Dear pro. Jasper,

We highly appreciated your review and constructive comments for our manuscript. We provide our responses to your queries below.

Kind regards, all authors

Comment: Review of "A Time-Varying Distributed Unit Hydrograph considering soil moisture content"

Summary: In this paper the authors propose an extension of the distributed unit hydrograph (DUH) method for routing of water in distributed hydrologic models. The default DUB approach takes into consideration the topography of the land surface in computation of surface runoff and river inflow but does not account for spatial variability of the soil moisture capacity within the catchment. The authors present a simple extension of the flow velocity equation which is thought to account for a varying moisture content.

Evaluation: I am not an expert on routing methods, in fact, never taken a course in surface hydrology, but I do know something about math, hydrology, and modeling. The authors presented in this paper are interesting, yet, I believe the paper warrants a major revision before it can be judged to making a significant contribution to hydrology and water resources. Specifically, the paper is not very well written, syntax and grammar need major improvements. I believe this is a first requirement before the paper is ready for detailed review. What is more, the methodology needs a much better physical underpinning citing properly past publications on this topic, presenting relevant units of variables, addressing sensitivity of results to variables such as gamma. Furthermore, the case study / demonstration of the methodology is not particularly convincing. I highlight my main comments below - not in a particular order of importance.

Response:

Thank you for your constructive comments.

First, the language of this paper will be revised by a native English-speaker.

Second, the methodology will be improved in the revised version. Some of the changes have been shown in the responses below. Some previous publications on this topic will be cited in the revised paper. Relevant units of variables will be added to the revised version.

Third, as for the case study / demonstration of the methodology, three new metrics will be introduced to evaluate the performance of the model. The sensitivity analysis to parameters $\gamma$ and $I_c$ will be added to the case study. Furthermore, comparisons between the traditional unit hydrograph and the proposed method will be added to the revised manuscript.

Point-by-point responses to the reviewers’ comments are shown below.

Comment #1: The authors should reference previous work on routing/modeling approaches. For example, Line 269, Eq. 10 does not provide a reference, whereas this equation is simply the Pareto distribution function, used by Moore (1985) to describe the spatial variability of the soil moisture storage capacity in the watershed.
This is just one example - this comment applies to many equations used by the authors; the continuity equation, Manning’s equation, etc.

Response:

Thanks for your comments. We will reference more previous work on routing and modeling approaches in the revised manuscript.

- The Soil Conservation Service (SCS) formula is expressed by (Haan et al., 1994)

\[
V = k \cdot S^2
\]  

(1)

where \( V \) (m/s) is the flow velocity; \( k \) (m/s) is Land use or flow type coefficient; and \( S \) (m/m) is the slope of the grid cell.

- The continuity equation for steady flow is given by (Chow et al., 1988)

\[
Q = V y = I L
\]  

(2)

where \( Q \) (m³/s) is the discharge per unit width; \( L \) (m) is the length of the overland flow; \( I \) (m/s) is the excess rainfall intensity; and \( y \) (m) is the depth of flow.

- The Manning formula is described by (Chow et al., 1988)

\[
V = \frac{1}{n} \cdot y^{2} \cdot S^{\frac{1}{3}}
\]  

(3)

where \( n \) (m\(^{1/3}\)s) is the Manning’s roughness coefficient.

- Then, the flow velocity, \( V \), through a grid cell can be computed by combing Eqs. (2) and (3), which is given by (Muzik, 1996)

\[
V = \frac{1}{n^{0.6}} \cdot I^{0.4} \cdot L^{0.4} \cdot S^{0.3}
\]  

(4)

- The generalized Pareto distribution was used for describing the spatial distribution of storage capacity (Moore, 1985), written as

\[
\alpha = 1 - \left( 1 - \frac{WM}{WMM} \right)^{b}
\]  

(5)
where $\alpha$ (unitless) represents the proportion of the pervious area of the basin whose tension water capacity is less than or equal to the value of the ordinate $WM$ (mm); the tension water capacity at a point, $WM$ varies from 0 to $WMM$; $WMM$ (mm) is maximum watershed soil storage capacity; $b$ (unitless) represents the degree of spatial variability of store capacity over the basin. The area under the curve represents the areal mean tension capacity of the entire basin.

References:


Comment #2: The derivation of the TDUH method needs significant improvement. Not well written - the derivation has been given in other manuscripts so either present it briefly and clearly, otherwise, I recommend the authors refer to the original sources as the present derivation brings up more questions than it answers. I am particularly bothered with the way the equations and variables are presented. It reads as a collection of some equations with some variables. Also, I am left wondering whether the equations presented are derived for the first time by the authors or whether they have been presented in the literature years ago? For example, Eqs. (6), (7), (8) and (9).

Response:

Thanks for your comments.

First, the derivation of the TDUH method was improved by referring the previous works.

The traditional DUH method can rout the variant spatially distributed rainfall to the watershed outlet (Grimaldi et al., 2010), and such a method is a lumped linear model of watershed response. However, many watersheds may display a nonlinear behavior over a wider range of net rainfall and discharge (Du et al., 2009). Muzik (1996) stated that a family of unit hydrographs should be derived for a considered watershed, with each unit hydrograph being applicable within a certain range of excess rainfall.

