



1 Effects of aquifer geometry on seawater intrusion in annulus segment island
2 aquifers

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21



22 **Abstract**

23 Seawater intrusion in island aquifers is considered analytically, specifically for annulus
24 segment aquifers (ASAs), i.e., aquifers that (in plan) have the shape of an annulus segment.
25 Based on the Ghijben-Herzberg and hillslope-storage Boussinesq equations, analytical
26 solutions are derived for steady-state seawater intrusion for ASAs, with a focus on the
27 freshwater-seawater interface and its corresponding watertable elevation. These analytical
28 solutions, after comparing their predictions with experimental data, are employed to investigate
29 the effects of aquifer geometry on seawater intrusion in island aquifers. Three different
30 geometries of ASA are compared: convergent (smaller side facing the lagoon), rectangular and
31 divergent (larger side facing the sea). The results show that the predictions from the analytical
32 solutions are in well agreement with the experimental data for both recharge events. In addition,
33 seawater intrusion is most extensive in divergent aquifers, and conversely for convergent
34 aquifers. Accordingly, the watertable elevation is lowest in divergent aquifers and highest in
35 convergent aquifers. Moreover, the effects of aquifer geometry on the freshwater-seawater
36 interface and watertable elevation vary with aquifer width and distance to the no-flow boundary.
37 Both a larger aquifer width and distance to the no-flow boundary weaken the effects of aquifer
38 geometry and hence lead to a smaller deviation of seawater intrusion between the three
39 geometries.

40

41 **Keywords:** sharp-interface; steady-state; convergent aquifer; divergent aquifer; rectangular
42 aquifer



43 **Key Points**

- 44 ➤ Analytical solutions of steady-state seawater intrusion are derived for annulus segment
45 aquifers
- 46 ➤ Among three different aquifer geometries, divergent aquifers have lowest watertable and
47 hence most extensive seawater intrusion
- 48 ➤ Aquifer geometry effects on seawater intrusion depend on the aquifer width and distance
49 from the circle center to the no-flow boundary



50 1. Introduction

51 Islands are extensively distributed in global oceans and are considered as vulnerable places
52 due to sea-level rise and human population growth. According to a recent estimate, there are
53 approximately 65 million people living in oceanic islands, where fresh groundwater may be the
54 only source of freshwater (Thomas et al., 2020). Fresh groundwater stored on oceanic islands
55 is mainly recharged from precipitation and its availability can be impacted by a variety of
56 factors, including island topography, rainfall patterns, tides, episodic storms and human
57 activities (White & Falkland, 2010; Storlazzi et al., 2018). Among these, seawater intrusion is
58 another important issue that greatly affects oceanic island freshwater storage and is thus of
59 considerable interest (e.g., Werner et al., 2017; Lu et al., 2019; Memari et al., 2020).

60 Different from coastal aquifers where seawater intrudes into freshwater from one direction
61 only, seawater intrusion occurs from two directions for narrow strip islands and from all
62 directions for circular islands. Over the past few decades, seawater intrusion in oceanic islands
63 has been extensively investigated in multiple ways, either directly by field observations (e.g.,
64 Röper et al., 2013; Post et al., 2019), or indirectly by experimental measurements (e.g., Stoeckl
65 et al., 2015; Bedekar et al., 2019; Memari et al., 2020), numerical simulations (e.g., Lam, 1974;
66 Gingerich et al., 2017; Liu & Tokunaga, 2019), and analytical solutions (e.g., Fetter, 1972;
67 Ketabchi et al., 2014; Lu et al., 2019). Among these, analytical solutions are effective tools to
68 assess the extent of seawater intrusion, despite that they cannot incorporate complex factors
69 (e.g., dispersive mixing and transient oceanic dynamics) (Werner et al., 2013). The advantages
70 of analytical solutions are that (1) they are rapidly and easily computed, and (2) they give



71 explicit relationships between parameters that influence seawater intrusion.

72 Based on the Dupuit-Forchheimer approximation (i.e., ignoring vertical flow) and the
73 Ghijben-Herzberg equation (Drabbe & Badon Ghijben, 1889, English translation given by Post
74 (2018); Herzberg, 1901), Fetter (1972) presented analytical solutions describing the freshwater-
75 seawater interface location and watertable elevation in a circular island. Bailey et al (2010)
76 further compared these single-layer analytical solutions with field measurements, indicating
77 that the analytical solutions perform well in estimating the freshwater-seawater interface
78 location and watertable elevation. Fetter's (1972) solutions formed the fundament for many
79 analytical studies aimed at seawater intrusion in island aquifers. Based on this single-layer
80 analytical theory, Chesnaux and Allen (2008) and Greskowiak et al. (2013) developed analytical
81 solutions to predict the steady-state groundwater age distribution in freshwater lenses. In
82 addition, using the single-layer analytical solutions, Morgan and Werner (2014) proposed
83 vulnerability indicators of freshwater lenses under sea-level rise and recharge change.

84 Since aquifers are usually heterogeneous in reality, the single-layer analytical solutions are
85 extended to two-layer island aquifers subsequently. Vacher (1988) derived solutions for the
86 freshwater-seawater interface location and watertable elevation for infinite-strip islands
87 composed of different layers. Dose et al. (2014) carried out laboratory experiments to validate
88 analytical solutions proposed by Fetter (1972) and Vacher (1988) and confirmed their reliability.
89 Ketabchi et al. (2014) extended Fetter (1972)'s analytical solutions to calculate the freshwater-
90 seawater interface location and watertable elevation in two-layer circular islands subject to sea-
91 level rise. Their results indicated that land-surface inundation caused by sea-level rise has a



92 considerable impact on the fresh groundwater lens. The analytical solutions of Ketabchi et al.
93 (2014) and Vacher (1998) assumed an island with two horizontal layers. More recently, Lu et
94 al. (2019) derived analytical solutions for the freshwater-seawater interface location and
95 watertable elevation for both strip and circular islands with two adjacent layers, i.e., a less
96 permeable slice along the shoreline of an island, and a more permeable zone inland. For more
97 studies associated with seawater intrusion in oceanic islands, readers are referred to the
98 comprehensive review of Werner et al. (2017).

99 All the abovementioned analytical solutions apply to either strip or circular islands.
100 According to the classification of sand dune developed by Stuyfzand (1993; 2017), there are
101 different layouts, e.g., Figure 1, where the shape of the island is an annulus segment, instead of
102 a strip or circle. This configuration is widely distributed in atolls (i.e., circular chains of islands
103 surrounding a central lagoon) as found in the Pacific and Indian Oceans (Werner et al., 2017;
104 Duvat, 2019). Nevertheless, no analytical solutions of seawater intrusion have been developed
105 for annulus segment aquifers (ASAs). In general, ASAs are conceptually treated as a 2D cross
106 section like strip islands (e.g., Ayers & Vacher, 1986; Underwood et al., 1992; Bailey et al.,
107 2009; Werner et al., 2017). Evidently, topography plays an important role in groundwater flow
108 and hence seawater intrusion (e.g., Zhang et al., 2016; Liu & Tokunaga, 2019). It remains
109 unclear whether the analytical solutions of seawater intrusion for strip islands are appropriate
110 for ASAs and how island geometry affects the freshwater-seawater interface location and
111 watertable elevation of ASAs.

