Classification of dissipation regimes

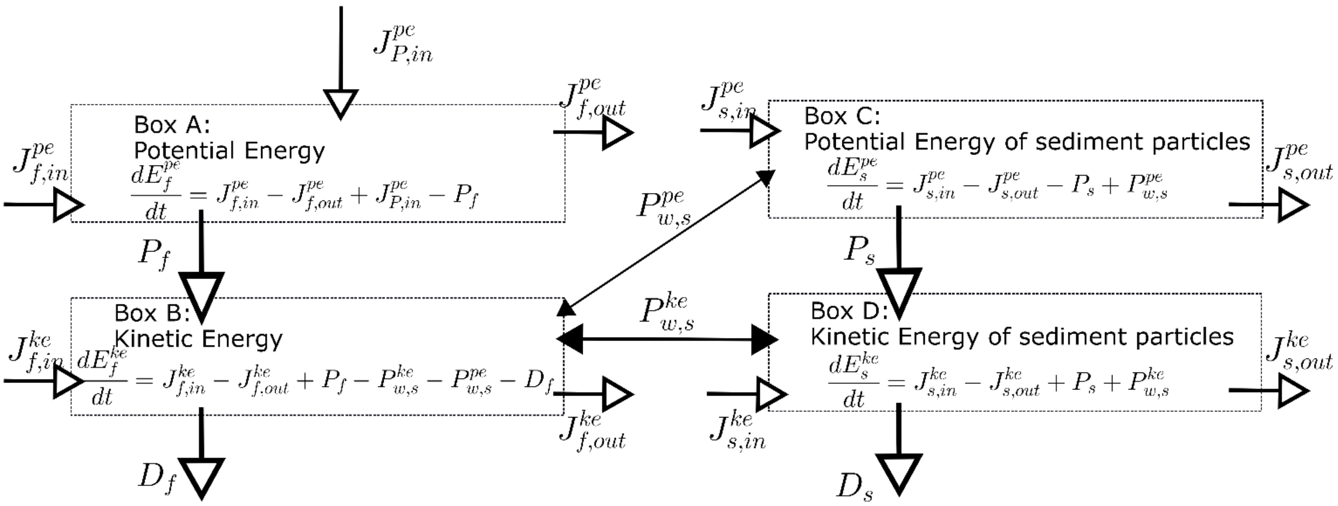

Figure 1: 4-Box energy scheme of water and sediment flow

For a flow that is comprised of water and sediment particles, the residual of the steady state energy balance can be calculated by combination of the energy balances of each medium. Potential energy of water is accumulated through rainfall, which converts into kinetic energy of water and heat (Eq. 1a). The magnitude of the dissipation of kinetic energy by viscous stress depends on flow regime (turbulent, laminar) as well as sediment transport. Sediment gains it's potential and kinetic energy from kinetic energy of the water, which is represented by the terms $P_{w,s}^{pe}$ and $P_{w,s}^{ke}$ (Eq. 1b). If these terms are positive, kinetic energy of water is converted into energy of sediment, which would be the case if water flow erodes the bed surface and picks up sediment particles. Reversely, surface runoff which already transports sediment might deposit some number of particles and sediments would lose potential and kinetic energy, leading to a negative sign of the terms $P_{w,s}^{pe}$ and $P_{w,s}^{ke}$.

Water energy balance:

$$\frac{dE_f^{pe}}{dt} = J_{f,net}^{pe} + J_{Peff}^{pe} - P_f \tag{1a}$$

$$\frac{dE_f^{ke}}{dt} = J_{f,net}^{ke} + P_f - D_f - P_{w,s}^{pe} - P_{w,s}^{ke} \tag{1b}$$

Sediment energy balance:

$$\frac{dE_s^{pe}}{dt} = J_{s,net}^{pe} - P_s + P_{w,s}^{pe} \tag{1c}$$

$$\frac{dE_s^{ke}}{dt} = J_{s,net}^{ke} + P_s - D_s + P_{w,s}^{ke} \tag{1d}$$

In steady state and with $P_s \approx D_s \approx 0$ (Ps represents the power of sediment to convert into movement of sediment and Ds is friction of grains against other grains) the residual of the water sediment energy balance is:

$$D_f = J_{f,net}^{pe} + J_{Peff}^{pe} + J_{s,net}^{pe} + J_{f,net}^{ke} + J_{s,net}^{ke} \tag{2}$$

Moreover, the flow regime might be of turbulent or laminar character, in the following indicated by superscripts T (turbulent flow regime) and L (laminar flow regime). With Eq. (2) We can now classify types of interactions between water and sediment regarding the energy residual D_f. In the case of this simple two box scheme, we consider D_f as dissipation of free energy into heat by the viscous stresses between water particles.