The rainfall intensity and soil moisture content were introduced to improve the flow velocity formula. Therefore, we can obtain a family of unit hydrographs corresponding to different rainfall intensity and soil water content. The general schematic of the TDUH method considering rainfall intensities and soil moisture contents is given in Figure 1.
The main procedures of this methods are given by

Step 1: Identification of the drainage network using advanced DEM pre-processing techniques. More details can be found in Grimaldi et al (2012).

Step 2: Estimation of flow path, which is measured for each grid cell along the flow directions to the outlet of basins.

Step 3: Calculation of the flow velocity. Three flow velocity equations were adopted to derive the DUHs respectively. Equation (6) is known as SCS formula (Haan et al., 1994). Equation (7) is time-varying flow velocity formula considering excess rainfall intensity (Kong et al., 2019). Equation (8) is time-varying flow velocity formula considering both rainfall and soil moisture content.

\[ V = k \cdot S^{\frac{1}{2}} \]  

(6)
\[ V = k \cdot S^2 \cdot \left( \frac{I_t}{I_c} \right)^{\frac{2}{3}} \]  
\[ V = k \cdot S^2 \cdot \left( \frac{I_t}{I_c} \right)^{\frac{2}{3}} \cdot \gamma \]  

where \( I_c \) (mm/h) is the reference excess rainfall intensity; \( \theta_i \) (unitless) represents the state of the soil moisture content of unsaturated areas; and \( \gamma \) (unitless) is an exponent smaller than unity.

Step 4: To compute the total travel time \( \tau_i \) of the flow from each cell \( i \) to the outlet, we added the retention times along the \( R_i \) cells belonging to the flow path starting at that cell, given by Eq. (10). Retention time in each grid cell can be calculated using Eq. (9).

\[ \Delta \tau_i = \frac{L}{V} \quad \text{or} \quad \Delta \tau_i = \frac{\sqrt{2L}}{V} \]  
\[ \tau_i = \sum_{i \in R_i} \Delta \tau_i \]  

where \( \Delta \tau_i \) is retention time in grid cell \( i \); and \( \tau_i \) is the total travel time along the flow path in grid cell \( i \).

Step 5: Develop a cumulative travel time map of the watershed based on cell by cell estimates for hillslope velocities. The cumulative travel time map is further divided into isochrones, which can be used to generate a time-area curve and the resulting unit hydrograph (Kilgore, 1997).

Second, the Eqs. (6), (7), (8), and (9) in the manuscript have been presented in the literature years ago, which can be found in Chow et al. (1988), Muzik (1996) and Kong et al. (2019) respectively.

References:


Grimaldi, S., Petroselli, A., Alonso, G., Nardi, F. Flow time estimation with spatially variable hillslope


Comment #3: Authors should give units of variables they use. This will make it easier for readers to digest the material, and for students to reproduce/implement the approach the authors have presented.

Response:

Thanks for your comments. The units of variables will be added to the revised manuscript.

Table 1. Comprehensive list of nomenclature used in this study.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Area ($m^2$)</td>
</tr>
<tr>
<td>$b$</td>
<td>The degree of spatial variability of store capacity over the basin</td>
</tr>
<tr>
<td>$E_{SE}$</td>
<td>Nash-Sutcliffe efficiency</td>
</tr>
<tr>
<td>$E_{KG}$</td>
<td>Kling-Gupta efficiency</td>
</tr>
<tr>
<td>$h$</td>
<td>Depth of flow ($m$)</td>
</tr>
<tr>
<td>$i$</td>
<td>Cell identification</td>
</tr>
<tr>
<td>$I_e$</td>
<td>Excess rainfall intensity at time $t$ ($mm/h$)</td>
</tr>
<tr>
<td>$I_c$</td>
<td>Reference rainfall intensity ($mm/h$)</td>
</tr>
<tr>
<td>$j$</td>
<td>Time interval identification</td>
</tr>
<tr>
<td>$k$</td>
<td>Land use or flow type coefficient ($m/s$)</td>
</tr>
<tr>
<td>$L$</td>
<td>Grid cell size ($m$)</td>
</tr>
<tr>
<td>$m$</td>
<td>Grid cell total numbers</td>
</tr>
<tr>
<td>$n$</td>
<td>Manning’s roughness coefficient ($s/m^{1/3}$)</td>
</tr>
<tr>
<td>$o$</td>
<td>Observed flood</td>
</tr>
<tr>
<td>$Q$</td>
<td>Discharge ($m^3/s$)</td>
</tr>
<tr>
<td>$r$</td>
<td>Correlation coefficient</td>
</tr>
<tr>
<td>$R_{SR}$</td>
<td>Root-mean-squared error to standard deviation ratio</td>
</tr>
<tr>
<td>$s$</td>
<td>Simulated flood</td>
</tr>
<tr>
<td>$S$</td>
<td>Slope of the watershed grid cell ($mm/mm$)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$t$</td>
<td>Time (s)</td>
</tr>
<tr>
<td>$UH$</td>
<td>Ordinate value of the DUH ($m^3/s$)</td>
</tr>
<tr>
<td>$V$</td>
<td>Flow velocity ($m/s$)</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Flow velocity corresponding to the reference rainfall ($m/s$)</td>
</tr>
<tr>
<td>$w_t$</td>
<td>Mean tension water storage of the unsaturated region at time $t$ ($m$)</td>
</tr>
<tr>
<td>$w_{\text{max},t}$</td>
<td>Maximum tension water storage of the unsaturated region at time $t$ ($m$)</td>
</tr>
<tr>
<td>$WM$</td>
<td>Tension water capacity at a point ($m$)</td>
</tr>
<tr>
<td>$WMM$</td>
<td>Maximum watershed soil storage capacity ($m$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The proportion of the pervious area of the basin whose tension water capacity is less than or equal to the value of the ordinate</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation values</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Power law related to the influence of soil moisture on the flow velocity</td>
</tr>
<tr>
<td>$\theta$</td>
<td>State of the soil moisture content of the unsaturated areas</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>Travel time from cell $i$ to the outlet of the basin (s)</td>
</tr>
<tr>
<td>$\Delta\tau$</td>
<td>Retention time in a grid cell (s)</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Duration of an excess rainfall pulse</td>
</tr>
</tbody>
</table>

