94 regime) is between 1 to 10. This finding was based on experimental data collected in packed
95 sand beds ([Ergun, 1952](#); [Fancher and Lewis, 1933](#); [Lindquist, 1933](#); [Scheidegger, 1960](#)).
96 [Schneebeli \(1955\)](#) and [Wright \(1968\)](#) experimentally measured the value of Re at the
97 beginning of turbulence and concluded that at very high velocities, the deviation from
98 Darcy's law is due to inertial effects followed by turbulent effects. In addition, [Dudgeon](#)
99 [\(1966\)](#) confirmed that Re_c is about 60~150 for relatively coarse particle medium including
100 river gravels, crushed rock particles and glass marbles with grain sizes from 16 mm to 152
101 mm. [Dudgeon \(1966\)](#) indicated that the deviation from Darcy's law was not entirely due to
102 turbulence, but in a large extent due to inertial forces. Besides, [Geertsma \(1974\)](#) proposed an
103 empirical relationship among the inertial coefficient, permeability and porosity by conducting
104 non-Darcian flow experiments in unconsolidated and consolidated sands. The laser
105 anemometry and flow visualization studies of fluid flow in porous structures were used by
106 [Dybbs and Edwards \(1984\)](#), and they observed the nonlinear behavior at Reynolds numbers
107 around 150. [Latifi et al. \(1989\)](#) found that the transition from unsteady-state laminar flow to
108 non-Darcian flow in packed beds of spheres was between Re values of 110 and 370. [Seguin et](#)
109 [al. \(1998\)](#) investigated the characterization of flow regimes in various porous media with
110 electrochemical techniques and found that the end of the Darcian flow regime in packed beds
111 of particles appeared at Re about 180. Besides, [Bu et al. \(2014\)](#) indicated that the Darcian
112 flow in the packed beds would end at Re around 100 by using electrochemical techniques.
113 [Sedghi-Asl et al. \(2014\)](#) found that the Darcy's law was usually not valid for rounded particle



114 sizes greater than 2.8 mm, according to the experimental results of flow in different sizes of
115 rounded aggregates. Our previous experimental research ([Li et al., 2017](#)) indicated that when
116 the particle size was smaller than 2.8 mm, the flow state gradually changed from pre-Darcy
117 flow to post-Darcy flow when the specific discharge increased. When the medium particle
118 sizes get even larger, such as 4.5 mm, 6.39 mm, 12.84 mm, and 16 mm ([Moutsopoulos et al.,](#)
119 [2009](#)), only the post-Darcy flow exists. Based on above analysis, we can see that many
120 previous experiments were carried out on homogeneous porous media, and the non-Darcy
121 flow characteristics are quite different in porous media with various particle sizes.

122 Among the numerous experimental studies on this issue, it is evident that most of them
123 focused on the effect of the mean particle size rather than the particle size distribution.
124 Recently, a few investigators recognized the importance of particle size heterogeneity in
125 understanding the transition of flow regimes, and have carried out a series of experiments to
126 address the issue. For instance, [Van Lopik et al. \(2017\)](#) provided new experimental data on
127 nonlinear flow behavior in various uniformly graded granular material for 20 samples,
128 ranging from medium sands ($d_{50} > 0.39$ mm) to gravel ($d_{50} > 6.34$ mm). In addition, they
129 investigated the nonlinear flow behavior through packed beds of 5 different types of natural
130 sand and gravel from unconsolidated aquifers, as well as 13 different composite mixtures of
131 uniformly graded filter sands at different grain size distributions and porosity values ([Van](#)
132 [Lopik et al., 2019](#)). We have also discussed the effect of particle size distribution on
133 Forchheimer flow and transition of flow regimes in a previous study ([Li et al., 2019](#)). And our
134 study showed that the uniformity coefficient of porous media (a term used to describe the
135 pore size distribution) is a critical factor for determining the flow regimes besides the mean
136 particle sizes. [Yang et al. \(2019\)](#) investigated the effects of the particle size distribution on the
137 seepage behavior of a sand particle mixture subjected and evaluated the validity of empirical
138 formulas of permeability and inertia factor used in engineering practice. [Shi et al. \(2020\)](#)



139 discussed the non-Darcy flow behavior of granular limestone with a wide range of porosity
140 from 0.242 to 0.449. Based on the experimental data, [Shi et al. \(2020\)](#) proposed an empirical
141 hydraulic conductivity-porosity relation as well as an expression of inertial coefficient.
142 Regardless of the media investigated are homogeneous or heterogeneous, the essence of the
143 water passing capacity of porous media is pore sizes. Thus, exploring the distribution of pores
144 in porous media is the basis of studying flow dynamics of Darcian and non-Darcian flows.

145 The purpose of this study is to provide a quantitative analysis on the effects of pore size
146 on the transition of flow regimes between Darcian and non-Darcian flows based on a series of
147 laboratory experiments. To meet the objectives, we have firstly carried out the seepage
148 experiments of permeable stones with four different particle sizes. After that, we have
149 conducted mercury injection experiments on permeable stones with four different particle
150 sizes, and the pore size distributions with different particle sizes are obtained. Finally, the
151 effect of pore size on the transition of flow regimes and Forchheimer coefficients are
152 discussed based on the experimental results.

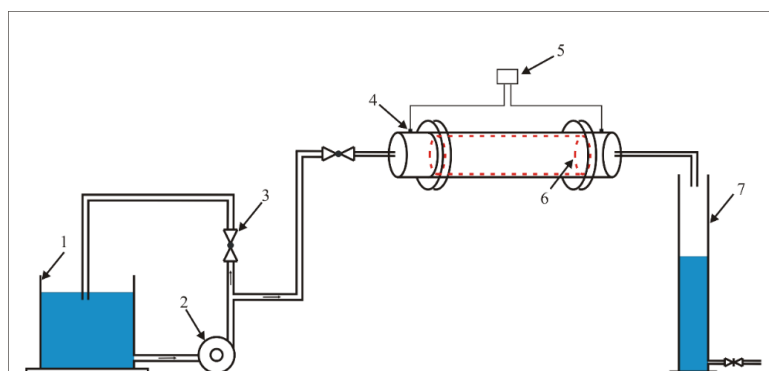
153 **2. Experimental methodology**

154 **2.1 Experimental setup and methods**

155 The experimental device is mainly composed of three parts: a water supply device, a
156 seepage experimental device and a measuring device. The schematic diagram of the
157 experimental apparatus is shown in Fig. 1. The water supply device consists of a tank, a
158 centrifugal pump and a flow regulating valve. The seepage experimental device consists of a
159 permeable stone and a plexiglass column. The measurement device monitors the real-time
160 water temperature and pressure. The water temperature is measured using a thermometer with
161 a precision of measurement of 0.1 °C. The water-level fluctuation is measured to calculate the
162 flow rate by a pressure transducer (CY201, Chengdu test LLC, China) in the range of 0–20
163 kPa with $\pm 0.1\%$ accuracy. The measuring device consists of a cylindrical tank and a pressure



164 transducer. The sample of permeable stone is 60 mm in length with a circular cross section of
165 51.3 mm in diameter. Two pressure transducers are set at the entrance and exit of the column
166 to measure the pressure drop. To minimize the boundary effects, the pressure transducer is
167 placed 30 mm away from either end of the column, and the way of pressure measurement is
168 consistent with our previous studies (Li et al., 2017; Li et al., 2019).



169
170 1) tank, 2) Pump, 3) Valve, 4) Pressure sensor, 5) Data collector, 6) Permeable stones, 7)
171 Measurement tank.

172 Fig. 1 The schematic diagram of experimental apparatus.

173 2.2 Experimental Materials and Procedures

174 Four different particle sizes of permeable stones are selected to carry out the seepage
175 experiment in this study. It is necessary to make a brief overview of the preparation process
176 of permeable stone, which is a type of artificially made tight porous medium formed by sand
177 grains and cementing compound. In the process of preparing permeable stone, a certain
178 particle size of sand and cementing compound is put in a mold, and is consolidated at room
179 temperature. We have carried out the seepage experiments of permeable stones with four
180 different mesh sizes, including 24 mesh size, 46 mesh size, 60 mesh size, and 80 mesh size,
181 and the mesh size is defined as the number of mesh elements (all in square shapes) in a one
182 inch by one inch square, thus a greater number of mesh size implies a smaller particle size.

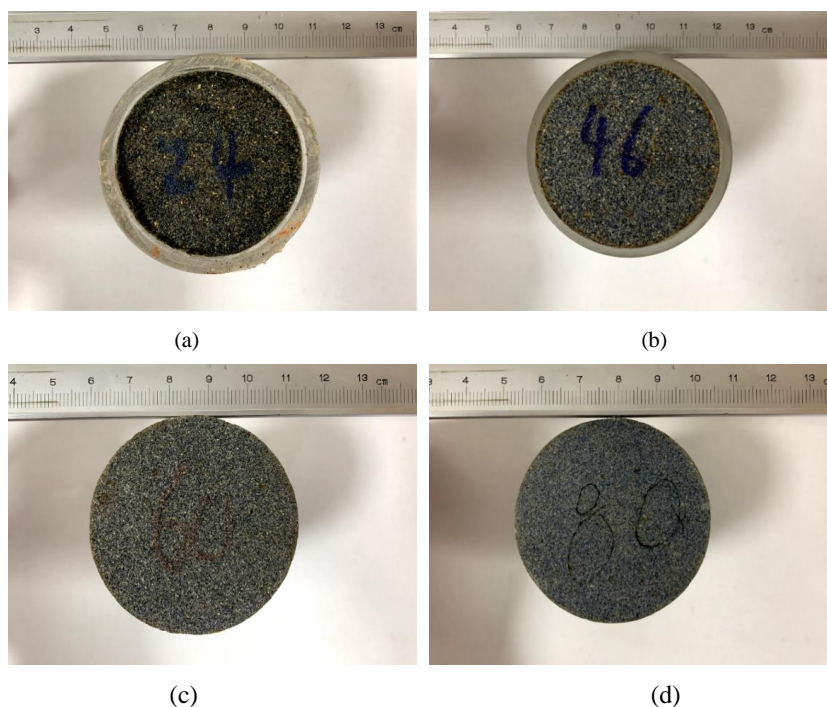


183 For instance, we can convert above four different mesh sizes of permeable stones into
184 corresponding particle sizes of 0.71 mm, 0.36 mm, 0.25 mm and 0.18 mm, respectively. The
185 pore structure of permeable rock will not change in the process of the seepage experiment
186 under room temperature, and the physical diagrams of four kinds of permeable stones with
187 different particle sizes are shown in Fig. 2 and Fig. 3.