As a starting point, we define the following assumptions:

1) $J_{f,net}^{pe} + J_{Peff}^{pe} = P_f = constant$
2) Velocity does increase due to flow accumulation: $J_{f,net}^{ke} \leq 0$
3) Sediment moves with velocity of discharge: $v_{sed} \ = \ v_{water}$
4) Sediment is distributed homogeneously in the vertical of the water column (no vertical gradient of sediment concentration)
5) Kinetic energy divergence is much smaller than potential energy divergence: $\left|J_{f,net}^{ke}\right| \ll \left|J_{f,net}^{pe}\right|$

First, we assume that the power of water to create kinetic energy is the same for each considered case (1) and second, we focus on cases where an influx of potential energy causes an increase in kinetic energy flow (2), combined with an increase of flow velocity. The sediment particles are assumed to move with the flow velocity (3) and are homogeneously distributed across the vertical of the water column (4). Also, most energy influx into the system is dissipated, meaning that the loss of geopotential energy is much larger than the gain of kinetic energy (5). These assumptions apply to a system which is not energy limited with large driving bed slopes and very shallow water depths.

In Figure 2 we plotted a flow chart diagram which separates the possible sediment water interactions. Starting at the top and continuing down a chart line the terms of the energy balance equation (Eq. 2) can be separated into different cases. Each line is named with a letter, resulting in a unique letter combination for each case. If no sediment is considered, the cases to distinguish (A) are just laminar and turbulent flow regime. The larger dissipative loss of turbulent flow leads to a slower flow velocity for turbulent flow than laminar flow and therefore $J_{f,net}^{ke,L} < J_{f,net}^{ke,T}$. All cases which include sediment transport start in the branch (B) of the flow chart. Branch (C) represents cases where $J_{Sed,net}^{pe} < 0$, which can only happen if sediment of the surface is eroded, and the total mass of sediment dissolved in the water flow is increasing. Further subdividing we find that the kinetic energy of sediment must increase as both v_{Sed} and m_{Sed} are increasing, meaning that $J_{Sed,net}^{ke}$ has to be smaller than zero, branch (E) is therefore not possible. If $J_{Sed,net}^{ke} < 0$ (branch (F)), we can again differentiate between laminar and turbulent flow regime. In contrast to cases with erosion only (C), branch (D) considers cases with deposition and erosion of sediment. The latter is however a special case as the sediment would have to lose more geopotential by slope difference than is gained by mass increase through erosion. Clarity exists if $J_{Sed,net}^{ke}$ is larger or equal to zero as this means that kinetic energy of sediment is decreasing, which can only be the case of deposition (mass decrease) as velocity increases (G). Contrarily, branch (H) is a collection of cases where the quantities of discharge, sediment concentration and magnitudes of slope and gradient determine whether sediment is eroded or deposited. These cases are either sediment mass decrease (deposition), which influences kinetic energy less than the increase of flow velocity, or little mass gain (erosion) in comparison to energy loss by slope. Both possibilities can be classified as little or no sediment mass change, therefore deposition and erosion being close to zero (H).