112 In this study, we derive analytical solutions of the steady-state seawater intrusion for ASAs,



113 with a focus on the freshwater-seawater interface location and its corresponding watertable
114 elevation. After comparing their predictions with experimental data compiled from Memari et
115 al. (2020), the analytical solutions are employed to investigate the effects of aquifer geometry
116 on the freshwater-seawater interface location and watertable elevation in ASAs.

117 **2. Conceptual Model**

118 Figure 2 shows the conceptual model of an ASA (a slice of atoll islands). The plan view
119 of the model domain is represented as a sector ($EFGH$) with an angle θ (Figure 2a). Radial
120 flow only is considered. The sea (EF) and lagoon (HG) boundaries are located at $L+L_0$ [L] and
121 L_0 [L] from the circle center, respectively. Since the longitudinal length is much longer than the
122 lateral length for an atoll island in reality (Werner et al., 2017), it can be expected that seawater
123 intrusion from lateral side would have negligible influence on that from longitudinal side,
124 especially for the middle part of an ASA. In order to facilitate analytical derivations, therefore,
125 EH and FG are treated as no-flow boundaries. It should be noted that assuming lateral boundary
126 as a no-flow boundary (e.g., EH and FG) has been widely adopted in previous studies related
127 to freshwater lenses on atoll islands (e.g., Ayers & Vacher, 1986; Underwood et al., 1992; Bailey
128 et al., 2009; Werner et al., 2017). The side view of the model domain is conceptualized as a
129 rectangle ($ABCD$) along the radial direction with dimensions of L [L] (width) \times d [L] (height)
130 (Figure 2b,c). AD and BC are impermeable base and land surface boundary, respectively.

131 To facilitate analytical derivations, both the sea and lagoon water levels are H_s [L],
132 which results in a no-flow boundary (water divide) between the sea and lagoon. For simplicity,
133 the segment between the sea and no-flow boundaries is referred to as Unit 1, whereas the



134 segment between the no-flow and lagoon boundaries is referred to as Unit 2. The widths of
135 Units 1 and 2 are l_1 [L] and l_2 [L], respectively. In addition, the flow is asymmetrical in Units
136 1 and 2, with divergent flow (the aquifer length w [L] increases along the flow direction) in
137 Unit 1 and convergent flow (the aquifer length w [L] decreases along the flow direction) in Unit
138 2.

139 The x - z coordinate origin is placed at the intersection of the no-flow boundary and
140 impermeable base, with the x -axis horizontally pointing to the circle center and z -axis vertically
141 upward. ϕ [L] is the watertable height. h [L] is the vertical distance between the watertable
142 and the interface. h_s [L] is the vertical distance between the sea level and the interface, and
143 $h_c = H_s - h_s$ [L] is the vertical distance from the impermeable base to the interface. Recharge
144 into the saturated zone is assumed to be uniform with value of N [LT^{-1}]. There are two
145 possibilities for the interface tip location: above the aquifer bed (Figure 2b) and on the aquifer
146 bed (Figure 2c). For the condition with the interface tip on the aquifer bed, the interface tip
147 locations in Units 1 and 2 are denoted as x_{t1} [L] and x_{t2} [L], respectively. Consistent with
148 previous studies (e.g., Ketabchi et al., 2014; Lu et al., 2016; 2019), the following assumptions
149 are made: (1) the flow system is steady state, (2) a sharp interface exists between freshwater
150 and seawater, (3) aquifer hydraulic properties are homogeneous and isotropic, (4) unsaturated
151 flow is neglected, (5) recharge is smaller than the saturated hydraulic conductivity (no ponding),
152 and (6) vertical flow is neglected in the saturated zone (i.e., Dupuit-Forchheimer
153 approximation).



154 3. Analytical Solutions Derivation

155 Groundwater flow in an ASA (Figure 2) can be described as (Paniconi et al., 2003; Troch
156 et al., 2003),

$$157 \quad -\frac{\partial}{\partial x}(wq) + Nw = \frac{\partial S}{\partial t} \quad (1)$$

158 where q [L^2T^{-1}] is the Darcy flux per unit length along the aquifer; S [L^2] is the total water
159 storage per unit distance along the aquifer, and t [T] is time. Equation (1) is derived from the
160 hillslope-storage Boussinesq equation reformulated in terms of soil water storage rather than
161 watertable elevation, as widely used previously (e.g., Stagnitti et al., 1986; Troch et al., 2003;
162 Hilberts et al., 2005; Kong et al., 2016; Luo et al., 2018). At steady state, equation (1) reduces
163 to,

$$164 \quad -\frac{\partial}{\partial x}(wq) + Nw = 0 \quad (2)$$

165 According to Darcy's law and the Dupuit-Forchheimer approximation, the freshwater flux
166 in the aquifer segment between the seaward boundary and interface tip can be calculated as,

$$167 \quad q = -\int_{h_c}^{\phi} K_s \frac{d\phi}{dx} dz \quad (3)$$

168 where K_s [LT^{-1}] is the saturated hydraulic conductivity.

169 3.1. Interface Tip above the Aquifer Bed

170 We first consider the situation where the interface tip is above the aquifer bed (Figure 2b).
171 In Unit 1 where $w = \theta(L_0 + l_2 - x)$, substituting equation (3) into equation (2) and then
172 integrating gives,

$$173 \quad -\frac{1}{2}[(L_0 + l_2 - x)^2 - (L_0 + l_2)^2]N = -(L_0 + l_2 - x) \int_{h_c}^{\phi} K_s \frac{d\phi}{dx} dz \quad (4)$$

174 According to the Ghijben-Herzberg equation, the vertical thickness of the freshwater zone



175 (h) in the interface zone is given by,

$$176 \quad h = (1 + \alpha)(\phi - H_s) \quad (5)$$

177 where $\alpha = \rho_f / (\rho_s - \rho_f)$ [-] is the dimensionless density difference, with ρ_f [ML^{-3}] and ρ_s
 178 [ML^{-3}] being the densities of freshwater and seawater, respectively.