188

189 Fig. 2 Physical drawing of permeable stones with four different particle sizes.



190

191

192

193

194 Fig. 3 Permeable stones with different particle sizes: (a) 24 mesh size or 0.71 mm, (b) 46

195 mesh size or 0.36 mm, (c) 60 mesh size or 0.25 mm, and (d) 80 mesh size or 0.18 mm.



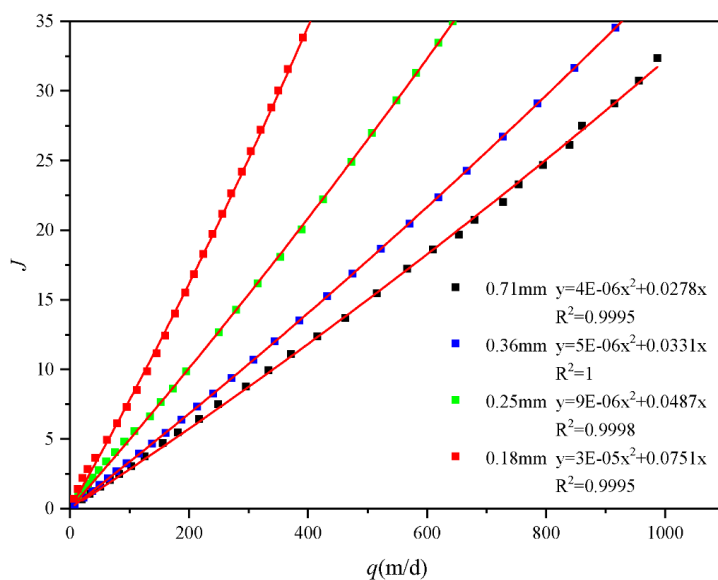
196 It is worth mentioning that the contact surface of the sample and the plexiglass column
197 is sealed to prevent any preferential flow through the wall of the plexiglass column. After the
198 permeable stone is inserted into the plexiglass column, both ends are sealed with silicone glue.
199 The water passing through the permeable stone is then collected by a cylindrical tank.
200 Moreover, the ratio of the internal diameter of the column to the particle size of permeable
201 stone is greater than 12, which can eliminate any possible wall effect on the seepage
202 according to [Beavers et al. \(1972\)](#). When carrying out the experiment, it usually takes about
203 two hours to saturate the permeable stone. For each packed sample, more than 25 tests with
204 different constant inlet pressures were conducted under steady-state flow condition. In
205 addition, for each group of permeable stone, repeated tests under the same experimental
206 condition were carried out 3-4 times to ensure the accuracy of the results.

207 **3. Results and discussion**

208 **3.1 Permeable stone seepage experiment**

209 In this study, we selected permeable stone with four different particle sizes as the
210 research objects, including 24 mesh size, 46 mesh size, 60 mesh size and 80 mesh size. The
211 mesh size is the number of holes per inch of screen mesh and the particle size is inversely
212 proportional to the mesh size. The mean particle sizes corresponding to the four different
213 mesh sizes are 0.71 mm, 0.36 mm, 0.25 mm, and 0.18 mm, respectively, where the mean
214 particle size is corresponding to 50% by weight hereinafter in this study. Such a definition of
215 mean particle size may be different from some other studies such as [Fetter \(2001\)](#) which has
216 used 10% by weight as the mean particle size. The relationship between the specific
217 discharge (q) and the hydraulic gradient (J) of permeable stones is plotted in Fig. 4. The units
218 of specific discharge mentioned in this study are all converted to meters per day (m/d).

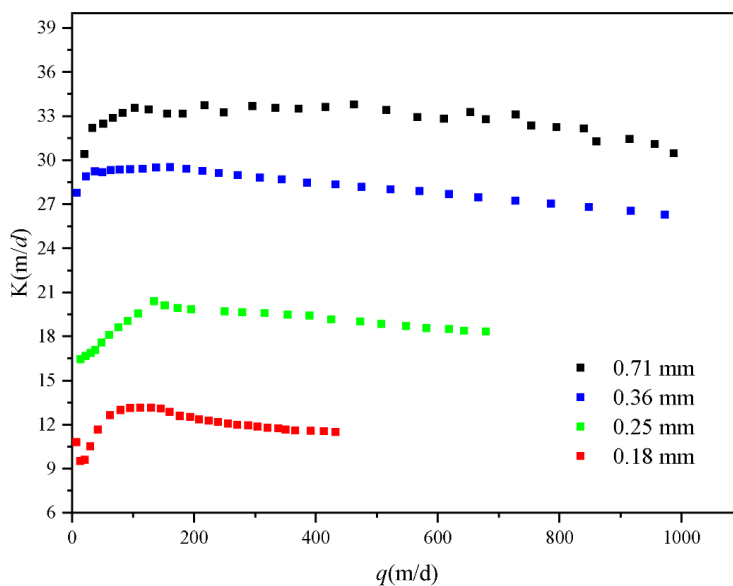
219



220

221 Fig. 4 Variation of J with q of four permeable stones with different particle sizes.

222 Fig. 4 shows that when q is somewhat the same, a larger mesh size (which means a
223 smaller particle size) will lead to a larger J . And the results are consistent with our previous
224 studies (Huang et al., 2013; Li et al., 2017; Li et al., 2019). However, the nonlinear
225 characteristics of q - J curve are not obvious due to the relatively small velocity range used in
226 the experiments. Nevertheless, the best-fitting results using the Forchheimer equation are
227 satisfactory. To analyze the influence of pore size on seepage flow regimes, we have obtained
228 the relationship between q and the “pseudo” hydraulic conductivity (K) (which is computed
229 using q/J) of four permeable stones with different particle sizes, as shown in Fig. 5. We
230 should point out that the “pseudo” hydraulic conductivity term discussed here for non-
231 Darcian flow is usually not a constant, thus it is different from the hydraulic conductivity
232 term used in Darcy’s law, which is a constant. It is obvious that the hydraulic conductivity is
233 not a constant with the increase of specific discharge, so it is called the “pseudo” hydraulic
234 conductivity (Li et al., 2019).



235

236 Fig. 5 Variation of K with q of four permeable stones with different particle sizes.

237 We can divide the q - K curve into two segments: for the first segment, K increases with
 238 the increase of q , which is denoted as the pre-Darcian flow. For the second segment, after q
 239 increases to a certain value, K begins to decrease with q , which is called the post-Darcian
 240 flow. When the hydraulic gradient is small (and q is small as well), a great portion of water is
 241 bounded (or becomes immobile) on the surface of solids due to the solid-liquid interfacial
 242 force, and only a small fraction of the water is mobile and free to flow through the pores. As
 243 the hydraulic gradient increases (and q increases as well), the initial threshold for mobilizing
 244 the previously immobile water near the solid-liquid surface is overcome and more water
 245 participates in the flow. For this reason, the "pseudo" hydraulic conductivity increases with
 246 the increase of hydraulic gradient and the specific discharge in the first segment. When the
 247 specific discharge increases to the critical specific discharge (q_c), the "pseudo" hydraulic
 248 conductivity is maximized. According to $K = \frac{q}{Aq + Bq^2} = \frac{1}{A + Bq}$ based on Eq. (1-2), we can
 249 find that the "pseudo" hydraulic conductivity begins to decrease as the specific discharge



250 continues to increase. Besides, the critical specific discharge corresponding to the transition
251 of flow regimes (from pre-Darcian to post-Darcian) increases with the increase of particle
252 sizes (or decrease of mesh sizes).

253 3.2 Mercury injection experiment

254 The particle size, different grain size distributions and degree of sorting are the main
255 factors that determine the size and shape of pores. And the shape of the pores determines the
256 tortuosity and distribution of flow paths, which are related to viscous and inertial flow
257 resistances. It is generally accepted in previous studies that the pore sizes of porous media
258 have an impact on the seepage law ([Maalal et al., 2021](#); [Zhou et al., 2019](#)). However, the
259 structure of natural porous media is very complex, and it is difficult to quantify the effects of
260 the arrangement of particles on the seepage law. The characteristics of pore size distribution
261 contains critical information for quantifying the flow regimes. The mercury intrusion
262 porosimetry and the nitrogen adsorption isotherm are two commonly used methods to
263 characterize the pore sizes and their distribution ([Rijfkogel et al., 2019](#)). Besides, other
264 techniques can also be used to derive the pore size distribution, such as small-angle neutron
265 and X-ray scattering measurements, CT images and nuclear magnetic resonance ([Anovitz and
266 Cole, 2015](#); [Hall et al., 1986](#); [Kate and Gokhale, 2006](#); [Lindquist et al., 2000](#)). In this study
267 we will use the mercury injection experiment to measure the pore size distribution of the four
268 permeable stones with different particle sizes and use the information to describe the flow
269 regimes.

270 To quantitatively study the pore size and pore throat distribution, we need to envisage a
271 physically based conceptual model to describe the pore structures of permeable stones. The
272 commonly used model is the so-called capillary model ([Pittman, 1992](#); [Rezaee et al., 2012](#);
273 [Schmitt et al., 2013](#)), which approximates the connected pores as many paralleled capillaries.
274 And the capillary forces are generated at the phase interface due to the surface tension

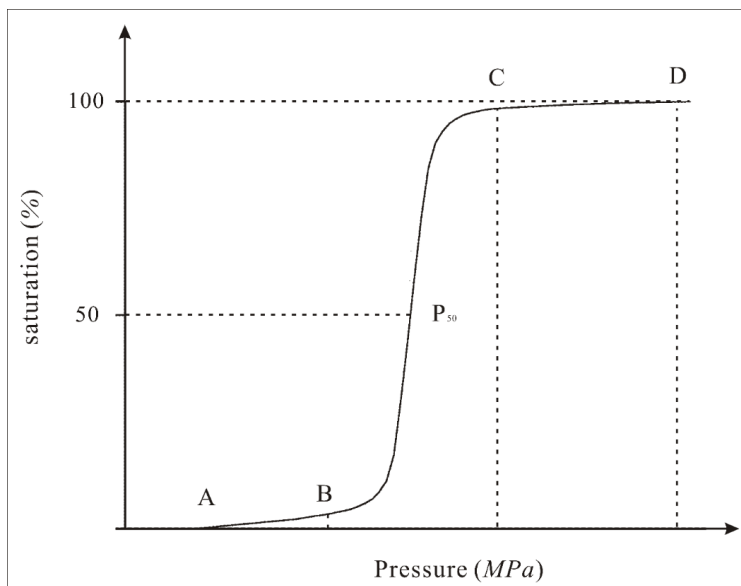


275 between the solid and liquid phases when liquid flows in a capillary. The capillary force is
276 directed toward the concave liquid level, and is shown as (Washburn, 1921):

$$P_c = \frac{2\sigma \cos \theta}{r} \quad (3-1)$$

277 where P_c is the capillary force, σ is the solid-liquid interfacial tension, θ is the wet angle
278 between the liquid and the solid surface, and r is the radius of curvature in capillary.