**Comment #4:** Line 208-209: Variable "m" is introduced but is not used in Equation (1) or (2). Introduce variables when they are used and not ahead of time, unless this makes sense to do.

**Response:**

Thank you for your comments. Variable "m" is used in Equation (3) and it will be corrected in the revised manuscript.

**Comment #5:** Line 261 - 302: The extension the authors propose, essentially applies the ideas of Moore (1985) to the unit hydrograph. Authors should do a much better job connecting what they do to the literature.

**Response:**

Thank you for your comments. The extension of the proposed method was that a soil moisture factor was introduced in the the flow velocity formula.

First, the ideas of Moore (1985) was applied to the unit hydrograph in this manuscript, because the runoff generation module of the Xinanjiang model (Zhao, 1977) was used. In this model, the Pareto distribution function was mostly used to characterize the non-uniform distribution of tension water capacity throughout the basin. More details about this distribution function can be found in Zhao (1992). This function was introduced to calculate a soil moisture content factor in this study. Of course, this soil moisture content factor can also be calculated with a more precise way such as completely distributed models. However, they are usually computationally intensive because they solve the momentum equation (Bunster et al., 2019). The purpose of the DUH method is trying to apply a more efficient distributed computing method in a large watershed.

Second, the reasons why we consider the soil moisture content in the runoff routing process can be summarized as follows. Hillslope flow velocity in each grid is related to soil moisture content. Fast
subsurface velocities and quick runoff responses to precipitation has been observed in many hillslopes (Hutchinson & Moore, 2000; Peters et al., 1995; Tani, 1997). The exact mechanisms that cause water to move through the preferential network are not well known, but it is often assumed that saturated soil provides the connection between preferential features (Sidle et al., 2001; Steenhuis et al., 1988). Many studies have also shown that antecedent moisture condition, precipitation intensity, precipitation amount, topography and so on play a significant role in this phenomenon (Sidle et al., 2000; Tsuboyama et al., 1994; Anderson et al., 2009).

References:


Comment #6: Equation (15) - is this equation a simple extension of Eq. (14) with "w_{t}" raised to gamma? or is this equation from Bhattacharya et al. (2012) and/or Bunster et al. (2019)?

Response:

Thank you for your comments. Equation (15) is an extension of Eq. (14) with "w_{t}" raised to gamma, which was inspired by the previous work and proposed by the authors.

The flood wave’s travel time expression from KW theory (Wong, 1995) is given by

$$\tau = \left( \frac{n l}{\sqrt{S}} \right)^{0.6} E^{-0.4} \left[ (\lambda + 1)^{0.6} - \lambda^{0.6} \right]$$

(11)

where $n$ is Manning roughness coefficient; and $\lambda$ is the dimensionless ratio relating upstream inflow and excess rainfall within the cell.

Gironás et al. (2009) developed a DUH model that uses Eq (1), and the variable $\lambda$ was assumed to be the ratio between the total upslope contributing area of the grid cell $A_u$ and the local area of the grid cell, which implies

$$\lambda = \frac{A_u}{A}$$

(12)

This assumption provides good reproduction of observed hydrographs when applied to a small urban watershed, but it may fail in other cases. Therefore, a power law $\phi$ was used to estimate $\lambda$ by many researchers

$$\lambda = \left( \frac{A_u}{A} \right)^{\phi}$$

(13)

Several values have been proposed for $\phi$ with 0.3 and 0.5 being the most typical (Bhattacharya et al., 2012; Rodriguez-Iturbe et al., 1992; Leopold et al., 2012).

Inspired by this, we made an extension of the Eq. (14), and sensitivity analysis to variable gamma will be added to the revised manuscript based on your suggestions.