179 Substitution of equation (5) into equation (4) yields,

$$180 \quad -\frac{1}{2}[(L_0 + l_2 - x)^2 - (L_0 + l_2)^2]N = -K_s(L_0 + l_2 - x)(1 + \alpha)(\phi - H_s)\frac{d\phi}{dx} \quad (6)$$

181 Rearranging equation (6) produces,

$$182 \quad -\frac{(L_0 + l_2 - x)N}{2} + \frac{N(L_0 + l_2)^2}{2(L_0 + l_2 - x)} = -K_s(1 + \alpha)(\phi - H_s)\frac{d\phi}{dx} \quad (7)$$

183 Integrating equation (7) leads to,

$$184 \quad -\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x) - \frac{1}{2}(L_0 + l_2)Nx + \frac{1}{4}Nx^2 + C_1 = -K_s(1 + \alpha)\frac{(\phi - H_s)^2}{2} \quad (8)$$

185 where C_1 is the integration constant and can be determined by the sea boundary condition (i.e.,
 186 $x = -l_1$, $\phi = H_s$),

$$187 \quad C_1 = \frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 + l_1) - \frac{1}{2}(L_0 + l_2)l_1N - \frac{1}{4}l_1^2N \quad (9)$$

188 The relation between h_s and ϕ is given by,

$$189 \quad h_s = \alpha(\phi - H_s) \quad (10)$$

190 Combining equation (8) with equation (10) and eliminating ϕ yield,

$$191 \quad -\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x) - \frac{1}{2}(L_0 + l_2)Nx + \frac{1}{4}Nx^2 + C_1 = -K_s(1 + \alpha)\frac{h_s^2}{2\alpha^2} \quad (11)$$

192 Equation (11) can be adopted to calculate the freshwater-seawater interface location in

193 Unit 1 if both l_1 and l_2 are determined.



194 Equation (8) can also apply to Unit 2 by replacing C_1 with C_2 ,

$$195 \quad -\frac{(L_0+l_2)^2 N}{2} \ln(L_0+l_2-x) - \frac{1}{2}(L_0+l_2)Nx + \frac{1}{4}Nx^2 + C_2 = -K_s(1+\alpha) \frac{(\phi-H_s)^2}{2} \quad (12)$$

196 where C_2 is chosen to satisfy the lagoon boundary condition ($x=l_2$, $\phi=H_s$),

$$197 \quad C_2 = \frac{(L_0+l_2)^2 N}{2} \ln(L_0) + \frac{1}{2}(L_0+l_2)l_2N - \frac{1}{4}l_2^2N \quad (13)$$

198 Combining equation (10) and equation (12) and eliminating ϕ yield,

$$199 \quad -\frac{(L_0+l_2)^2 N}{2} \ln(L_0+l_2-x) - \frac{1}{2}(L_0+l_2)Nx + \frac{1}{4}Nx^2 + C_2 = -K_s(1+\alpha) \frac{h_s^2}{2\alpha^2} \quad (14)$$

200 Similarly, equation (14) can be adopted to calculate the freshwater-seawater interface
 201 location in Unit 2 if l_2 is determined. As mentioned before, since the sea level and lagoon
 202 water level are assumed to be the same, a no-flow boundary exists between the sea and lagoon,
 203 i.e.,

$$204 \quad x=0, (h_s)_{unit1} = (h_s)_{unit2} \quad (15)$$

205 Combining equation (11) and equation (14) with equation (15) leads to expressions for l_1
 206 and l_2 , respectively,

$$207 \quad l_1 = L + L_0 - \sqrt{\frac{2LL_0 + L^2}{2\ln(L+L_0) - 2\ln(L_0)}} \quad (16)$$

$$208 \quad l_2 = \sqrt{\frac{2LL_0 + L^2}{2\ln(L+L_0) - 2\ln(L_0)}} - L_0 \quad (17)$$

209 As indicated by equations (16) and (17), the no-flow boundary between the sea and lagoon
 210 only depends on L and L_0 under steady state. For known l_1 and l_2 , equations (11) and (14)
 211 can be employed to predict the freshwater-seawater interface location in Units 1 and 2,
 212 respectively. Once the interface location is determined, we can further calculate h and ϕ as



213 follows,

$$214 \quad h = \frac{1+\alpha}{\alpha} h_s \quad (18)$$

$$215 \quad \phi = \frac{h_s}{\alpha} + H_s \quad (19)$$

216 3.2. Interface Tip on the Aquifer Bed

217 When the interface tip is on the aquifer bed, the location of the no-flow boundary remains
218 the same as for the interface tip above the aquifer bed. The freshwater-seawater interface for
219 Units 1 and 2 can be respectively determined by equations (11) and (14). Then we can calculate
220 h at the aquifer segment between the sea boundary and the interface tip according to equation
221 (18).

222 In order to calculate h for the aquifer segment between the interface tip and the no-flow
223 boundary, the interface tip location should be determined first. At the interface tip ($x = x_{i1}$) of
224 Unit 1, we have,

$$225 \quad h_s = H_s \quad (20)$$

$$226 \quad \phi = \frac{1+\alpha}{\alpha} H_s \quad (21)$$

227 By substituting equation (21) into equation (11), the interface tip location of Unit 1 can be
228 obtained as,

$$229 \quad -\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x_{i1}) - \frac{1}{2}(L_0 + l_2) N x_{i1} + \frac{1}{4} N x_{i1}^2 = -C_1 - K_s (1 + \alpha) \frac{H_s^2}{2\alpha^2} \quad (22)$$

230 Let,

$$231 \quad a = \frac{1}{4} N \quad (23a)$$

$$232 \quad b = -\frac{1}{2}(L_0 + l_2) N \quad (23b)$$



$$233 \quad c = -\frac{(L_0 + l_2)^2 N}{2} \quad (23c)$$

$$234 \quad m = -C_1 - K_s (1 + \alpha) \frac{H_s^2}{2\alpha^2} \quad (23d)$$

235 then equation (22) becomes,

$$236 \quad ax_{i1}^2 + bx_{i1} + c \ln(L_0 + l_2 - x_{i1}) = m \quad (24)$$

237 Equation (24) can be easily solved by a root-finding method.

238 The freshwater discharge for the aquifer segment between the interface tip and the no-flow
 239 boundary can be calculated as,

$$240 \quad -\frac{1}{2} \left[(L_0 + l_2 - x)^2 - (L_0 + l_2)^2 \right] N = -(L_0 + l_2 - x) \int_0^\phi K_s \frac{d\phi}{dx} dz \quad (25)$$

241 Repeating the steps from equations (4) to (8) gives,

$$242 \quad -\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x) - \frac{1}{2} (L_0 + l_2) Nx + \frac{1}{4} Nx^2 + C_3 = -\frac{K_s}{2} \phi^2 \quad (26)$$

243 where C_3 can be determined by substituting equation (21) to equation (26). Subsequently,
 244 equation (26) can be adopted to calculate h for the segment between the interface tip and the
 245 no-flow boundary where $h = \phi$.

246 Similarly, the interface tip location of Unit 2 is obtained by substituting equation (21a) into
 247 equation (14). Then, the value of h for the aquifer segment between the interface tip and no-
 248 flow boundary for Unit 2 is computed by repeating the steps from equations (22) to (26).

249 4. Results and Discussion

250 4.1. Validation of Analytical Solutions

251 The analytical solutions were validated by comparing their predictions with experimental
 252 data compiled from Memari et al. (2020). Their experiments were carried out using a 15° radial



253 tank that can be considered as an ASA. The radial tank contained three distinct chambers: no-
254 flow boundary condition, porous medium and seaward boundary condition. The no-flow and
255 seaward boundaries were respectively located at 10 cm and 55.5 cm from the circle center, i.e.,
256 45.5 cm from the no-flow boundary to seaward boundary along the radial direction (L^*). Note
257 that the experimental tank only corresponds to Unit 1 of the radial aquifer with $l_1 = L^*$ while
258 $l_2 = 0$ cm. The thickness of the porous medium and sea level were 28 and 25 cm, respectively.
259 The sand used in experiments had a saturated hydraulic conductivity of 1.23×10^{-2} m/s and an
260 effective porosity of 0.40. The measured saltwater and freshwater densities were respectively
261 1.015 and 0.999 g/ml, leading to $\alpha = 62$. Two different recharge events, 2.46×10^{-4} m/s and
262 1.08×10^{-4} m/s, were considered in the experiments. For clarity, experimental parameters are
263 summarized in Table 1.