279 Since mercury is a nonwetting phase to solids, so to get mercury into the pores of the
280 permeable stone, an external force (or displacement pressure) must be applied to overcome
281 the capillary force. When a greater pressure is applied, mercury can enter smaller pores.
282 When a certain pressure is applied, the injection pressure is equivalent to the capillary
283 pressure in the corresponding pore. Then we can calculate the corresponding capillary radius
284 according to Eq. (3-1), and the volume of mercury injected is the pore volume.



285

286 Fig. 6 Schematic diagram of pressure changes with saturation: the initial stage (A-B), the
287 intermediate mercury entry stage (B-C), and the end stage (C-D).



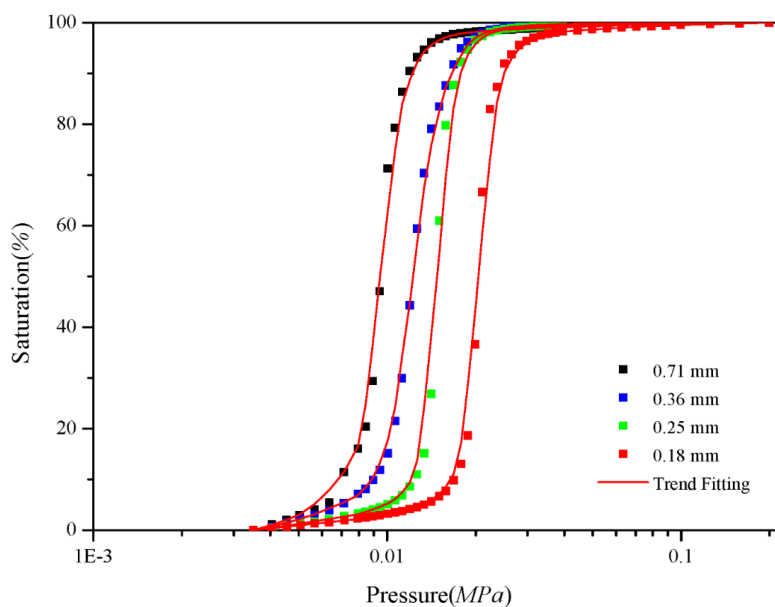
288 By continuously increasing the injection pressure, one can obtain the curve of injection
289 pressure and the volume of injected mercury, from which one can also obtain the pore-throat
290 distribution curve and capillary pressure curve. According to the amount of mercury injected
291 at different injection pressures, the relation between the injection pressure and the injection
292 saturation is shown in Fig. 6.

293 Fig. 6 shows that the mercury injection curve can be divided into three stages. Firstly,
294 during the initial stage (A-B) which has a very mild slope, the intake pressure is very small
295 and the intake saturation is also very low. With the increasing of the injection pressure, the
296 intake saturation slowly increases. Secondly, during the intermediate mercury entry stage (B-
297 C) which has a steep slope, a small pressure change will lead to a significant saturation
298 change. This means that the pores are relatively uniform and the differences in pore sizes are
299 small. Hence, we can use the pressure ratios of B and C (P_C/P_B) to reflect the inhomogeneity
300 of the pore size in the porous media. Besides, when the saturation reaches 50%, the
301 corresponding pressure value (P_{50}) reflects the characteristics of the mean pore size, and a
302 larger P_{50} leads to a larger mean pore size. Finally, during the end stage (C-D) which has a
303 very mild slope as well, the amount of mercury will not increase considerably when the
304 injection pressure increases. This indicates that nearly all the pores are essentially filled with
305 mercury, and the mercury injection experiment is completed. After completing the mercury
306 injection experiments, we have obtained the mercury injection curves of four permeable
307 stones with different particle sizes, as shown in Fig. 7.

308 We can make a number of interesting observations based on Fig. 7. Firstly, the pressure
309 at the starting point (when the saturation begins to increase), denoted as P_A , increases as the
310 mean particle size decreases. This means that the maximum pore size in permeable stone
311 decreases with the decrease of the mean particle sizes. Secondly, the mercury injection curves
312 of four permeable stones all include steep intermediate stages, indicating that the pore size



313 distributions are all relatively uniform. And the corresponding pressure values at points B and
314 C increase as the mean particle sizes decreases. Moreover, the pressure ratios corresponding
315 to points B and C (P_C/P_B) also decrease with the decrease of particle sizes, suggesting even
316 more uniform pore size distributions with decreasing particle sizes. Thirdly, the intermediate
317 mercury entry stages gradually shift to the right with the decrease of particle sizes. When the
318 saturation reaches 50%, the corresponding pressure (the median pressure) decreases with the
319 increase of the mean particle sizes. Fourthly, the mercury injection curves of these four
320 permeable stones with different particle sizes all approach 100% saturation with very mild
321 slopes, indicating that there are few small pores in the permeable stones. We have extracted
322 the key pressure characteristic values of mercury injection experiment of Fig. 7, and listed the
323 results in Table 1.



324

325 Fig. 7 Variation of pressure with saturation of four permeable stones with different particle

326

sizes.

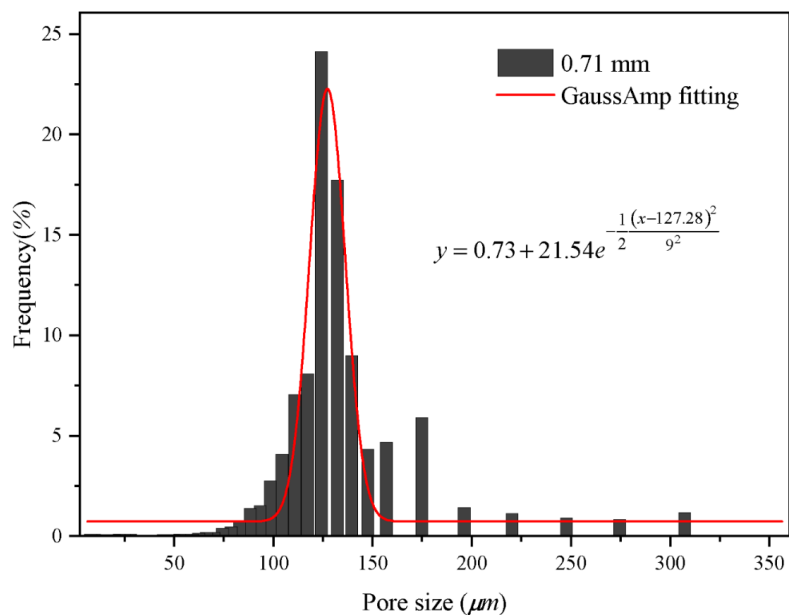
327



328 Table 1. Pressure characteristic values of four permeable stones with different particle sizes.

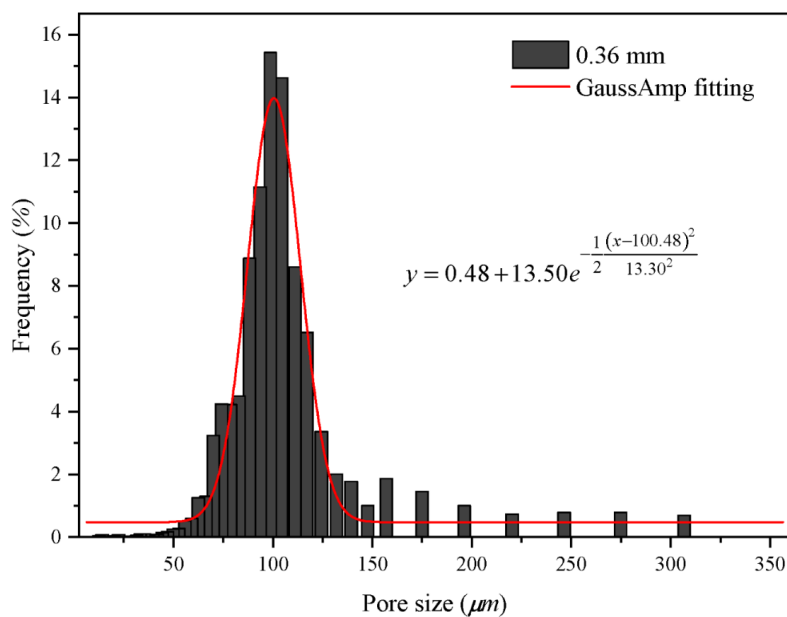
Mesh size	$P_A(MPa)$	$P_B(MPa)$	$P_C(MPa)$	$P_{50}(MPa)$	P_C/P_B
24	0.0041	0.0064	0.0133	0.0094	2.0987
46	0.0045	0.0071	0.0188	0.0119	2.6374
60	0.0051	0.0112	0.0211	0.0150	1.8764
80	0.0057	0.0158	0.0281	0.0211	1.7758

329 To observe the pore size distributions of the four permeable stones with different particle
 330 sizes in more details, we can calculate the percentages of different pore sizes in permeable
 331 stones according to the mercury injection curves, as shown in Figs. 8-11.



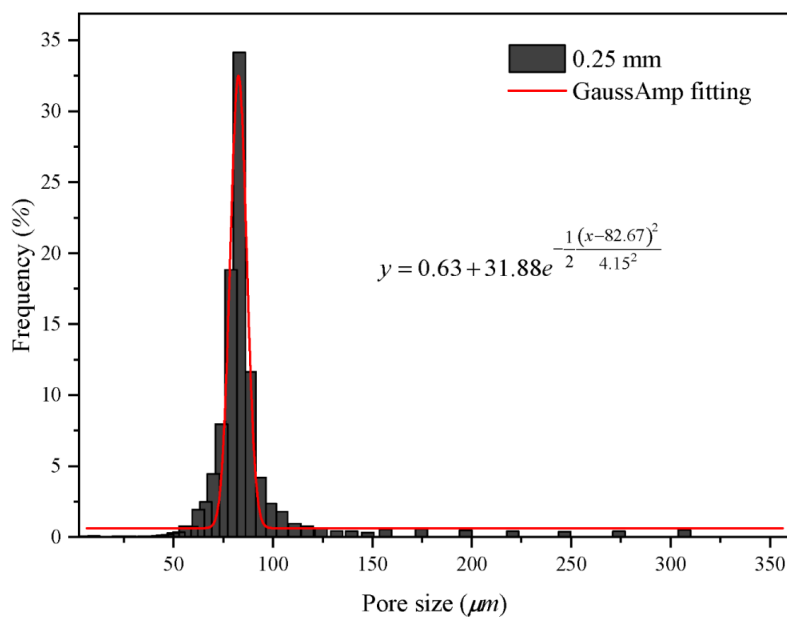
333 Fig. 8 Histogram of pore size distribution of permeable stone with diameter of 0.71 mm.