References:


Comment #7: The authors state that their routing method takes account of soil moisture content, but soil moisture content does not appear in any of the equations. Instead, they use the Pareto distribution function to express the spatial variability of the soil moisture capacity. They assume that this capacity represents temporal variations in soil moisture content. Furthermore, the representative volume of the soil moisture content is unknown. Are we considering the topsoil moisture content, or the moisture content of the first 50 or 100 cm of the profile? I guess I am looking for a better physical underpinning of the presented method.

Response:

Thanks for your comments.

First, the soil moisture content factor \( \theta_i \) presents in Eq. (15) in the manuscript.

\[
V = k \cdot S^2 \cdot \left( \frac{I}{I_c} \right)^{\frac{2}{3}} \cdot \theta_i \quad (14)
\]

Among the equation, the state of the soil moisture content \( \theta_i \) is expressed by

\[
\theta_i = \frac{w_i}{w_{\text{max},i}} \quad (15)
\]

where \( w_i \) (m) is the mean tension water storage of the unsaturated region at time \( t \), and \( w_{\text{max},i} \) (m) is the maximum tension water storage of the unsaturated region at time \( t \).

Second, as for the representative volume of the soil moisture content.
The Xinanjiang model was used for the calculation of runoff production, in which, runoff production at a point, occurs only on repletion of the tension water storage at that point (Zhao, 1992). To provide for a non-uniform distribution of tension water capacity throughout the basin, the Pareto distribution function was mostly used to express the spatial variability of the soil moisture capacity (Moore, 1985), which is shown in Figure 2.

As shown in the Figure 2, $\alpha$ represents the proportion of the basin area where the tension water capacity is less than or equal to the value of the ordinate $WM$. The tension water capacity at a point, $WM$ varies form 0 to a maximum $WMM$ according to the equation. The area below this curve represents the mean tension water capacity of the entire basin.

$$\alpha = 1 - \left(1 - \frac{WM}{WMM}\right)^b$$ (16)

where $b$ represents the degree of spatial variability of store capacity over the basin.

The state of the catchment, at any time, is assumed to be represented by a point $x$ on the curved line of Figure 2. The area to the right and below the point $x$ is proportional to the areal mean tension water storage (not capacity). This assumption implies that each point in the basin is either at capacity tension (points to the left of $x$) or at a constant tension (points to the right of $x$) (Zhao, 1992).

By introducing the tension water capacity curve above, the representative volume of the soil moisture content includes the capacities of the upper, lower, and deepest layers of the basin.

References:

Comment #8: Line 261 - 268: I read this paragraph several times, and it is still not clear to me. What is $B_{t}$? Is "$A_{t}$" the same $A$ as used in the continuity equation? Why not use symbol theta for moisture content? Why do I need to compute the ratio of $A$ and $A+B$?

Response:

Thank you for your comments.

$A_{t}$ will be replaced by $w_{t}$, and $(A_{t}+B_{t})$ is replaced by $w_{\text{max},t}$ in the revised manuscript.

First, $w_{t}$ (m) is the mean tension water storage of the unsaturated region at time $t$ and $w_{\text{max},t}$ (m) (as shown in Fig. 3) is the maximum tension water storage of the unsaturated region at time $t$.

![Figure 3. Watershed storage capacity curve](image)

Second, this variable $(A_{t})$ is different with that in the continuity equation and we have corrected it to $w_{t}$. In the continuity equation, $A$ is the area of the grid cell and we will distinguish them in the revised manuscript.

We will use $\theta_{t}$ ($w_{t}/w_{\text{max},t}$) for the soil moisture content state in the revised manuscript. The ratio of $w_{t}$ and $w_{\text{max},t}$ was calculated to identify the soil moisture storage status of the unsaturated areas in the watershed. Then, we assumed the flow velocity of the grid cell is proportional to $(\theta_{t})^{\gamma}$ for the reasons as follows.

Fast subsurface velocities and quick runoff responses to precipitation has been observed in many hillslopes (Hutchinson & Moore, 2000; Peters et al., 1995; Tani, 1997). The exact mechanisms that cause water to move through the preferential network are not well known. But it is often assumed that saturated
soil provides the connection between preferential features (Sidle et al., 2001; Steenhuis et al., 1988). Many studies have also shown that antecedent moisture condition, precipitation intensity, precipitation amount, topography and so on play a significant role in this phenomenon (Sidle et al., 2000; Tsuboyama et al., 1994; Anderson et al., 2009).

References:


Comment #9: Why does WM’ use the prime symbol?

Response:

Thank you for your comments. It will be changed to WM in the revised manuscript.

Comment #10: Line 261 - 268: We have A for moisture content, B for the maximum soil moisture storage, W for current soil moisture storage. Why using so many different variables for essentially the same thing? Also, what is the unit of storage and moisture content? Are they similar, or different (as they should be).
Is soil moisture storage not simply equal to soil moisture content \( x \) depth of profile. Why not use theta for soil moisture content and \( S \) for soil moisture storage, where \( S = L \cdot \theta \), where \( L \) is the depth of the profile?

Response:

Thank you for your comments. The unit for each variable will be added to the revised paper and the variables have been corrected.