264 Figure 3 shows the comparison between analytical and experimental results of the
265 freshwater-seawater interface for different recharge events. In general, the analytical solution
266 performs well in predicting the freshwater-seawater interface for both recharge events, despite
267 that there are small differences between analytical results and measurements, particularly in the
268 zone near the sea boundary. These deviations are likely due to assumptions made in the
269 analytical solution, i.e., (i) a sharp freshwater-seawater interface, (ii) ignoring the effect of
270 freshwater discharge, and (iii) neglecting the vertical flow.

271 **4.2. Effects of Aquifer Geometry on Seawater Intrusion**

272 Previous studies indicated that boundary conditions play a critical role in affecting
273 seawater intrusion (Werner & Simmons, 2009; Lu et al., 2016). Therefore, the no-flow



274 boundary between the sea and lagoon was examined for ASAs. As indicated by equations (16)
275 and (17), this no-flow boundary only depends on L and L_0 . The values of l_1 and l_2 calculated
276 respectively from equations (16) and (17) are shown in Figure 4 for three typical values of L
277 (500, 1000 and 2000 m) with varying L_0 from 10^2 to 10^6 m. In general, the no-flow boundary
278 deviates from the middle of the ASA. When L_0 is smaller than 10^5 m, l_1 is larger than l_2 for
279 the three different values of L , indicating a no-flow boundary closer to the lagoon boundary.
280 For example, assuming $L = 2000$ m and $L_0 = 100$ m leads to $l_1 = 1240$ m and $l_2 = 760$ m,
281 with a deviation of 240 m (12% of 2000 m) from the middle of the ASA. When L_0 exceeds 10^5
282 m, however, the no-flow boundary can be approximated to be at the middle of the ASA for all
283 considered values of L . This is in contrast to strip and circular aquifers where the no-flow
284 boundary is in the middle of aquifers due to symmetric flow. Moreover, l_1 approaches l_2 with
285 increasing L_0 , which suggests a movement of the no-flow boundary to the middle of the ASA.
286 This is because, as L_0 increases, the island shape approaches to be rectangular and hence leading
287 to the flow parallel with EH and FG . By comparison, at a given L_0 smaller than 10^5 m, the no-
288 flow boundary location deviates more from the middle of the ASA with increasing L .

289 Since the no-flow boundary location between the sea and lagoon deviates from the middle
290 of the ASA, we expect aquifer geometry to play a significant role in controlling seawater
291 intrusion. As mentioned previously, ASAs can be convergent (Unit 1) or divergent aquifers
292 (Unit 2) where the extent of seawater intrusion may be different. However, for strip aquifers,
293 both Units 1 and 2 are rectangular with the same extent of seawater intrusion. Therefore, three
294 geometries were compared in this study: convergent, rectangular and divergent (Figure 5).



295 These geometries have been widely examined in hillslope hydrology regrading to the effects of
296 aquifer geometry on runoff generation (Troch et al., 2003; Kong et al., 2016; Luo et al., 2018).
297 For the sake of simplicity, we replaced the x - z coordinate origin at the intersection of the
298 seawater boundary and the impermeable base, with the x -axis horizontally pointing to the no-
299 flow boundary and the z -axis vertically upward.

300 Following previous studies (e.g., Lu et al., 2016; 2019), hypothesized cases were designed
301 to show the effects of aquifer geometry on seawater intrusion (Cases 1 and 2 in Table 1).
302 According to Werner et al. (2017), the width of atoll islands generally varies from 100 to 1500
303 m along the radial direction. In order to focus on the effects of aquifer geometry on seawater
304 intrusion, we assumed the same L^* and L_0 for the three aquifers, with L^* and L_0 equal to 1000
305 and 200 m, respectively. The sand characteristics were the same as in the experiments of
306 Memari et al. (2020). Two recharge events were considered (Cases 1 and 2, Table 1). The
307 freshwater-seawater interface was calculated using the analytical solutions for the three
308 different aquifers. Note that Appendix A presents analytical solutions for seawater intrusion in
309 strip aquifers deduced from Lu et al. (2019).

310 Figure 6 shows the freshwater-seawater interface calculated for Cases 1 and 2. As can be
311 seen, the extent of seawater intrusion is greatly different for the three different aquifers. For the
312 high recharge, the interface tip is located at around 500 m for the divergent aquifer, about twice
313 the value of the rectangular aquifer and six times the value for the convergent aquifer (Figure
314 6a). When the recharge decreases to 3×10^{-7} m/s, the interface tip moves more landward for
315 three aquifers as expected, but the difference between results of three aquifers is still great



316 (Figure 6b). The interface tip rises above the aquifer bed for both rectangular and divergent
317 aquifers, while it remains on the aquifer bed for the convergent aquifer. Regardless of the
318 recharge rate, the most landward freshwater-seawater interface occurs in divergent aquifers and
319 vice versa for convergent aquifers. This underlines that aquifer geometry plays a significant
320 role in controlling seawater intrusion and hence it is necessary to account for the effects of
321 aquifer geometry in analytical solutions of seawater intrusion.

322 **4.3. Sensitivity Analysis**

323 A sensitivity analysis was conducted to investigate at which degree of curvature the
324 deviation of seawater intrusion between three different aquifers becomes significant. Since we
325 focus on the effects of aquifer geometry on seawater intrusion, values of L_0 and L^* were varied,
326 with other parameters kept constant. When conducting the sensitivity analysis of L_0 , L^* was
327 fixed at 1000 m that is a typical value for ASAs (Werner et al., 2017). Figure 7 shows the
328 sensitivity of the freshwater-seawater interface and watertable elevation to changes in L_0 (Case
329 3, Table 1). As expected, the freshwater-seawater interface and watertable elevation are
330 independent of L_0 for rectangular aquifers. However, the freshwater-seawater interface and
331 watertable elevation differ greatly when varying L_0 for both convergent and divergent aquifers,
332 highlighting that L_0 plays an important role in affecting seawater intrusion. Specifically, as L_0
333 increases, the freshwater-seawater interface moves more landward (Figure 7a) and its
334 corresponding watertable elevation decreases (Figure 7c) for convergent aquifers. This is
335 contrary to divergent aquifers where the freshwater-seawater interface moves more seaward
336 (Figure 7b) and its corresponding watertable elevation increases (Figure 7d) with increasing L_0 .



337 For a given L_0 , divergent aquifers have the largest extent of seawater intrusion and lowest
338 watertable elevation, and conversely for convergent aquifers (Figure 7). Consistent with the
339 experimental results of Hilberts et al. (2005), the steady-state watertable elevation is lower in
340 the divergent aquifer than in the convergent aquifer for the same rainfall.