334



335

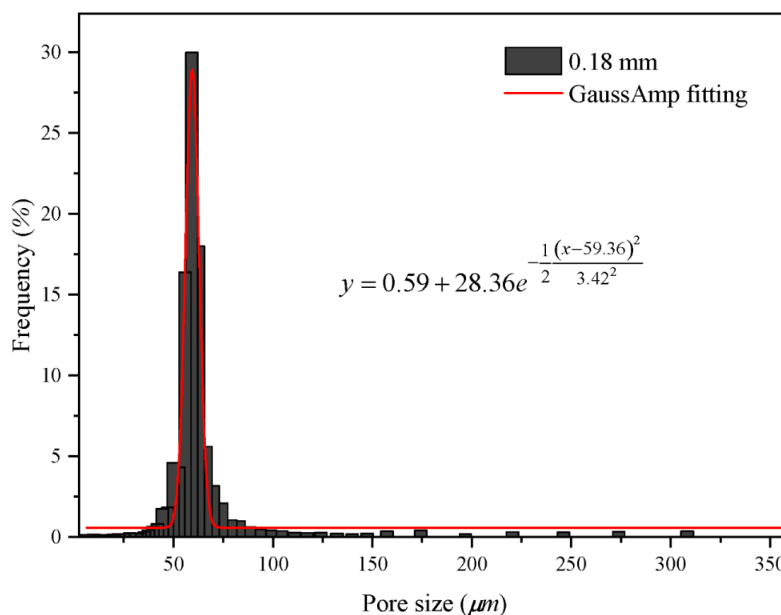
336 Fig. 9 Histogram of pore size distribution of permeable stone with diameter of 0.36 mm.



337

338 Fig. 10 Histogram of pore size distribution of permeable stone with diameter of 0.25 mm.

339



340

341 Fig. 11 Histogram of pore size distribution of permeable stone with diameter of 0.18 mm.

342 From Fig. 8 to Fig. 11 we can find that the pore sizes of the four permeable stones are
 343 uniform and fall within narrow ranges. The pore size distributions of four different particle
 344 sizes show a skewed normal distribution. Besides, the pore maximum proportion (the peak of
 345 the curve, see Figs. 8-11) of permeable stones with different particle sizes are different, which
 346 are 124 μm , 99 μm , 83 μm and 59 μm , respectively. The Gaussian function is widely used to
 347 characterize the pore system and classify the petrophysical rock (Harlan et al., 1995; Jeon et
 348 al., 2014; Xu and Torres-Verdín, 2013), and the general form of the Gauss function is shown
 349 below:

$$y = y_0 + He^{-\frac{(x-x_c)^2}{2w^2}} \quad (3-2)$$

350 where H is the height of the peak of the mercury injection curve, x_c is the abscissa
 351 corresponding to the peak of the curve (the pore size), w is the standard variance, which
 352 represents the width of the curve. To characterize the distribution of pore structure of four



353 different permeable stones, we best-fit the Gaussian curve of the pore distribution of four
354 permeable stones with different particle sizes, and the best-fitted parameters are shown in
355 Table 2. We can make several interesting observations from Table 2. Firstly, the expected
356 value (x_c) decreases with decreasing particle sizes of permeable stone, and the x_c values of
357 different permeable stones are almost the same. Secondly, the standard variance (w)
358 corresponding to the permeable stone of 0.18 mm is the smallest, indicating that the pore size
359 distribution is more concentrated (or relatively homogeneous). For comparison, the pore size
360 distribution of 0.36 mm permeable stone is the widest with the greatest variance. Finally,
361 different values of H represent different proportions of pore sizes, among which the highest
362 proportion can reach 34.04%. It will be desirable to establish a correlation between the
363 parameters used in the pore-size distribution of Eq. (3-2) with the two Forchheimer
364 coefficients A and B . This objective may be achieved using high-resolution pore-scale fluid
365 mechanics simulations, which are out of the scope of this study. Further research is needed to
366 address this issue in the future.

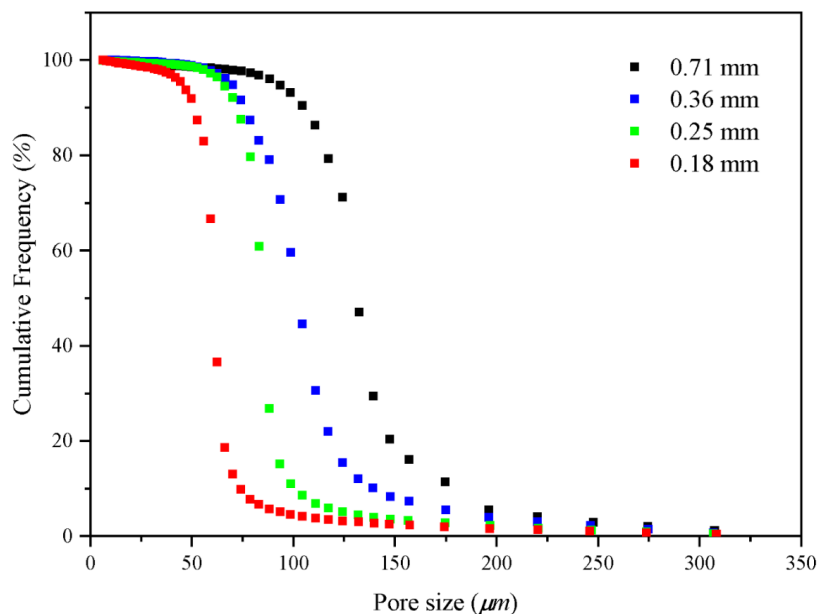
367 Table 2. Gaussian function characteristic values of four permeable stones with different
368 particle sizes.

Mesh size	particle size (mm)	y_0	H	x_c	w
24	0.71	0.73	21.54	127.28	9.00
46	0.36	0.48	13.49	100.48	13.30
60	0.25	0.63	31.88	82.67	4.15
80	0.18	0.59	28.36	59.36	3.42

369



370 The pore size distributions fall within ever narrower ranges with mesh sizes become
371 larger. Moreover, the cumulative percentage frequency curves of the pore size distributions
372 with different particle sizes are exhibited in Fig. 12 and the results are shown in Table 3.



373

374 Fig. 12. The cumulative frequency curve of pore size distribution.

375 Fig. 12 shows that R_{50} (the pore size corresponding to the median pressure P_{50}) increases
376 with the increase of permeable stone particle size, and the mean pore diameter (R_m) also
377 increases. In general, the pore size corresponding to the median pressure (denoted as R_{50})
378 may be slightly different than the mean pore diameter (R_m) which has been defined in
379 different ways by various investigators when analyzing the pore size distributions (Hea and
380 Zhangb, 2015; Zhen-Hua et al., 2007; Zhihong et al., 2000). As R_{50} is easily identifiable in
381 the mercury injection experiments, it is used in this study as a representative of the mean pore
382 diameter (R_m) of the permeable stone. Besides, the seepage law of permeable stone is closely
383 related to the pore size, and the smaller average pore size will result in a larger hydraulic
384 gradient under the condition of the same specific discharge (see Fig. 4). The pore size



385 characteristic values with different particle sizes are listed in Table 3. We find that the
386 porosity decreases as the particle size increases while the mean pore diameter increases. And
387 the mean pore size can reflect the influence of particle diameter, sorting degree and
388 arrangement mode of porous medium on seepage parameters.

389 Table 3. Pore size characteristic values of four permeable stones with different particle sizes.

Mesh size	Mean particle size (<i>mm</i>)	Porosity (%)	R_m (μm)	R_{50} (μm)
24	0.71	32.35	131.31	131.34
46	0.36	36.69	102.56	103.42
60	0.25	40.82	84.73	85.09
80	0.18	42.88	60.97	61.12

390 *Note:* R_m is the mean pore diameter, R_{50} is the pore diameter corresponding to the median
391 pressure P_{50} .

392 3.3 Analysis of influencing factors of Forchheimer equation coefficients

393 3.3.1 Influence of particle size on equation coefficient

394 The analysis of non-Darcy coefficient has always been of interest to many researchers
395 working in different disciplines of porous media flow ([Moutsopoulos et al., 2009](#); [Sedghi-Asl
396 et al., 2014](#); [Shi et al., 2020](#)). Various studies have suggested expressions for Forchheimer
397 coefficients, [Ward \(1964\)](#) proposed the estimation formula of Forchheimer coefficients A and
398 B by analyzing the experimental data of 20 different porous media:

$$A = \frac{360}{gd^2} \quad B = \frac{10.44}{gd} \quad (3-3)$$



399 where d is the particle diameter hereinafter, g is the acceleration due to gravity. Based on the
400 mixed model of parallel capillary tubes, [Blick \(1966\)](#) proposed a new form of Forchheimer
401 coefficients:

$$A = \frac{32}{gn^2d^2} \quad B = \frac{C_D}{2gn^2d} \quad (3-4)$$

402 where n is the porosity of the medium, C_D is an appropriate phenomenological coefficient.