The unit of soil moisture storage is \( m \), and the soil moisture content is unitless.

\[ \theta_t \text{ (unitless)} = \frac{w_t}{w_{\text{max},t}}, \]

which represents the soil moisture content of the unsaturated region at time \( t \). \( w_t \) (m) is the mean tension water storage of the unsaturated region at time \( t \), and \( w_{\text{max},t} \) (m) is the maximum tension water storage of the unsaturated region at time \( t \).

In this study, soil moisture storage is equal to the soil moisture content \( x \) depth of profile. In this case, \( w_t = w_{\text{max},t} \cdot \theta_t \), where \( \theta_t \) (unitless) is the soil moisture content in unsaturated areas.

Comment #11: Line 269 - 289: I have a hard time following the different steps of the methodology. I think the authors unnecessarily confuse readers - the methodology can be presented in much easier to understand language - and in a much more coherent style.

Response:

Thank you for your comments. This part has been revised as follows.

The generalized Pareto distribution has been widely used for describing the spatial distribution of storage capacity in many previous works (Moore, 1985; Zhao, 1977; Zhao, 1992; Wood, 1992; Wang, 2018). This function provides a non-uniform distribution of tension water capacity throughout the watershed, and tension water capacity curve is given in Figure 4.
For the tension water storage capacity curve, the specific formula (Moore, 1985) is given by
\[
\alpha = 1 - \left(1 - \frac{W_M}{W_{MM}}\right)^b
\]
where \(\alpha\) (unitless) represents the proportion of the pervious area of the basin whose tension water capacity is less than or equal to the value of the ordinate \(W_M\) (m); the tension water capacity at a point, \(W_M\) varies from 0 to \(W_{MM}\); \(W_{MM}\) (m) is maximum watershed soil storage capacity; \(b\) represents the degree of spatial variability of store capacity over the basin; and the area under the curve represents the areal mean tension capacity of the entire basin.

The state of the catchment at any time \(t\), can be represented by a point \(x(\alpha, W_M(t))\) on the curved line of Figure 4 (Zhao, 1992), which implies
\[
\alpha_t = 1 - \left(1 - \frac{W_{Mt}}{W_{MM}}\right)^b
\]

The area to the right and below the point \(x\) is proportional to the areal mean tension water storage (not capacity). Thus, \(W_{Mt}\) (m) the ordinate of the point \(x\) represents the tension water storage capacity in the basin at time \(t\); \(w_t\) (m) can be assumed to represent the mean tension water storage of the unsaturated region, and \(w_{max,t}\) (m) represents the maximum tension water storage of the unsaturated region at time \(t\). The expressions are given by
\[
w_t = (1 - \alpha_t) \cdot W_M(t)
\]
\[
w_{max,t} = \int_{\alpha_t}^{1} W_{MM} \left[1 - (1 - \alpha)^b\right] d\alpha
\]

The state of the soil moisture content \(\theta_t\) of the unsaturated areas is the ratio \(w_t\) and \(w_{max,t}\), which implies
\[
\theta_t = \frac{w_t}{w_{max,t}} = \frac{(1 - \alpha_t) \cdot W_M(t)}{\int_{\alpha_t}^{1} W_{MM} \left[1 - (1 - \alpha)^b\right] d\alpha}
\]
Substitute Eq. (18) to Eq. (21), which yields

\[
\theta_t = \frac{(1-\alpha_t \cdot WM_t)}{WMMM \left[ 1 - \frac{b}{b+1} (1-\alpha_t) \right]} = \frac{(b+1)WM_t}{WMMM + bWM_t}
\]  

Equation (22)

where \( \theta_t \) (unitless) represents the state of the soil moisture content of the unsaturated areas at time \( t \).

References:


Comment #12: Equation (10) presents how we compute alpha, but then in a next equation, alpha is a function of time. Please make clear in your entire derivation which variables are constant (scalars), which ones vary as function of time/space (scalars) and, if necessary, which are vectors and/or matrices. WM' varies as function of time? Otherwise alpha is constant.

Response:

Thank you for your comments. Variables and constants will be distinguished in the revised manuscript as follows.

The generalized Pareto distribution (Moore, 1985) is

\[
\alpha = 1 - \left(1 - \frac{WM}{WMM} \right)^b
\]

Equation (23)

For the Eq. (23), \( \alpha \) varies as function of \( WM \), and \( WM \) is time-varying. Then, the state of the catchment at any time \( t \), can be represented by a point \( x \) on the curved line given by (Zhao, 1992), which
yields

\[ \alpha_t = 1 - \left(1 - \frac{W_{M_t}}{W_{MM}}\right)^b \]  

(24)

References:


Comment #13: The exponent \( b \) of the Pareto distribution function. Is this constant, or varies throughout your watershed?

Response:

Thanks for your comments. The exponent \( b \) of the Pareto distribution function is constant for a watershed, which controls the degree of spatial variability of store capacity over the basin (Moore, 1985). The purpose of this manuscript is to explore the effects of different routing methods. Thus, the same parameters were used for the runoff generation processes in the entire watershed.

