Review of Hess-2021-442

General Assessment: As the title already suggests, this manuscript considers the inference of hydrological laws based on a combination of physics, boundary conditions, and self-similarity arguments (complete and incomplete) using the Buckingham Pi theorem. The illustrations selected focus on processes with rich complexity that simply prevent the use of first principles. In this sense, they offer a strong test case for the use (and usefulness) of the Buckingham Pi theorem. To be clear, the work here is precisely in the spirit of what the Buckingham Pi theorem intends to achieve – a compact representation of a wealth of data (i.e. dimension reduction) in the form of a similarity law. The illustrations here include 3 examples – the Budyko curve (multiple version depending on the complexity of rainfall representation), weathering rates and mineral partitioning, and the spectral properties of landscapes (that is an insightful analysis of a PDE representing processes necessitating closure schemes that can be derived from the Buckingham Pi). The work concludes with analogies between landscape and turbulent spectral cascades and the equivalent role of the Reynolds number in elevation profiles (i.e. channelization index) thereby opening up new vistas to the study of landscape evolution.

The introduction part is well crafted and elegantly summarizes a number of topics based on complete and incomplete similarity arguments, as well as an insightful explanation of directional dimensional analysis. The paper concludes with conjectures about how the Buckingham Pi theorem be used in an era where data generation is overwhelming the ability of data interpretation. It is safe to state that this new perspective differs from the text-book illustration of the Buckingham Pi in hydrology and hydraulics – which is the drag coefficient as a function of Reynolds numbers for flow over a sphere or the Darcy-Weisbach friction factor as a function of bulk Reynolds number and relative roughness in pipes. The diversity of sources and references is quite complete – and covers depth and breadth for students and researchers alike. For this reason, I am recommending that the manuscript be accepted with technical comments and friendly suggestions.

Minor comments:

1. The introduction: the dichotomy between the promise of dimensional analysis and the number of results in the geosciences derived from the Pi theorem is certainly illuminating. But, perhaps it is also wise to point out that a large number of problems in the geosciences – when tackled in their most general form, lead to a large number of Pi groups. Simply put, the usage of the Pi theorem becomes tenuous when $n-k>>1$. In these cases, a restricted version of the problem must be sought that may enable the Buckingham Pi to be applied as an effective dimension reduction approach. The cases selected here for illustration are clearly chosen so that $n-k$ is a small number (order unity). Even an experienced chef may have hard time predicting the taste of a dish when the number of ingredients is large and the number of steps involved in a recipe is no less large!. In those situations, focusing on certain statistics or long time scales or restricted range of length scales (or modes of activity) must be sought. Whether these length or time scales are the pertinent ones remains problem specific. This is where the 'art' of using the Buckingham Pi comes into play. This point was alluded to in lines 230-238 as well as lines 329-335, and lines 341-345, but warrants a general statement early on.

2. Example 3.3 takes on a disproportionate space in the paper and serves as an illustration when the continuous space-time dynamics are known through an approximated PDE (eq. 27). I wonder whether this section can benefit from some re-arrangements. To arrive at the PDE, a number of assumptions were invoked that themselves could have benefited from the Buckingham Pi (e.g. the erosion term). Hence, the Buckingham Pi can be used as a 'closure' scheme to construct the governing PDE of elevation. Next, certain aspects of this resulting PDE can then be analyzed using the Buckingham Pi. This would be similar to what Prandtl did when invoking K-theory to close the turbulent stress term in the Reynolds-averaged Navier-Stokes equation thereby arriving at an advection-diffusion equation for turbulent boundary layer flows. This averaging and closure scheme enabled Prandtl to convert the elliptic nature of the instantaneous (and unsolvable) Navier-Stokes equation to a parabolic PDE (when time averaged) that can be solved. In essence, the erosion term is way too complicated to derive from first principle but the closure used here resulted in an approximated but closed PDE (i.e. eq. 27) with coefficients that are difficult to derive solely from first principles (much like eddy diffusion) that require experiments. The remaining sections become an analysis of solutions after some averaging and homogenizations are applied.

   For example, section 3.3.1 proceeds to deals with simplifications leading to bulk expressions whereas section 3.3.3 considers the scaling laws and spectral properties of the stationary state in a restricted region.

   This example can be a prototypical case study of how an approximated PDE can be constructed to describe the space-time evolution of the system when adopting certain closure schemes (derivable from Buckingham Pi). Then, proceed to show how one may proceed to derive different physical laws depending on the a priori assumptions made about the space and time scales and regions of the solution. The point here is that the tactic used to arrive at the approximated PDE and the method of analysis of this PDE can serve as a methodological complement to the elegant justification given in Figure 1.