403 Besides, [Ergun \(1952\)](#) suggested the new expressions by extending the classic Kozeny-
404 Carman model:

$$A = \frac{150(1-n)^2}{gn^3d^2} \quad B = \frac{1.75(1-n)}{gn^3d} \quad (3-5)$$

405 [Kovács \(1981\)](#) analyzed 300 data in the range of $10 < Re < 100$, and derived a similar
406 expression for the case of dispersed spherical particles, and the coefficients of Forchheimer
407 are as follows:

$$A = \frac{144(1-n)^2}{gn^3d^2} \quad B = \frac{2.4(1-n)}{gn^3d} \quad (3-6)$$

408 [Kadlec and Knight \(1996\)](#) also proposed the following Forchheimer coefficients:

$$A = \frac{255(1-n)^2}{gn^{3.7}d^2} \quad B = \frac{2(1-n)}{gn^3d} \quad (3-7)$$

409 [Sidiropoulou et al. \(2007\)](#) focused on the determination of the Forchheimer coefficients
410 for non-Darcian flow in porous media and evaluated the original theoretical equations above
411 and the validity of these equations was checked using existing experimental data. In addition,
412 the Root Mean Square Error (RMSE) was used as a criterion to quantitatively evaluate the



413 coefficients, and the RMSE was defined as $RMES = \sqrt{\frac{\sum_{i=1}^N (x_i - y_i)^2}{N}}$, where x_i were the
414 experimental values of Forchheimer coefficients, y_i were the values computed by different
415 equations above, and N was the total number of experimental points (Moutsopoulos et al.,
416 2009). The different forms of Forchheimer coefficients described above are based on different
417 assumptions and simplifications of pore structure. Consequently, these series of coefficients
418 are applicable under specific conditions with different degrees of accuracy.

419 According to Eq. (1-2), the hydraulic gradient (J) is composed of a viscous force-related
420 component (J_n) and an inertia force-related component (J_r), as below:

$$J_n = Aq = \frac{\alpha\mu}{\rho g} \frac{1}{d^2} q \quad J_r = \frac{\beta}{g} \frac{1}{d} q^2 \quad (3-8)$$

421 We can see from Eq. (3-8) that the J_n is inversely proportional to the square of the particle
422 size, and the J_r is inversely proportional to the particle size when the specific discharge
423 remains the same. Both J_n and J_r are closely related to specific surface area and sizes of pores.
424 Therefore, the particle size is an important factor affecting the Forchheimer coefficient,
425 Huang et al. (2013) carried out the experimental investigation on water flow in four columns
426 with cubic arrays of acrylic balls in diameter 3 mm, 5 mm, 8 mm and 10 mm, where all the
427 acrylic balls are arranged in regular cubes. Accordingly, the coefficients A and B can be
428 written as follows:

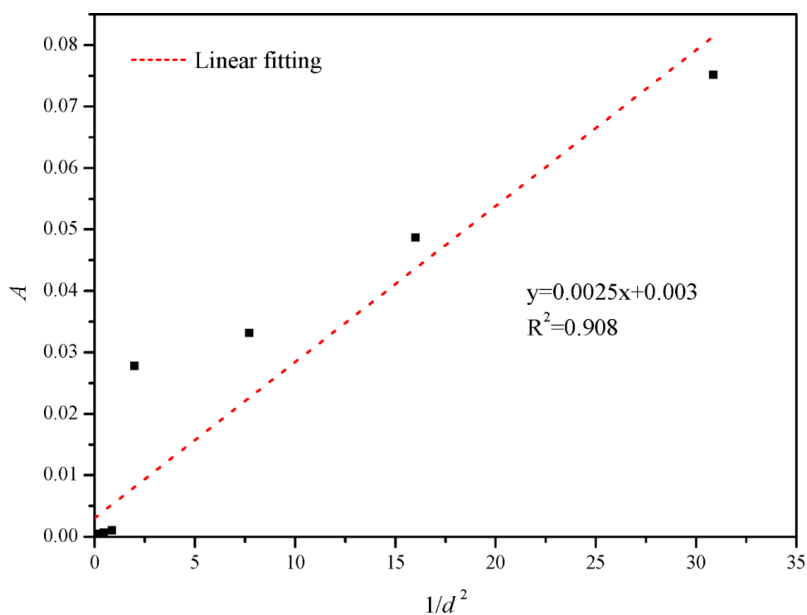
$$A = \frac{\alpha\mu}{\rho g} \frac{1}{d^2} \quad B = \frac{\beta}{g} \frac{1}{d} \quad (3-9)$$

429 where α and β are constants related to the shape, sorting, and arrangement of the particles,
430 and the specific derivation process is detailed in the previous study (Huang, 2012). The
431 experimental results showed that the coefficient A was inversely proportional to the particle
432 diameter square (d^2) and coefficient B was inversely proportional to the particle size (d)



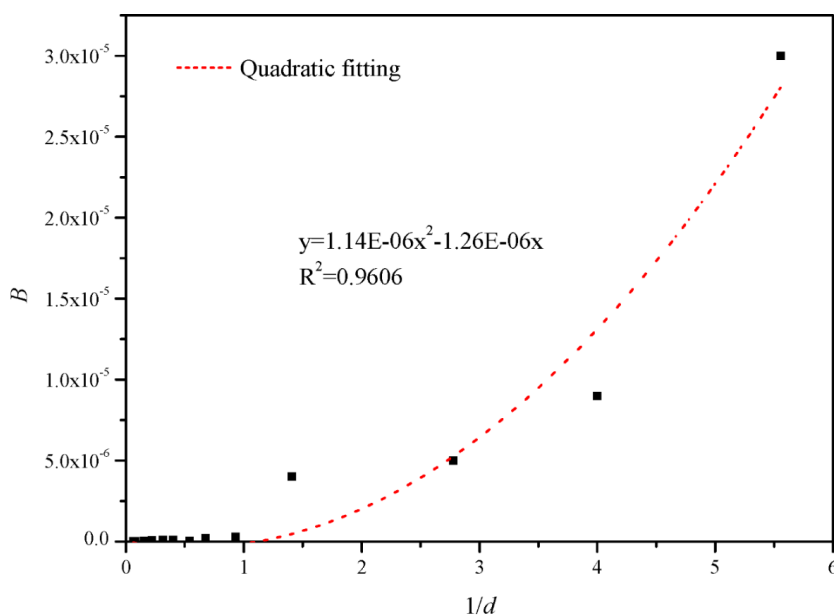
433 ([Huang et al., 2013](#)).

434 The uniform diameter cubic arrangement of porous media mentioned above is a rather
435 ideal medium. The shape and arrangement of particles of natural pore aquifers are usually
436 irregular. Therefore, the above-mentioned linear correlations between A and $1/d^2$, and
437 between B and $1/d$ should be examined specifically. For this purpose, we collect the
438 experimental data of homogeneous porous media, including the previous research results and
439 the results of other scholars. Among them, samples P1-P4 are the permeable stones selected
440 in this study, samples L1-L5 are from previous studies ([Li et al., 2017](#)), and the experimental
441 data of samples M1-M4 are from [Moutsopoulos et al. \(2009\)](#). The fitting coefficients are
442 shown in Table 4. Furthermore, we can identify nice correlations between the Forchheimer
443 coefficient A and $1/d^2$ and between the Forchheimer coefficient B and $1/d$, which are shown
444 in Fig. 13 and Fig. 14, respectively.



445

446 Fig. 13. Variation of A with $1/d^2$ of different homogeneous particle sizes.



447

448 Fig. 14. Variation of B with $1/d$ of different homogeneous particle sizes.

449 We can see from Fig. 13 that the coefficient A is linearly related to $1/d^2$ and the
450 relationship between coefficient A and is given as $A = 0.0025(1/d^2) + 0.003$. And the
451 relationship between coefficient B and $1/d$ is completely different from the linear correlation
452 as reported before. Fig. 14 shows that the coefficient B is quadratic related to $1/d$ and the
453 relationship between coefficient B and $1/d$ is given as $B = 1.14E-06(1/d)^2 - 1.26E-06(1/d)$.
454 That is to say, the relationship between coefficient A and $1/d^2$ is consistent with the law of
455 simple cubic arrangement porous media, but the relationship between coefficient B and $1/d$ is
456 not consistent with the law of simple cubic arrangement porous media. The structure of
457 porous medium arranged in cubes is different from the permeable stone. The porosity of the
458 porous media with spheres arranged in cubic is close to 0.48, independent of the diameter of
459 spheres. While the particle shape, arrangement and tightness of permeable stone are different,
460 and the porosity of permeable stone with different particle size is also different (see Table 3).



461 Table 4. Experimental fitting coefficient of different homogeneous particle sizes.

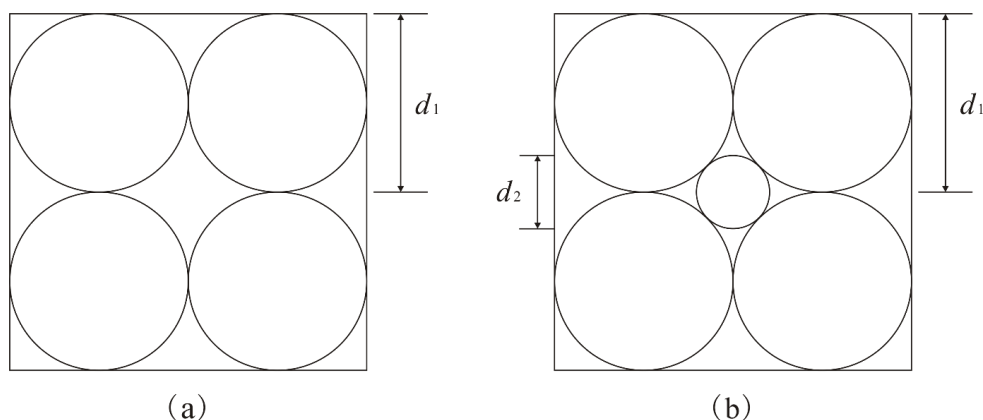
Sample	Particle size (mm)	Fitting equation	A	B	The correlation
P1	0.18	$y=0.0751x+3E-05x^2$	0.0751	3E-05	0.9995
P2	0.25	$y=0.0487x+9E-06x^2$	0.0487	9E-06	0.9998
P3	0.36	$y=0.0331x+5E-06x^2$	0.0331	5E-06	1
P4	0.71	$y=0.0278x+4E-06x^2$	0.0278	4E-06	0.9995
L1	1.075	$y=0.001x+3E-07x^2$	0.001	3E-07	0.9999
L2	1.475	$y=0.0007x+2E-07x^2$	0.0007	2E-07	0.9998
L3	1.85	$y=0.0005x+5E-08x^2$	0.0005	5E-08	0.9998
L4	2.5	$y=0.0005x+9E-08x^2$	0.0005	9E-08	0.9997
L5	3.17	$y=0.0004x+1E-07x^2$	0.0004	1E-07	0.9998
M1	4.5	$y=3E-05x+7E-08x^2$	3E-05	7E-08	0.9913
M2	6.39	$y=3E-05x+3E-08x^2$	3E-05	3E-08	0.9984
M3	12.84	$y=1E-05x+2E-08x^2$	1E-05	2E-08	0.9977
M4	16	$y=1E-05x+2E-08x^2$	1E-05	2E-08	0.998

462 **3.3.2 Influence of porosity on equation coefficient**

463 In above sections, we have analyzed the influence of particle sizes on seepage
 464 coefficient. Furthermore, the pore size and pore specific surface area are also related to the
 465 arrangement and sorting degree of particles, that is, to the porosity of porous media. To
 466 explore the effect of sorting degree on seepage coefficient, we draw a schematic diagram of



467 different sorting degree of particles, as shown in Fig. 15. The degree of particle sorting is one
468 of the important factors affecting the pore size. In porous media with a poor sorting degree,
469 the pore size is usually determined by the diameter of the smallest particle. We can see from
470 Fig. 15 that the pores between the larger particles are filled by smaller particles, resulting in
471 even smaller pores. In addition, the poorer sorting degree of particles leads to the larger pore
472 specific surface area and stronger viscous force of flow, which can lead to a larger coefficient
473 A.