References:


Comment #14: Line 283: Mention that \( w_t \) varies between 0 and 1, thus, \( w_t \in (0,1] \) (in latex). I assume that \( w_t \) cannot be zero as soil can never be entirely be depleted from water.

Response:

Thanks for your comments. The range of the \( w_t \) will be corrected to \( (0,1] \) in the revised manuscript.

Comment #15: Equation 13: The denominator may need further explanation. Either solve analytically the integral of Eq. (12) and substitute this in Eq. (13), or do the analytic integration explicitly in Eq. 13.

Response:

Thanks for your comments. The answers can be found in responses to comment #11.

Comment #16: The denominator of Eq. 13 - last step - is wrongly formulated. Do we do \( 1 - (b/(b+1)) * (1/\alpha_t^{1/b}) \) or \( 1 - (b/(b+1)) * (1 - \alpha_t^{1/b}) \)? If the first then remove the 1 to get \( 1 - (b/(b+1)) - \alpha_t^{1/b} \), etc. The present formulation is unclear.

Response:

Thanks for your comments. The answers can be found in responses to comment #11.
Response:

Thanks for your comments. Eq. (13) is wrongly formulated due to the loss of parentheses in the process of converting .docx to .pdf file. It will be corrected as

$$
\theta_i = \frac{(1 - \alpha_i) \cdot WM_i}{WMM \left[ 1 - \frac{b}{b+1} \left( 1 - \alpha_i \right)^{\frac{1}{b}} \right]} = \frac{(b+1)WM_i}{WMM + bWM_i}
$$

(25)

Comment #17: Line 296: The variable ‘gamma’, is this constant for the entire watershed, or varies per sub-catchment or grid cell or? I recommend that for each parameter the text explains how this parameter is treated, besides its units, a description of what the parameter represents, etc.

Response:

Thanks for your comments. We thought the variable ‘gamma’ is treated the same with the power law $\phi$. It should be a typical constant for the entire watershed. Additionally, the sensitivity analysis for the variable ‘gamma’ and how we deal with each parameter will be added to the revised manuscript.

Comment #18: The authors use an aggregated objective function. Why use a single aggregate objective function? I would advise analyzing the performance metrics separately. The SCE_UA method can do three separate trials - each using a different objective. Then you can compare the results of the proposed method against existing unit hydrograph routing methods proposed in the literature. If so desired, you can even consider the aggregate objective function - but then as fourth option.

Response:

Thanks for your comments. We will do four separate trials corresponding to three metrics and the aggregate objective function as fourth option.

Based on literature review, three metrics were selected to calibrate the model, including the Nash-Sutcliffe efficiency ($E_{NS}$) (Nash and Sutcliffe, 1970), the Kling-Gupta efficiency ($E_{KG}$) (Gupta et al., 2009) and the root-mean-squared error to standard deviation ratio ($R_{SR}$). Moreover, the new aggregated objective function (Brunner et al., 2021) targeted at optimizing flood characteristics composed of these three metrics, in which $E_{KG}$ focuses on high flows (Mizukami et al., 2019), log($E_{NS}$) emphasizes the low flows and $R_{SR}$ quantifies volume errors. Three metrics and the aggregated objective function are expressed by

$$
E_{NS} = 1 - \frac{\sum_{t=1}^{T} |Q'_t - Q''_t|}{\sum_{t=1}^{T} |Q'_t - Q''_t|}
$$

(26)
$$E_{\text{KG}} = 1 - \sqrt{(r-1)^2 + \left(\frac{\sigma_s}{\sigma_o} - 1\right)^2 + \left(\frac{\mu_s}{\mu_o} - 1\right)^2} \quad (27)$$

$$R_{\text{SR}} = \sqrt{\frac{\sum_{t=1}^{T} (\overline{Q}_t - \overline{Q})^2}{\sum_{t=1}^{T} (Q'_t - \overline{Q}_o)^2}} \quad (28)$$

$$M = 0.5 \times (1 - E_{\text{NS}}) + 0.25 \times (1 - E_{\text{KG}}) + 0.15 \times (1 - \log(E_{\text{NS}})) + 0.1 \times R_{\text{SR}} \quad (29)$$

where $Q'_o$ is observed discharge at time $t$; $Q'_s$ is simulated discharge at time $t$; $\overline{Q}_o$ is the mean of observed discharges; $T$ is duration of the flood event; $r$ is correlation coefficient between the observed and simulated flood; $\sigma_s$ and $\sigma_o$ are the standard deviation values for the simulated and observed responses, respectively, and $\mu_s$ and $\mu_o$ are the corresponding mean values.

References:


Comment #19: What is the definition of flood peak? Need more information to compute the first objective function; are you looking at the peaks of the measured discharge record? and then use the exact same indices of the simulated record to compute the objective function? Or are you getting the indices of the peaks from the simulated record? and then use the corresponding measured values to compute the OF? This difference in implementation may seem insignificant, but can lead to widely different results.

Response:

Thank you for your comments. The definition of the flood peak is shown as below.
Figure 5. Schematic diagram of the flood peak definition

As shown in the Figure 5, the peak error is computed with the peaks of simulated record and measured discharge record.