341 By comparison, the deviation between rectangular aquifers and divergent or convergent
342 aquifers, regardless of the freshwater-seawater interface and watertable elevation, is significant
343 when L_0 is less than 2000 m (Figure 7). For example, the interface toe location is 262 m for the
344 rectangular aquifer at $L_0 = 200$ m, whereas it is 78 m (31% of that in the rectangular aquifer)
345 and 500 m (191% of that in the rectangular aquifer) for the convergent and divergent aquifers,
346 respectively. As L_0 increase, the deviation between three aquifers decreases. When $L_0 = 2000$
347 m, the interface toe location is 262, 209 (80% of that in the rectangular aquifer) and 318 m (121%
348 of that in the rectangular aquifer), respectively. As L_0 continues to increase to 6000 m, the
349 freshwater-seawater interface and watertable elevation of both convergent and divergent
350 aquifers tend to those of rectangular aquifers, i.e., geometry effects decrease with increasing L_0 .
351 These highlight the critical role played by the shape of aquifers. As a result, ignoring geometry
352 effects may lead to an inappropriate management strategy for fresh groundwater resource in
353 atoll islands.

354 The sensitivity of the freshwater-seawater interface and watertable elevation to L^* was
355 further conducted with varying L^* from 600 to 1600 m while fixing L_0 at 200 m (Case 4, Table
356 1). As shown in Figure 8, in contrary to L_0 , the freshwater-seawater interface and watertable
357 elevation in all three topographies is related to L^* . Again, seawater intrusion is greatest in



358 divergent aquifers and least in convergent aquifers for given L^* . When L^* increases, the
359 freshwater-seawater interface moves more seaward and the watertable elevation increases,
360 regardless of aquifer geometry, i.e., the seawater intrusion decreases (Figure 8a-c). This is
361 because the total freshwater flux increases with increasing L^* , leading to a higher hydraulic
362 gradient and hence less seawater intrusion (Figures 8d-f). Moreover, an increase in L^* induces
363 a smaller deviation of seawater intrusion between three geometries, i.e., geometry effects on
364 seawater intrusion are more significant at small L^* . However, even at the maximum L^* given in
365 this study (1600 m), the deviation between three aquifers is significant: the interface toe location
366 is about 148 m for the rectangular aquifer, whereas it is about 32 (22% of that in the rectangular
367 aquifer) and 278 m (188% of that in the rectangular aquifer) for the convergent and divergent
368 aquifers, respectively. Both L_0 and L^* can greatly impact seawater intrusion for divergent and
369 convergent aquifers, highlighting the necessity to include geometry effects into analytical
370 solutions of seawater intrusion.

371 **4.4. Limitations of This Study and Future Work**

372 The foregoing results show that aquifer geometry affects the freshwater-seawater interface
373 and watertable elevation, and is directly related to the aquifer width, the distance from the circle
374 center to the no-flow boundary, and aquifer shape, i.e., whether the aquifer is convergent or
375 divergent. Of the aquifer shapes considered, divergent aquifers are the most vulnerable to
376 seawater intrusion and convergent aquifers have the least seawater intrusion. Correspondingly,
377 the watertable elevation is the lowest in divergent aquifers and highest in convergent aquifers,
378 which is consistent with rainfall-runoff generation in unconfined hillslope aquifers (Hilberts et



379 al., 2005; Hazenberg et al., 2015; Kong et al., 2016). The assumption of a rectangular aquifer
380 may lead to inaccurate estimates of freshwater storage on oceanic islands and hence future
381 efforts should be devoted to establishing digital elevation models with high accuracy for
382 oceanic islands. Therefore, it is necessary to build the hydrology database for oceanic islands
383 including soil properties, rainfall, sea level, water level of lagoon and others for estimating
384 seawater intrusion accurately. Although some existing codes such as SEAWAT (Langevin et al.,
385 2008) and SUTRA (Voss & Provost, 2008) can be used to simulate seawater intrusion in ASAs,
386 they are computationally expensive, especially for divergent and convergent aquifers where the
387 model domain cannot reduce to a two-dimensional section. However, the new analytical
388 solutions, validated against experiments, can be used as a fast tool to estimate seawater intrusion
389 in ASAs. Due to lack of data, the new analytical solutions have not been applied to estimate
390 seawater intrusion in the field. Despite of this, it can be expected that the analytical solutions
391 can be easily used once we have known island geometry and corresponding soil properties.

392 In line with previous studies, a variety of assumptions were made to facilitate deriving
393 analytical solutions the freshwater-seawater interface and watertable elevation (e.g., Fetter,
394 1972; Strack, 1976; Lu et al., 2019). In addition to the assumptions mentioned previously, the
395 temporal and spatial variability in recharge is not considered into analytical solutions. Previous
396 field monitoring and numerical modelling studies in St. Georgia Island indicate that the
397 freshwater lens morphology is controlled by spatial variability of recharge values, while the
398 response of the freshwater lens to monthly variability in recharge is insignificant (Schneider &
399 Kruse, 2006). In addition, the lagoon water level usually lags behind the sea level, leading to



400 changes of the no-flow boundary condition that greatly affects seawater intrusion. Furthermore,
401 global mean sea level has increased with an apparent rate acceleration during the last decade
402 (Dieng et al., 2017; Kim et al., 2019), which further results in uncertainties to predict seawater
403 intrusion in oceanic islands. The effects of temporal and spatial variability in recharge, water
404 level difference between sea and lagoon and sea level rise on seawater intrusion in ASAs should
405 be further explored in the future work.

406 **5. Conclusions**

407 Based on the Ghijben-Herzberg and hillslope-storage Boussinesq equations, we derived
408 analytical solutions of steady-state seawater intrusion for ASAs, with a focus on the freshwater-
409 seawater interface and its corresponding watertable elevation. After comparing with
410 experimental data of Memari et al. (2020), the analytical solutions were employed to examine
411 the effects of aquifer geometry on seawater intrusion in island aquifers. Three different shapes
412 of island aquifer were compared: convergent, rectangular and divergent. The results lead to the
413 following conclusions:

414 (1) The presented analytical solutions perform well in predicting the experimental
415 freshwater-seawater interface for both recharge events, suggesting that these analytical
416 solutions can predict seawater intrusion reasonably in different aquifer geometries.

417 (2) Island geometry plays a significant role in affecting the freshwater-seawater interface
418 and watertable elevation. The extent of seawater intrusion is the most serious in divergent
419 aquifers, while the lightest in convergent aquifers. In contrast, the watertable elevation is the
420 lowest in divergent aquifers while the highest in convergent aquifers.



421 (3) The effects of aquifer geometry on seawater intrusion are dependent of the aquifer
422 width and distance from the circle center to the no-flow boundary. Both a larger of the aquifer
423 width and distance from the circle center to the no-flow boundary can weaken the role played
424 by aquifer geometry and hence lead to a smaller deviation of the extent of seawater intrusion
425 between three topographies.

426 Real island aquifers are expected to exhibit more complexity than considered here with
427 regards to their topographies and subjecting to transient flow conditions caused by tide and
428 wave. Nevertheless, based on steady-state analytical solutions, the present results provide the
429 overall behavior of the effects of aquifer geometry on seawater intrusion. As the results show,
430 the freshwater-seawater interface and watertable elevation within island aquifers can be
431 markedly affected by aquifer geometry.