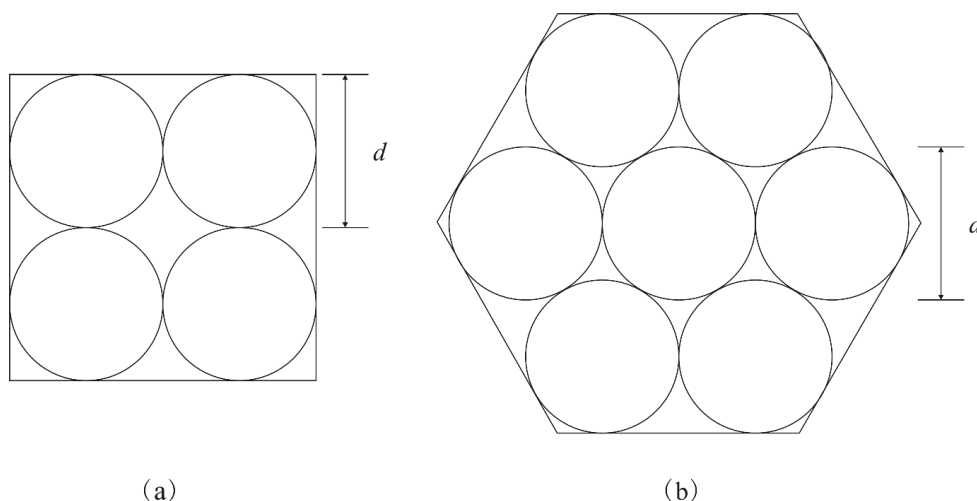


475 Fig. 15. The schematic diagram of particle sorting degree in different types.

476 Furthermore, we have also provided the schematic diagrams of spherical particles with
477 equal size in two simple arrangements, namely cube arrangement and hexahedron
478 arrangement, as shown in Fig. 16. And the cube arrangement is the less compact arrangement
479 with a pore diameter of $0.414d$, while the hexahedron arrangement is the more compact
480 arrangement with a pore diameter of $0.155d$. The characteristic value of pore structure in
481 different arrangement with the same particle size are shown in Table 5. We can see that
482 different arrangement modes will substantially affect the pore specific surface area and pore
483 size of porous media. The more compactly packed particles lead to the larger pore specific
484 surface area and stronger viscous force. Meanwhile, the smaller pore diameter is associated



485 with stronger effect of viscous force and inertia force. In summary, the better sorting degree
 486 of particles leads to the weaker viscous and inertial forces, then the coefficients A and B will
 487 be smaller. As the better sorting degree and the less compact (or looser) arrangement particles
 488 mean the larger porosity, so we can conclude that the larger porosity leads to the smaller
 489 coefficients A and B under the condition of the same particle size.



490

(a)

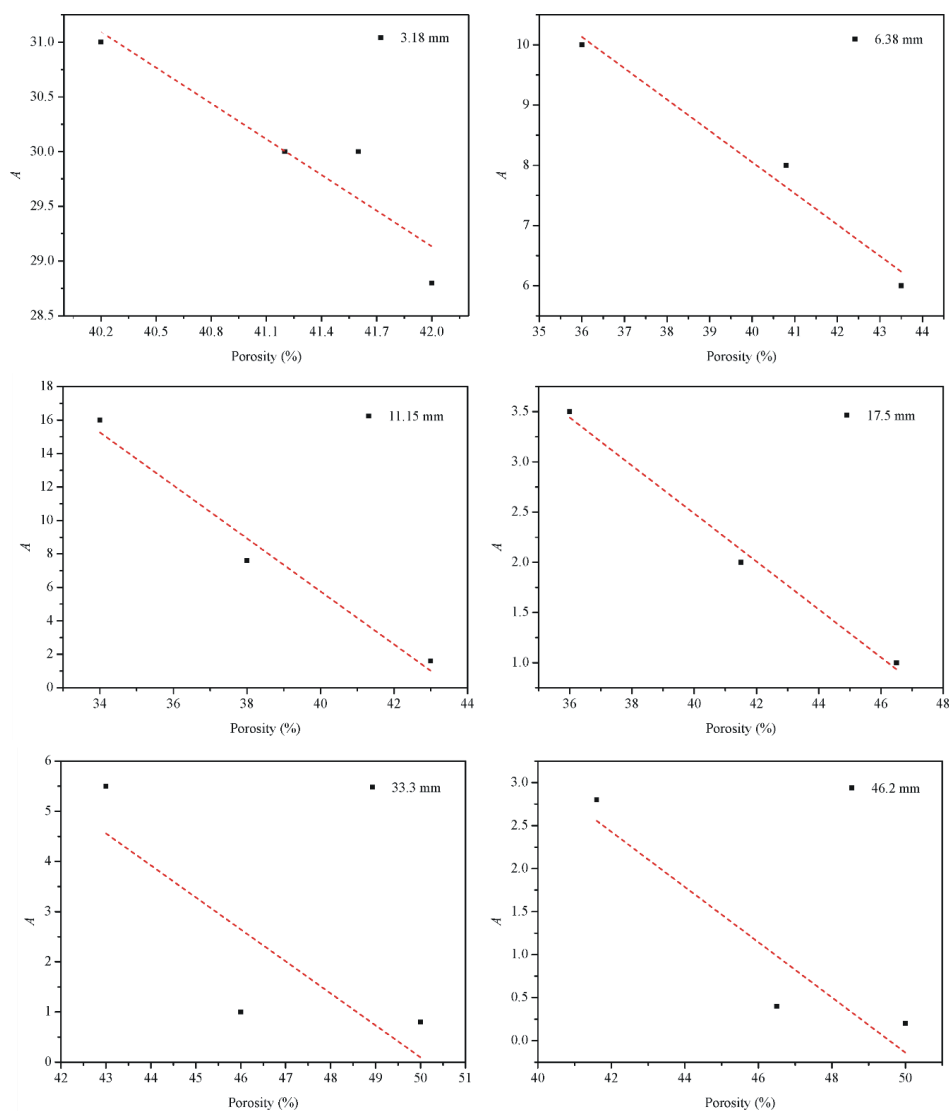
(b)

491 Fig. 16. The schematic diagram of particle arrangement in different types.

492 Table 5. Characteristic value of pore structure in different arrangement with the same particle
 493 size.

Arrangement mode	Side length	Porosity (%)	Specific surface area
Cube	$2d$	47.60	3.142
Hexahedron	$1.577d$	43.30	3.402

494 However, the structure of natural porous media is much more complex and
 495 heterogeneous than what has been shown in Figure 16, so it is difficult to quantitatively
 496 describe the effect of sorting degree and arrangement on seepage law.



497

498

Fig. 17. Variation of A with n of six gravels with different particle sizes.

499

500

501

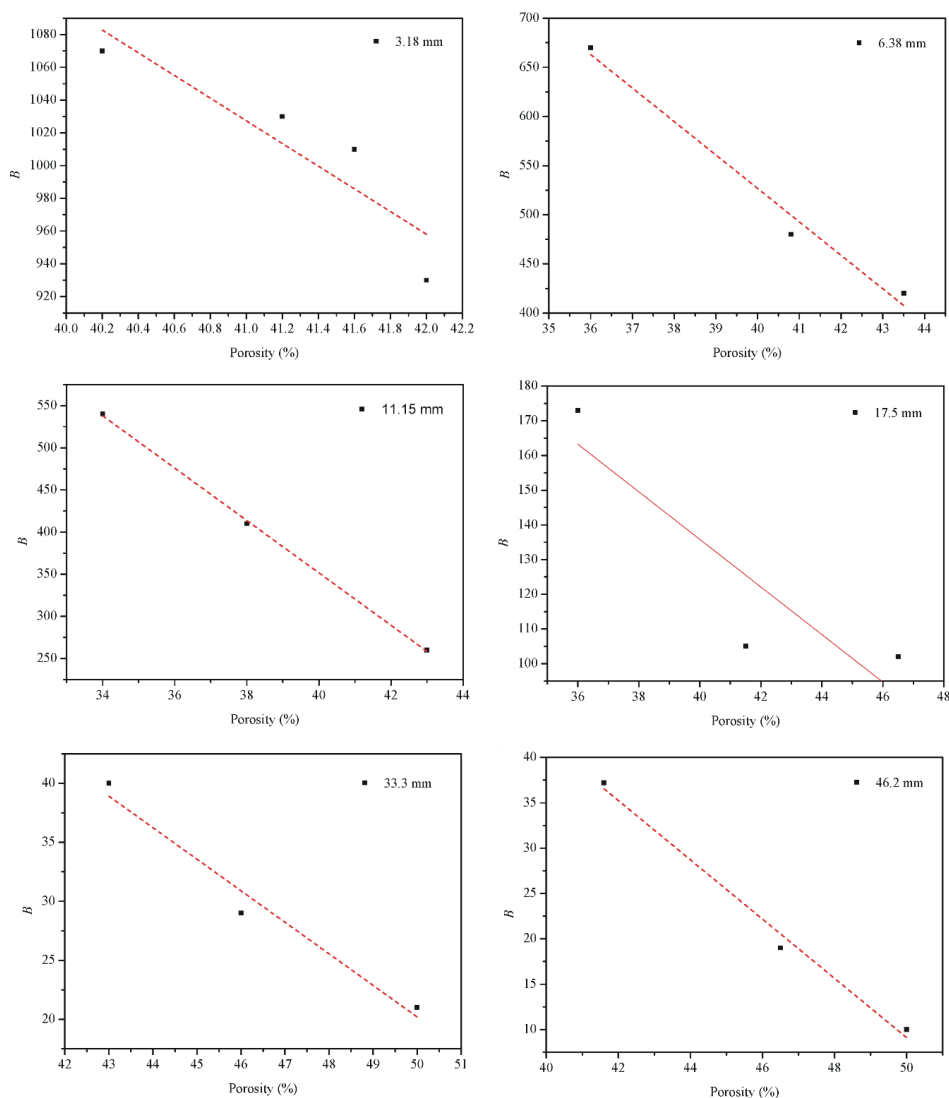
502

503

In view of this, we can use a macro parameter porosity (n) to reveal the effect of sorting degree and arrangement on seepage coefficient. In order to verify the correctness of the above analysis results, we selected the seepage experiment results of [Niranjan \(1973\)](#) for further validation. [Niranjan \(1973\)](#) chose gravel of the same size but different porosity and carried out seepage experiments. We selected the experimental results of six different particle sizes



504 with 3.18 mm, 6.38 mm, 11.15 mm, 17.5 mm, 33.3 mm and 46.2 mm from [Niranjan \(1973\)](#),
505 and drew the relationship between coefficient A and B and porosity respectively, as shown in
506 Fig. 17 and Fig. 18. We can see that the coefficients A and B of the six groups of experimental
507 data of [Niranjan \(1973\)](#) decrease with the increase of porosity, which is consistent with our
508 theoretical analysis of this investigation.