In the revised manuscript, the Kling-Gupta efficiency will be used for assessment of high flow. Then, the ratio between the simulated and measured peak discharges \( \frac{Q_p^s}{Q_p^o} \) will be calculated to evaluate the flood peak discharge error.

Comment #20: Why define the peak error as bias? Why is this preferred over a standard squared residual metric? L2-norm versus L1 norm. Same question for the timing error - and see above comment as well for this 2nd metric.

Response:

Thank you for your comments. Three new metrics discussed above will be adopted in the revised manuscript. The ratio between the simulated and observed peak discharges \( \frac{Q_p^s}{Q_p^o} \) and the error between the simulated and observed time to peak \( |t_p^s - t_p^o| \) can be further calculated to evaluate the forecast effects (Bunster et al., 2019).

References:


Comment #21: Why not use a metric such as the sum of squared residuals to compare the measured and simulated discharge records of the different routing methods?
Response:

Thank you for your comments. The metric such as the sum of squared residuals, is commonly used for the evaluation of different routing methods. Nevertheless, this metric cannot characterize how well the model performed. Flow magnitude of different basins presents significant differences. For example, in the large basin, the mean flow may be 10000 m³/s, and in the small basin, it may be only several hundreds. Using the sum of squared residuals as a performance assessment index, it is difficult to easily demonstrate that which flood forecast model is better for the two basins. The root-mean-squared error to standard deviation ratio ($R_{SR}$) will be used in the revised manuscript.

Comment #22: I doubt that the improvement of the new routing method proposed by the authors is related to incorporating what the authors believe to be soil moisture content. What is key to the performance of the new routing method is what is done to the parameter gamma. The authors articulate what they have done with gamma on Lines 365 - 373. The value of gamma will determine the results of the new method; hence, is the performance improvement related to the gamma parameter, simply as this provides additional flexibility to routing?

Response:

Thank you for your comments. The sensitivity analysis for variable gamma will be added in the revised manuscript. Here, we take a specific grid cell for example, whose slope of the grid cell is set to 0.34 m/m. The coefficient of the flow velocity $k$ and reference rainfall intensity $I_c$ are assumed to be 1.5 m/s and 20 mm/h. When the parameter gamma is 0.1, 0.5 and 1 respectively, the hillslope flow velocity values corresponding to different rainfall and soil moisture content using the proposed velocity formula are given in Figure 6. Three flow velocity formulas are given by

\[ V_t = k \cdot S^{\frac{1}{2}} \]  
\[ V_t = k \cdot S^{\frac{1}{2}} \cdot \left( \frac{I_t}{I_c} \right)^{\frac{2}{3}} \]  
\[ V_t = k \cdot S^{\frac{1}{2}} \cdot \left( \frac{I_t}{I_c} \right)^{\frac{2}{3}} \theta_i^{\gamma} \]
Figure 6. Time-varying flow velocity values corresponding to different parameters

It can be seen from Figure 1 that when $\theta_i$ equaling to 1, the proposed equation turns to Eq. (31). No matter what the parameter $\gamma$ is, the flow velocity values are ranging from 0.663 to 1.154 m/s corresponding to different rainfall intensities.

When $I_t$ equaling to the reference rainfall $I_c$, Eq. (31) turns to Eq. (30), and the flow velocity is 0.875 m/s.

After introducing a soil moisture content factor to the flow velocity formula, the flow velocity values range from 0.133 m/s to 1.154 m/s when $\gamma$ equaling 1. The flow velocity values are significantly different corresponding to different values of parameter $\gamma$. Thus, what is done to the parameter gamma significantly affects the performance of the new routing method.

The influence of soil moisture content $\theta_i$ on flow velocity cannot be ignored too. As shown in Figure 1, the flow velocity values under different soil water contents are also significantly different when $\gamma$ is a constant. In the revised manuscript, the sensitivity analysis for variable $\gamma$ will be added.

In practical flood forecasting, the parameter $\gamma$ should be a constant once it was determined in this study. This is similar with the influence of upstream contributions to the flow velocity formula (Leopold & Miller, 1956; Rodriguez-Iturbe et al., 1992; Rodriguez-Iturbe & Rinaldo, 1997; Leopold et al., 2012; Bhattacharya et al., 2012).

References:


Comment #23: The different routing methods amount to a model selection problem - and proper techniques such as information criteria or the marginal likelihood should be used to compare the different routing methods. I strongly doubt that what is shown in this paper is the result of soil moisture. The methodology and statistical analysis should be much improved to inspire confidence in this conclusion. As it stands right now, there are many other reasons so as to why the new method outperforms the existing routing methods. One of which is the parameter gamma. If nothing else, the analysis should show the sensitivity of the new routing method to the choice of gamma. The same should be done for parameter k in the existing formulation.