432 **Appendix A: Analytical Solutions for Rectangular Aquifers**

433 For rectangular aquifers, seawater intrusion in Units 1 is identical to that in Unit 2 because
434 of symmetrical groundwater flow. With the interface tip on the aquifer bed, analytical solutions
435 of the freshwater-seawater interface, watertable elevation, and interface tip in Unit 2 can be
436 respectively written as (Lu et al., 2019),

437
$$h_s = \alpha \sqrt{\frac{N}{(1+\alpha)K_s} \left(\frac{L^2}{4} - x^2 \right)} \quad (\text{A1})$$

438
$$h = \sqrt{\frac{N}{(1+\alpha)K_s} \left(\frac{L^2}{4} - x^2 \right)} + H_s \left(x_t < x \leq \frac{L}{2} \right) \quad (\text{A2})$$

439
$$h = \sqrt{\frac{N}{K_s} (x_t^2 - x^2)} + \left(\frac{H_s}{\alpha} + H_s \right) \left(0 \leq x \leq x_t \right) \quad (\text{A3})$$

440
$$x_t = \sqrt{\frac{L^2}{4} - \frac{(1+\alpha)K_s}{N} \left(\frac{H_s^2}{\alpha^2} \right)} \quad (\text{A4})$$

441 When the interface tip is above the aquifer bed, the analytical solution of the freshwater-
442 seawater interface location and watertable elevation in Unit 2 are the same as equations (A1)
443 and (A2), respectively.

444



445 **Code/Data availability**

446 The paper is theoretical, and experimental data used in this study can be found in Memari

447 et al. (2020).



448 **Author contribution**

449 All authors contributed to the design of the research. ZL carried out data collation,
450 developed the analytical solutions and prepared the manuscript with contributions from all co-
451 authors. All authors contributed to the interpretation of the results and provided feedback.



452 **Competing interests**

453 The authors declare that they have no conflict of interest.



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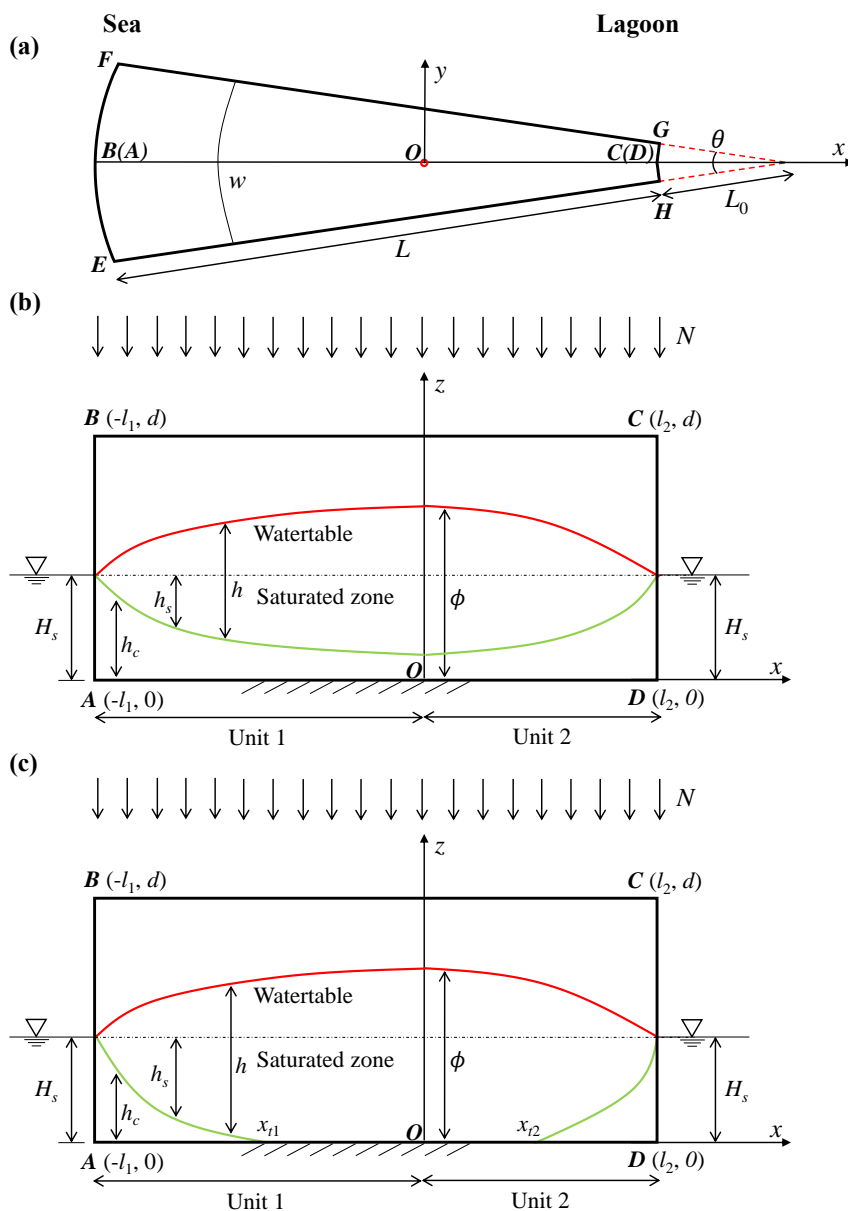


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614

615 **Figure 1.** Island with an annulus segment in the Namu Atoll (© Google Earth).

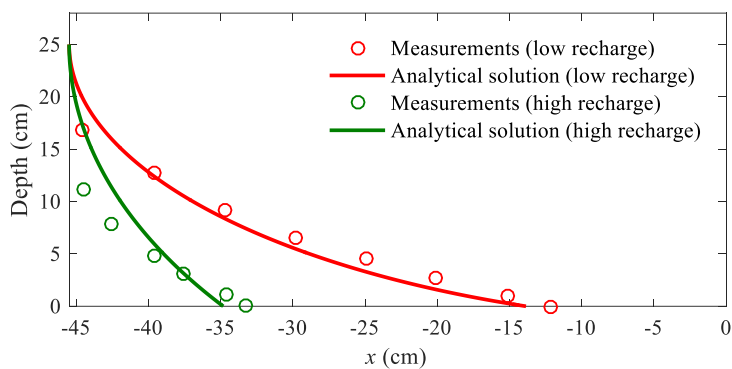


616

617 **Figure 2.** Conceptual model of an annulus segment aquifer (a slice of atoll islands). (a) Plain

618 view and (b, c) side view with the saltwater interface tip (b) above the aquifer bed and (c) on

619 the aquifer bed.