509

510 Fig. 18. Variation of B with n of six gravels with different particle sizes.



511 **4. Summary and conclusions**

512 This study presents experimental results of Forchheimer flow in four different
513 permeable stones with different mesh sizes, including 24 mesh size (0.71 mm), 46 mesh size
514 (0.36 mm), 60 mesh size (0.25 mm), 80 mesh size (0.18 mm). The effects of mean pore size
515 and pore size distribution on the transition of flow regimes (from pre-Darcian to post-Darcian)
516 are discussed. In addition, the mercury injection experiment is proposed to investigate the
517 pore distribution of the permeable stones. In addition, the Forchheimer coefficients are
518 specifically discussed. The main conclusions can be summarized as follows:

519 1) The relationships between specific discharge (q) and the “pseudo” hydraulic conductivity
520 (K) (which is computed as a ratio of q and hydraulic gradient, J) of permeable stones show
521 that deviation from Darcian flow regime is clearly visible. In addition, the critical specific
522 discharge corresponding to the transition of flow regimes (from pre-Darcian to post-Darcian)
523 increases with the increase of mean particle size.

524 2) When the specific discharge is small, only a small fraction of the water flowing through
525 the pores. The rest of the water adheres to the surface of the solid particles (immobile),
526 partially blocking the flow pathways. As the specific discharge increases, more water
527 becomes mobile and participates in flow. Hence, the “pseudo” hydraulic conductivity
528 increases with the increase of specific discharge. When the specific discharge increases to the
529 critical specific discharge (q_c), the “pseudo” hydraulic conductivity is maximized, and then it
530 begins to decrease as the specific discharge continues to increase.

531 3) The mercury injection experiment results show that the mercury injection curve can be
532 divided into three segments. The beginning and end segments of the mercury injection curve
533 of the four permeable stones with different particle sizes are very gentle, while the main (or
534 intermediate) mercury injection curve is steep, indicating that the pore size distribution falls
535 within a narrow range, and the proportions of large pores and small pores are relatively small.



536 4) The porosity decreases as the mean particle size of permeable stone increases while the
537 mean pore diameter increases. And the porosity can reflect the influence of particle diameter,
538 sorting degree and arrangement mode of porous medium on seepage parameters. The larger
539 porosity leads to the smaller coefficients A and B under the condition of the same particle size.
540 5) The coefficient A is linearly related to $1/d^2$ and the relationship between coefficient A and
541 $1/d^2$ is given as $A = 0.0025(1/d^2) + 0.003$. The coefficient B is not linearly related to $1/d$,
542 instead it is quadratic related to $1/d$ as $B = 1.14E-06(1/d)^2 - 1.26E-06(1/d)$. The particle
543 shape and arrangement of permeable stone have imposed great influences on the seepage
544 parameters.

545 **Notation**

546	q	The specific discharge, m/d.
547	K	The Hydraulic conductivity, m/d.
548	J	The dimensionless parameter defined as hydraulic gradient.
549	A	The Forchheimer equation coefficient (viscous force item), sm^{-1} .
550	B	The Forchheimer equation coefficient (Inertia force item), s^2m^{-2} .
551	P_c	The capillary force, Pa .
552	P_{50}	The corresponding pressure value when the saturation reaches 50%, MPa .
553	P_A, P_B, P_C	The pressure corresponding to different stages on mercury injection curve, MPa .
554	σ	The solid-liquid interfacial tension.
555	θ	The wet angle between the liquid and the solid surface.
556	r	The radius of curvature in capillary, mm.
557	d	The particle size, mm.



- 558 d_{50} The mean particle sizes (50% by weight), mm.
- 559 R_m The mean pore diameter, μm .
- 560 R_{50} The pore diameter corresponding to the median pressure P_{50} , μm .
- 561 H The height of the peak of the mercury injection curve.
- 562 x_c The abscissa corresponding to the peak of the curve (the pore size).
- 563 w The standard variance.
- 564 n The porosity.
- 565 J_n The viscous force-related component.
- 566 J_r The inertia force-related component.
- 567 **Authors contributions**
- 568 Zhongxia Li: Experiment, Writing original draft. Junwei Wan: Methodology,
569 Conceptualization. Tao Xiong: Data curation, Investigation, Experiment. Hongbin Zhan:
570 Methodology, Writing, Review & Editing. Linqing He: Experiment, Methodology. Kun
571 Huang: Funding acquisition, Investigation
- 572 **Competing interests**
- 573 The authors declare that they have no conflict of interest.
- 574 **Acknowledgements**
- 575 This study was supported by the National Natural Science Foundation of China (Grant
576 Nos. 41402204), the National Key Research and Development Program of China
577 (No. 2018YFC0604202) and the Fundamental Research Funds for National Universities,
578 China University of Geosciences (Wuhan). Thank Zhongzhi Shen of China University of
579 Geosciences for his great help in developing the experimental set up. And the authors want to



580 express their sincere appreciation of the constructive comments made by the three
581 anonymous reviewers and Associate Editor for improving the quality of the manuscript.

582 **References**

- 583 Anovitz, L. M. and Cole, D. R.: Characterization and Analysis of Porosity and Pore Structures, *Reviews in*
584 *Mineralogy and Geochemistry*, 80, 61-164, <https://doi.org/10.2138/rmg.2015.80.04>, 2015.
- 585 Bear, J.: *Dynamics of Fluids in Porous Media*, American Elsevier Pub. Co., New York, N.Y., and Amsterdam,
586 1972.
- 587 Beavers, G. S., Sparrow, E., and Rodenz, D. E.: Influence of Bed Size on the Flow Characteristics and Porosity
588 of Randomly Packed Beds of Spheres, *Journal of Applied Mechanics*, 40, 655-660,
589 <https://doi.org/10.1115/1.3423067>, 1972.
- 590 Blick, E.: Capillary-Orifice Model for High-Speed Flow through Porous Media, *Industrial Engineering*
591 *Chemistry Process Design Development*, 5, 90-94, <https://doi.org/10.1021/i260017a019>, 1966.
- 592 Bu, S., Yang, J., Dong, Q., and Wang, Q.: Experimental study of transition flow in packed beds of spheres with
593 different particle sizes based on electrochemical microelectrodes measurement, *Applied Thermal Engineering*,
594 73, 1525-1532, 2014.
- 595 Darcy, H.: *Recherches expérimentales relatives au mouvement de l'eau dans les tuyaux*, Impr. Impériale, Paris,
596 France, 1857.
- 597 Dudgeon, C. R.: An experimental study of the flow of water through coarse granular media, *La Houille Blanche*,
598 785-801, <https://doi.org/10.1051/lhb/1966049>, 1966.
- 599 Dybbs, A. and Edwards, R.: A new look at porous media fluid mechanics—Darcy to turbulent, in: *Fundamentals*
600 *of transport phenomena in porous media*, Springer, 199-256, 1984.
- 601 Ergun, S.: Fluid flow through packed columns, *Chemical Engineering Progress*, 89-94,
602 [https://doi.org/10.1016/0009-2509\(53\)80048-5](https://doi.org/10.1016/0009-2509(53)80048-5), 1952.
- 603 Fancher, G. H. and Lewis, J. A.: Flow of simple fluids through porous materials, *Industrial & Engineering*
604 *Chemistry*, 25, 1139-1147, <https://doi.org/10.1021/ie50286a020>, 1933.
- 605 Fetter, C. W.: *Applied Hydrogeology: International Edition*, Prentice Hall, Pearson, Englewood Cliffs, 2001.
- 606 Forchheimer, P.: *Wasserbewegung durch boden*, Z. Ver. Deutsch, Ing., 45, 1728-1782, 1901.
- 607 Geertsma, J.: Estimating the Coefficient of Inertial Resistance in Fluid Flow Through Porous Media, *Society of*
608 *Petroleum Engineers Journal*, 14, 445-450, <https://doi.org/10.2118/4706-PA>, 1974.
- 609 Hall, P. L., Mildner, D., and Borst, R. L.: Small-angle scattering studies of the pore spaces of shaly rocks,
610 *Journal of Geophysical Research Atmospheres*, 91, 2183-2192, <https://doi.org/10.1029/JB091iB02p02183>, 1986.
- 611 Harlan, J., Picot, D., Loll, P., and Garavito, R.: Calibration of size-exclusion chromatography: use of a double
612 Gaussian distribution function to describe pore sizes, *Analytical biochemistry*, 224, 557-563,
613 <https://doi.org/10.1006/abio.1995.1087>, 1995.
- 614 Hea, X. and Zhangb, Z.: Microscopic Pore Structural Characteristics in Coal Particles, *International Conference*
615 *on Material*, Guangzhou, China,
- 616 Huang, K.: Exploration of the basic seepage equation in porous media, Ph.D., dissertation, China University of
617 Geosciences, Wuhan, 2012.
- 618 Huang, K., Wan, J., Chen, C., Linqing, H., Mei, W., and Zhang, M.: Experimental investigation on water flow in