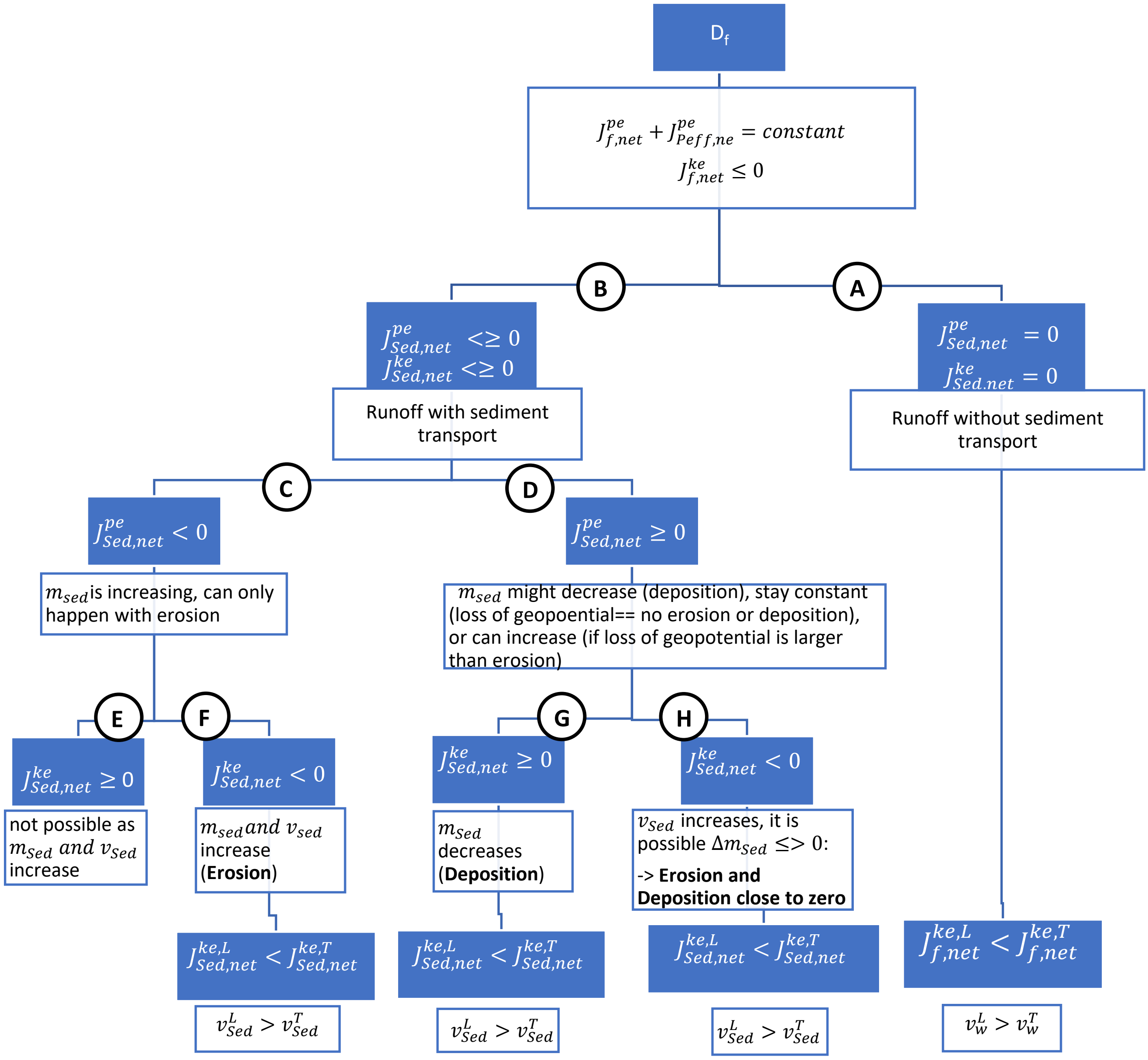

Figure 2: Flowchart for dissipation (D_f) regimes

The classification into dissipation regimes of water and sediment flow is summarized in Table 1. We can distinguish four principal cases: 1) Water and sediment flow with sediment mass increase (Erosion), 2) Water and sediment flow with little sediment mass in- or decrease, 3) Water flow without sediment transport, and 4) Water and sediment flow with sediment mass decrease (Deposition). Cases 2) and 3) might energetically be quite similar and could be considered a single case (No deposition and no erosion). Comparing the signs of the terms of Eq. 2 dissipation is lowest for erosion (case 1) and highest for deposition (case 3). For all cases turbulent flow is more dissipative than laminar flow ($v^L_w > v^T_w$ and $v^L_{Sed} > v^T_{Sed}$).

Table 1: Dissipation regimes

Case	Route (Figure 2)	$J^{pe}_{f,net} + J^{pe}_{Peff,net}$	$J^{pe}_{Sed,net}$	$J^{ke}_{Sed,net}$	$J^{ke}_{f,net}$	$Erosion, Deposition: \Delta m_{Sed}$	Dissipation D_f
1	B-C-F	Const.	< 0	< 0	< 0	> 0	Low
2	B-D-H	Const.	≥ 0	< 0	< 0	~0	↓
3	A	Const.	0	0	< 0	0	
4	B-D-G	Const.	≥ 0	≥ 0	< 0	< 0	High