Response:

Thank you for your comments. The accuracy of the proposed routing method is mainly related to the flow velocity. In this manuscript, two parameters (\( \gamma \) and \( \theta \)) were added to the current flow velocity formula, which mainly affected the accuracy of the proposed method. The criteria for selecting routing methods are usually depend on the performance of the model because it is hard to measure the flow velocity of each grid. For instance, Bunster et al, (2019) developed a new DUH method that accounts for dynamic upstream contributions, and they made a comparison between their method and a fully distributed KW numerical method. Similar approaches have been applied by Du et al (2009); Gironás et al (2009) and so on. Therefore, the sensitivity of the new routing method to the choice of gamma will be added to the revised paper.

As for the parameter k, it is a coefficient of the current SCS formula, which is related to the land cover or flow type. It can be determined based on different underlying surface types or different flow states, and the values have been given by Ajward & Muzik, (2000) and Grimaldi et al. (2010).

Furthermore, the same will be done for parameter \( I_c \) in the revised paper.

A review of velocity formulas is summarized as below. In previous works, several flow velocity formulas are commonly used for deriving the spatially distributed unit hydrograph (DUH), such as the Manning’ formula (Chow et al., 1988), Soil Conservation Service (SCS) formula (Haan et al., 1994), Darcy-Weisbach formula (Katz et al., 1995) and Maidment et al. (1996) uniform flow equation.

<table>
<thead>
<tr>
<th>Flow velocity formula</th>
<th>Equation</th>
<th>Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manning’s formula</td>
<td>( V = \frac{\gamma^3 S^{\frac{1}{2}}}{n} )</td>
<td>Commonly used for turbulent flow.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Formula</th>
<th>Equation</th>
<th>Validity Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darcy-Weisbach formula</td>
<td>( V = \frac{8gSy^2}{K\Phi} )</td>
<td>Valid for flow regime, including laminar, transitional and turbulent.</td>
</tr>
<tr>
<td>SCS formula</td>
<td>( V = k \cdot S^1 )</td>
<td>Applicable for overland and shallow channel flow.</td>
</tr>
<tr>
<td>Maidment et al. uniform flow equation</td>
<td>( V = \frac{S^a A_u^b}{[S^a A_u]^m} )</td>
<td>Both used in hillslopes and channel cells.</td>
</tr>
</tbody>
</table>

where,

- \( V \) = velocity (m/s)
- \( y \) = Overland flow depth (m)
- \( S \) = Slope (m/m)
- \( n \) = Manning’s roughness coefficient (m\(^{1/3}\)/s)
- \( g \) = Acceleration of gravity (m/s\(^2\))
- \( K \) = Coefficient related to land use and rainfall intensity (unitless)
- \( \Phi \) = Kinematic viscosity (m\(^2\)/s)
- \( k \) = Land use or flow type coefficient (m/s)
- \( V_m \) = Average velocity for all the cells in the watershed (m/s)
- \( A_u \) = Upslope or upstream contributing area (m\(^2\))
- \( [S^a A_u^b]^m \) = Watershed average value of the slope-area term (m\(^2\))
- \( a, b \) = Calibration coefficients (unitless)

Grimaldi et al. (2010) found that the SCS formula can be well used to define the basin flow time. This formula was also used by NRCS (1997) and Gimaldi (2012) et al., which verified the rationality of this equation. The SCS flow velocity formula is time invariant. The time-varying rainfall intensity should be considered. The usage of the theory has been proved by Wong (1995), Muzik (1996), Bedient and Huber (2002), Gironas et al. (2009), Du et al. (2009) and Kong et al., (2019) et al.

\[
V_t = k \cdot S^1 \cdot \left( \frac{I_t}{I_c} \right)^{\frac{2}{3}}
\]

where \( I_t \) represents the excess rainfall intensity at time \( t \); and \( I_c \) represents the reference excess rainfall intensity of the basin.

Based on this formula, a time-varying soil moisture factor was introduced to it, which is expressed by

\[
V_t = k \cdot S^1 \cdot \left( \frac{I_t}{I_c} \right)^{\frac{2}{3}} \theta_t
\]

Above three flow velocity formulas were used for deriving the DUH, TDUH and TDUH considering soil moisture content respectively.
References:


Comment #24: Some section names confuse the reader; for example, section 4.2 is labeled "Derivation of TDUH considering time-varying soil moisture content", but the derivation has already been presented. In 4.2, the authors simply apply their method to a case study. Unless "derivation" has another meaning and refers to the computation of the TDUH.

Response:

Thank you for your comments. Section name of 4.2 will be corrected as “Computation of TDUH considering time-varying soil moisture content”.

General Comment: I leave it with this for now as further review will reiterate similar points. I very much appreciate the efforts of the authors in trying to improve the description of the unit hydrograph for distributed hydrologic modeling. Yet, the present paper needs a major revision before it can be judged to making a significant contribution to the field and warrant publication in HESS/HESS-D. As it stands right now, the methodology needs to be much better embedded into the literature and cite relevant papers when using existing equations, etc. and improve considerably the physical underpinning of the presented method. The authors should also revisit their calibration method - and provide compelling information about the sensitivity of their findings to the choice of gamma and k. Furthermore, the presentation and writing need considerable improvement.

I hope my comments are useful to improve the presentation and description of the methodology, including its application in a case study.

We highly appreciate your constructive comments. We hope the responses above could address your questions. We shall make a detailed revision based on your comments.