- 619 cubic arrays of spheres, *Journal of Hydrology*, 492, 61-68, <https://doi.org/10.1016/j.jhydrol.2013.03.039>, 2013.
- 620 Izbash, S.: *O Filtracii v Kroponozernstom Materiale*, Leningrad, USSR, 1931.
- 621 Jeon, H., Cho, H., Kim, J., and Sung, B.: Non-Gaussian rotational diffusion in heterogeneous media, *Physical*
622 *Review E Statistical Nonlinear & Soft Matter Physics*, 90, 042105, <https://doi.org/10.1103/PhysRevE.90.042105>,
623 2014.
- 624 Kadlec, R. H. and Knight, R. L.: *Treatment Wetlands*, Lewis Pub, Boca Raton, 1996.
- 625 Kate, J. M. and Gokhale, C. S.: A simple method to estimate complete pore size distribution of rocks,
626 *Engineering Geology*, 84, 48-69, <https://doi.org/10.1016/j.enggeo.2005.11.009>, 2006.
- 627 Koch, D. and Ladd, A.: Moderate Reynolds number flows through periodic and random arrays of aligned
628 cylinders, *Journal of Fluid Mechanics*, 349, 31-66, <https://doi.org/10.1017/S002211209700671X>, 1996.
- 629 Kovács, G.: *Seepage Hydraulics*, Development in Water Sciences. Elsevier: New York, 1981.
- 630 Latifi, M., Midoux, N., Storck, A., and Gence, J.: The use of micro-electrodes in the study of the flow regimes in
631 a packed bed reactor with single phase liquid flow, *Chemical engineering science*, 44, 2501-2508, 1989.
- 632 Li, Z., Wan, J., Huang, K., Chang, W., and He, Y.: Effects of particle diameter on flow characteristics in sand
633 columns, *International Journal of Heat & Mass Transfer*, 104, 533-536,
634 <https://doi.org/10.1016/j.jheatmasstransfer.2016.08.085>, 2017.
- 635 Li, Z., Wan, J., Zhan, H., Cheng, X., Chang, W., and Huang, K.: Particle size distribution on Forchheimer flow
636 and transition of flow regimes in porous media, *Journal of Hydrology*, 574, 1-11,
637 <https://doi.org/10.1016/j.jhydrol.2019.04.026>, 2019.
- 638 Lindquist, E.: On the flow of water through porous soil, *Premier Congres des grands barrages (Stockholm)*, 81-
639 101,
- 640 Lindquist, W. B., Venkatarangan, A., Dunsmuir, J., and Wong, T. F.: Pore and throat size distributions measured
641 from synchrotron X-ray tomographic images of Fontainebleau sandstones, *Journal of Geophysical Research*
642 *Solid Earth*, 105, 21509-21527, <https://doi.org/10.1029/2000JB900208>, 2000.
- 643 Maalal, O., Prat, M., Peinador, R., and Lasseux, D.: Determination of the throat size distribution of a porous
644 medium as an inverse optimization problem combining pore network modeling and genetic and hill climbing
645 algorithms, *Physical Review E*, 103, 023303, <https://doi.org/10.1103/PhysRevE.103.023303>, 2021.
- 646 Moutsopoulos, K. N., Papaspyros, I. N., and Tsihrintzis, V. A.: Experimental investigation of inertial flow
647 processes in porous media, *Journal of hydrology*, 374, 242-254, <https://doi.org/10.1016/j.jhydrol.2009.06.015>,
648 2009.
- 649 Niranjana, H.: *Non-Darcy flow through porous media*, M.S., dissertation, ITT, Kanpur, India, 1973.
- 650 Panfilov, M. and Fourar, M.: Physical splitting of nonlinear effects in high-velocity stable flow through porous
651 media, *Advances in Water Resources*, 29, 30-41, <https://doi.org/10.1016/j.advwatres.2005.05.002>, 2006.
- 652 Pitman, E. D.: Relationship of porosity and permeability to various parameters derived from mercury injection-
653 capillary pressure curves for sandstone (1), *AAPG bulletin*, 76, 191-198, [https://doi.org/10.1306/BDF87A4-
654 1718-11D7-8645000102C1865D](https://doi.org/10.1306/BDF87A4-1718-11D7-8645000102C1865D), 1992.
- 655 Rezaee, R., Saedi, A., and Clennell, B.: Tight gas sands permeability estimation from mercury injection
656 capillary pressure and nuclear magnetic resonance data, *Journal of Petroleum Science and Engineering*, 88, 92-
657 99, <https://doi.org/10.1016/j.petrol.2011.12.014>, 2012.
- 658 Rijfkoogel, L. S., Ghanbarian, B., Hu, Q., and Liu, H. H.: Clarifying pore diameter, pore width, and their
659 relationship through pressure measurements: A critical study, *Marine and Petroleum Geology*, 107, 142-148,
660 <https://doi.org/10.1016/j.marpetgeo.2019.05.019>, 2019.



- 661 Scheidegger, A.: The physics of flow through porous media,
662 Scheidegger, A. E.: On the stability of displacement fronts in porous media: a discussion of the muskat-
663 aronofsky model, *Canadian Journal of Physics*, 38, 153-162, <https://doi.org/10.1139/p60-017>, 1960.
664 Schmitt, M., Fernandes, C. P., da Cunha Neto, J. A., Wolf, F. G., and dos Santos, V. S.: Characterization of pore
665 systems in seal rocks using nitrogen gas adsorption combined with mercury injection capillary pressure
666 techniques, *Marine and Petroleum Geology*, 39, 138-149, <https://doi.org/10.1016/j.marpetgeo.2012.09.001>, 2013.
667 Schneebeli, G.: Experiences sur la limite de validite de la loi de Darcy et l'apparition de la turbulence dans un
668 ecoulement de filtration, *La Huille Blanche*, 2, 141-149, <https://doi.org/10.1051/lhb/1955030>, 1955.
669 Sedghi-Asl, M., Rahimi, H., and Salehi, R.: Non-Darcy Flow of Water Through a Packed Column Test,
670 *Transport in Porous Media*, 101, 215-227, <https://doi.org/10.1007/s11242-013-0240-0>, 2014.
671 Seguin, D., Montillet, A., Comiti, J., and Huet, F.: Experimental characterization of flow regimes in various
672 porous media—II: Transition to turbulent regime, *Chemical engineering science*, 53, 3897-3909, 1998.
673 Shi, W., Yang, T., and Yu, S.: Experimental Investigation on Non-Darcy Flow Behavior of Granular Limestone
674 with Different Porosity, *Journal of Hydrologic Engineering*, 25, 06020004,
675 [https://doi.org/10.1061/\(ASCE\)HE.1943-5584.0001966](https://doi.org/10.1061/(ASCE)HE.1943-5584.0001966), 2020.
676 Sidiropoulou, M. G., Moutsopoulos, K. N., and Tsihrintzis, V.: Determination of Forchheimer equation
677 coefficients a and b, *Hydrological Processes*, 21, 534-554, <https://doi.org/10.1002/hyp.6264>, 2007.
678 Skjetne, E., Hansen, A., and Gudmundsson, J.: High-velocity flow in a rough fracture, *Journal of Fluid*
679 *Mechanics*, 383, 1-28, <https://doi.org/10.1017/S0022112098002444>, 1999.
680 Souto, H. P. A. and Moyne, C.: Dispersion in two-dimensional periodic porous media. Part I. Hydrodynamics,
681 *Physics of Fluids*, 9, 2243-2252, <https://doi.org/10.1063/1.869365>, 1997.
682 Van Lopik, J. H., Zazai, L., Hartog, N., and Schotting, R.: Nonlinear Flow Behavior in Packed Beds of Natural
683 and Variably Graded Granular Materials, *Transport in Porous Media*, 131, 957-983,
684 <https://doi.org/10.1007/s11242-019-01373-0>, 2019.
685 Van Lopik, J. H., Snoeijers, R., van Dooren, T. C. G. W., Raoof, A., and Schotting, R. J.: The Effect of Grain
686 Size Distribution on Nonlinear Flow Behavior in Sandy Porous Media, *Transport in Porous Media*, 120, 1-30,
687 <https://doi.org/10.1007/s11242-017-0903-3>, 2017.
688 Ward, J. C.: Turbulent Flow in Porous Media, *Journal of Hydraulic Engineering*, 90, 1-12,
689 [http://dx.doi.org/10.1016/S0301-9322\(02\)00051-4](http://dx.doi.org/10.1016/S0301-9322(02)00051-4), 1964.
690 Washburn, E. W.: The Dynamics of Capillary Flow, *Physical Review*, 17, 273-283,
691 <https://doi.org/10.1103/PhysRev.17.273>, 1921.
692 Wright, D.: Nonlinear Flow Through Granular Media, *Journal of Hydraulic Engineering*, 94, 851-872,
693 <https://doi.org/10.1061/JYCEAJ.0001858>, 1968.
694 Xu, C. and Torres-Verdín, C.: Pore System Characterization and Petrophysical Rock Classification Using a
695 Bimodal Gaussian Density Function, *Mathematical Geosciences*, 45, 753-771, [https://doi.org/10.1007/s11004-](https://doi.org/10.1007/s11004-013-9473-2)
696 [013-9473-2](https://doi.org/10.1007/s11004-013-9473-2), 2013.
697 Yang, B., Yang, T., Xu, Z., Liu, H., Yang, X., and Shi, W.: Impact of Particle-Size Distribution on Flow
698 Properties of a Packed Column, *Journal of Hydrologic Engineering*, 24, 04018070,
699 [https://doi.org/10.1061/\(ASCE\)HE.1943-5584.0001735](https://doi.org/10.1061/(ASCE)HE.1943-5584.0001735), 2019.
700 Zhen-hua, MIN, and, Min, CAO, and, Shu, ZHANG, and, and Xiu-dan: Effect of precursor on the pore structure
701 of carbon foams, *New Carbon Materials*, 22, 75-79, [https://doi.org/10.1016/S1872-5805\(07\)60009-2](https://doi.org/10.1016/S1872-5805(07)60009-2), 2007.
702 Zhihong, L. I., Jihong, S., Dong, W. U., Yuhan, S., Liu, Y. I., Wenjun, S., and Baozhong, D.: Determination of



703 average pore diameter of SiO₂ xerogels by small angle X-ray scattering, ACTA Physica sinica, 49, 1312-1315,
704 <https://doi.org/10.3321/j.issn:1000-3290.2000.07.020>, 2000.
705 Zhou, H., Fang, Y.-g., Chen, M., Gu, R.-g., and Li, W.: Experimental and analytical study on electro-osmosis in
706 low-permeability soil considering the pore size effect, Geotechnique, 71, 141-152,
707 <https://doi.org/10.1680/jgeot.18.p.362>, 2019.
708
709
710