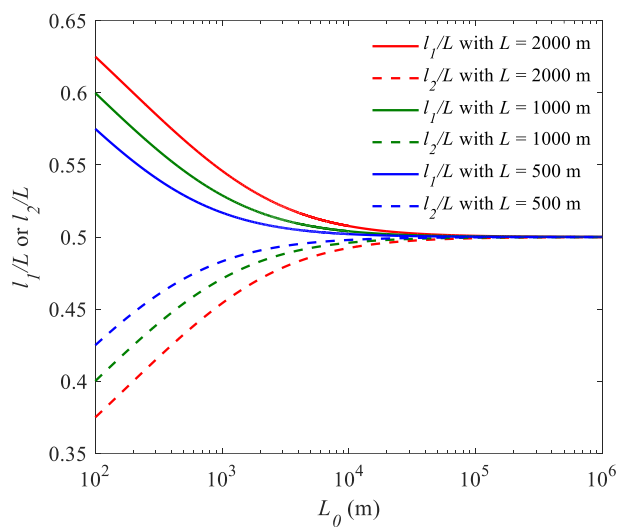


620

621 **Figure 3.** Comparison between analytical and experimental results of the freshwater-seawater

622 interface for different recharge events. Note that the left and right sides are the sea and no-flow

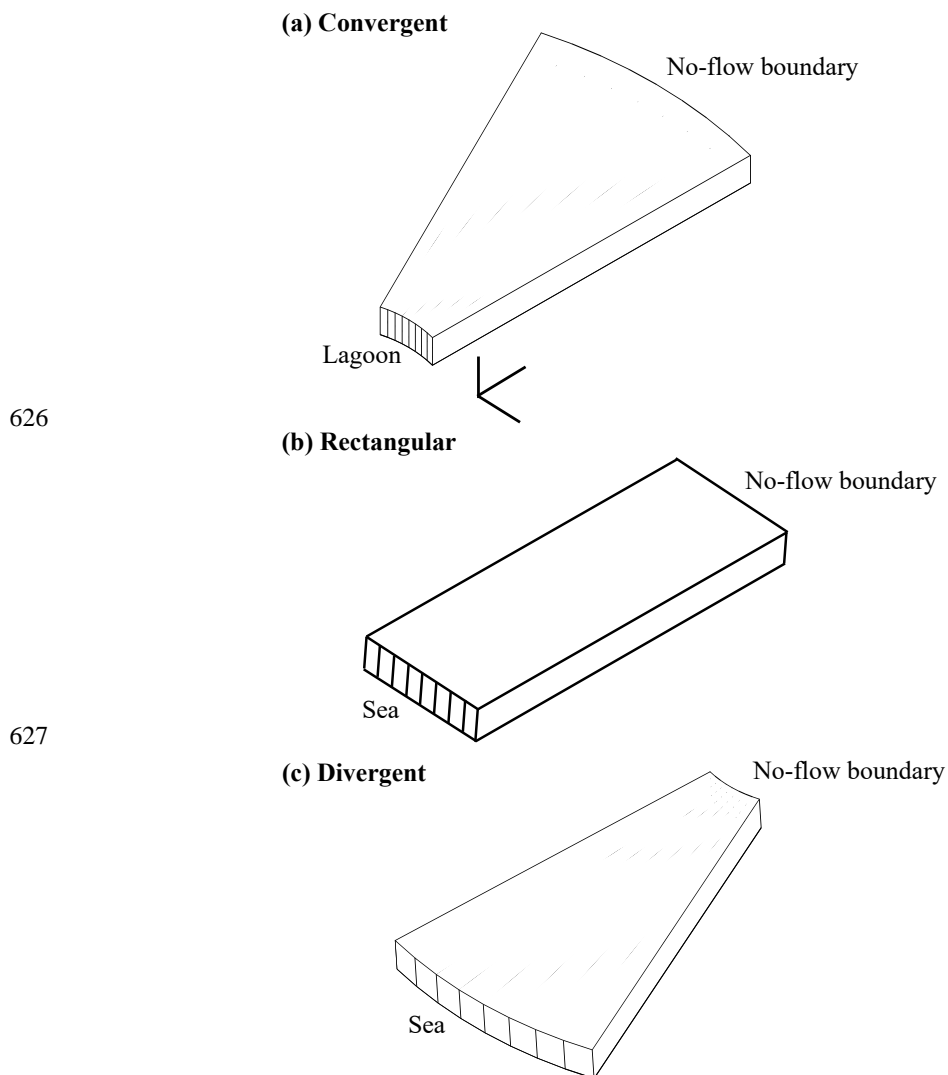
623 boundaries, respectively.



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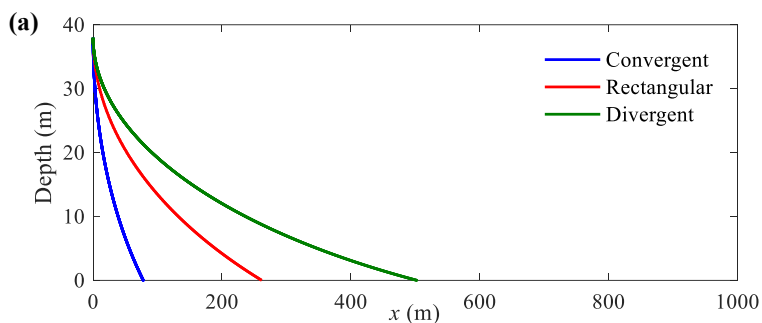
Figure 4. Widths of Unit 1 and Unit 2 versus L_0 for aquifers with different total width L .



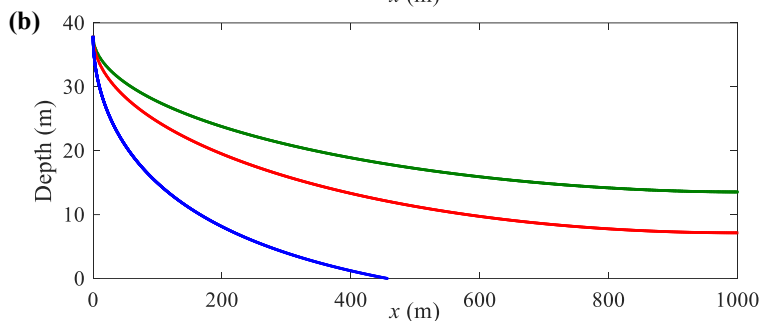
628
629 **Figure 5.** Three-dimensional view of (a) convergent (smaller side facing the lagoon), (b)
630 rectangular and (c) divergent aquifers (larger side facing the sea) compared in this study. Note
631 that the left and right sides of the aquifer are the sea and no-flow boundaries, respectively.
632



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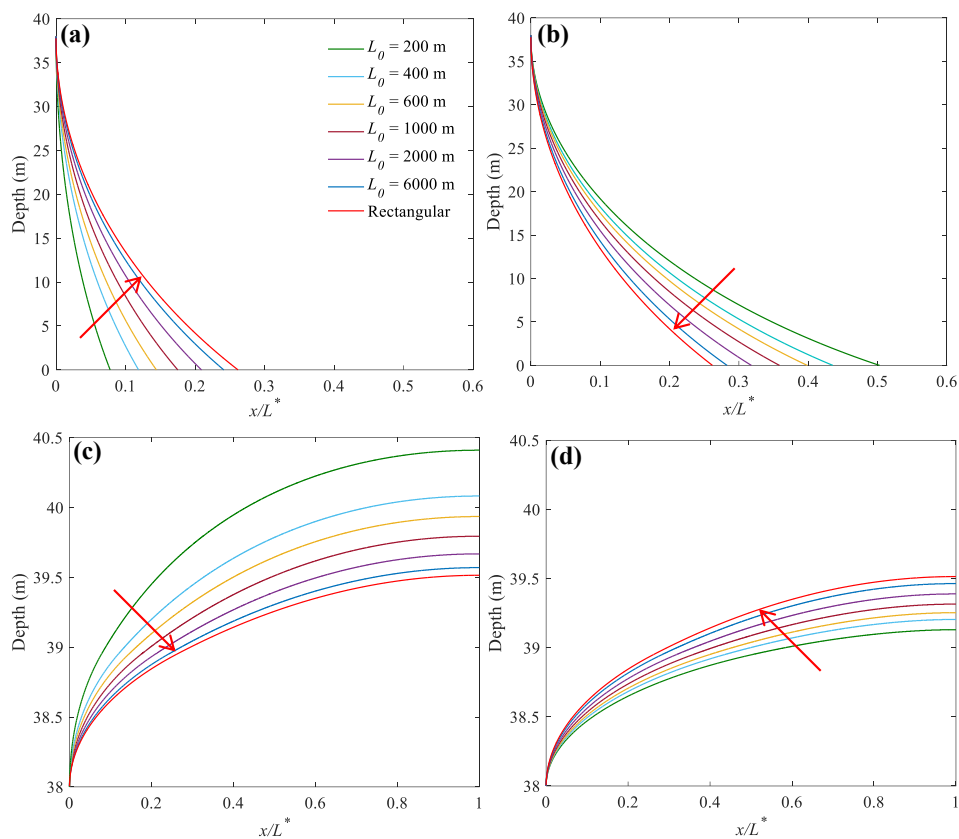


634



635 **Figure 6.** Freshwater-seawater interface predicted by analytical solutions for three different
636 aquifers with (a) high and (b) low recharge.

637



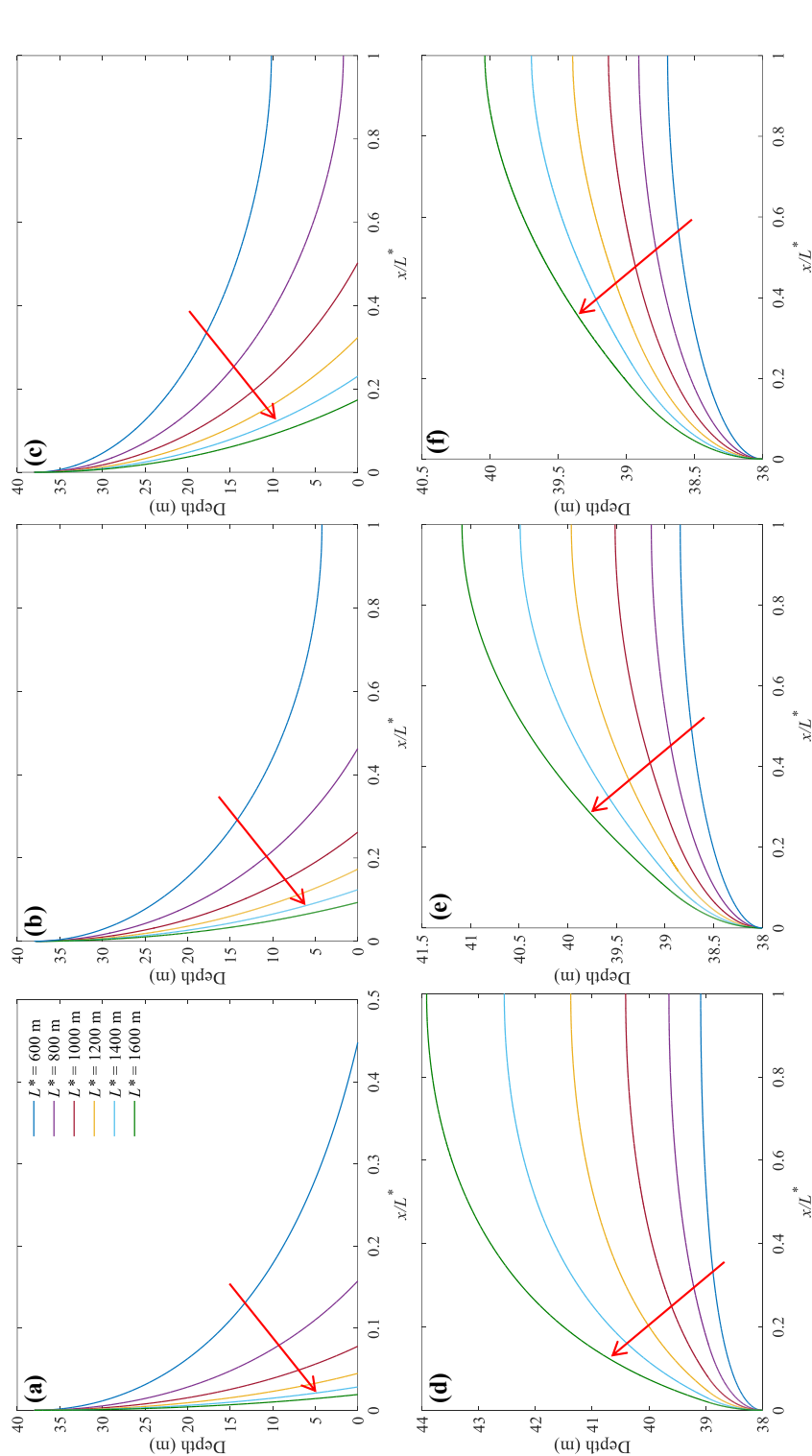
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639

640 **Figure 7.** Sensitivity of (a, b) the freshwater-seawater interface and (c, d) watertable to L_0 for

641 convergent (left panel) and divergent (right panel) aquifers. Note that red arrows point to the

642 increase of L_0 .



643

644

645 **Figure 8.** Sensitivity of (a-c) the freshwater-seawater interface and (d-f) watertable to L^* for convergent (left panel), rectangular (middle panel)

646 and divergent (right panel) aquifers. Note that red arrows point to the increase of L^* .



647 **Table 1.** List of parameters.

	NO.	L^* (m)	L_0 (m)	H_s (m)	d (m)	α (-)	n_e (-)	K_s (m/s)	N (m/s)
Experiments	1	0.455	0.1	0.25	0.28	62	0.4	1.23×10^{-2}	2.46×10^{-4}
	2	0.455	0.1	0.25	0.28	62	0.4	1.23×10^{-2}	1.08×10^{-4}
Hypothesized	1	1000	200	38	45	40	0.4	1.23×10^{-2}	1×10^{-6}
	2	1000	200	38	45	40	0.4	1.23×10^{-2}	3×10^{-7}
Cases	3	1000	-	38	45	40	0.4	1.23×10^{-2}	1×10^{-6}
	4	-	200	38	45	40	0.4	1.23×10^{-2}	1×10^{-6}

648 Note: - means that the parameter is varied.