

1                   Effects of aquifer geometry on seawater intrusion in annulus  
2   segment island aquifers

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## 22 **Abstract**

23         Seawater intrusion in island aquifers was considered analytically, specifically for annulus  
24 segment aquifers (ASAs), i.e., aquifers that (in plan) have the shape of an annulus segment.  
25         Based on the Ghijben-Herzberg and hillslope-storage Boussinesq equations, analytical  
26 solutions were derived for steady-state seawater intrusion in ASAs, with a focus on the  
27 freshwater-seawater interface and its corresponding watertable elevation. Predictions of the  
28 analytical solutions compared well with experimental data, and so they were employed to  
29 investigate the effects of aquifer geometry on seawater intrusion in island aquifers. Three  
30 different ASA geometries were compared: convergent (smaller side facing the lagoon),  
31 rectangular and divergent (larger side facing the sea). Depending on the aquifer geometry,  
32 seawater intrusion was found to vary greatly, such that the assumption of a rectangular aquifer  
33 to model an ASA can lead to poor estimates of seawater intrusion. Other factors being equal,  
34 compared with rectangular aquifers, seawater intrusion is more extensive and watertable  
35 elevation is lower in divergent aquifers, with the opposite tendency in convergent aquifers.  
36         Sensitivity analysis further indicated that the effects of aquifer geometry on seawater intrusion  
37 and watertable elevation vary with aquifer width and distance from the circle center to the  
38 inner arc (the lagoon boundary for convergent aquifers while the internal no-flow boundary  
39 for divergent aquifers). A larger aquifer width and distance from the circle center to the inner  
40 arc weaken the effects of aquifer geometry and hence differences in predictions for the three  
41 geometries become less pronounced.

42 **Keywords:** sharp-interface; steady-state analytical solution; atoll aquifer; annulus segment  
43 aquifer, seawater intrusion

44 **Key Points**

- 45 ➤ Analytical solutions of steady-state seawater intrusion were derived for annulus segment  
46 aquifers
- 47 ➤ Among three different aquifer geometries, divergent aquifers have the lowest watertable  
48 and hence the most extensive seawater intrusion
- 49 ➤ Aquifer geometry effects on seawater intrusion depend on the aquifer width and distance  
50 from the circle center to the inner arc

## 51 1. Introduction

52 Islands are extensively distributed throughout the world's oceans. Unfortunately, their  
53 groundwater resources are impacted by sea-level rise and increased demands. According to a  
54 recent estimate, there are approximately 65 million people living in oceanic islands where  
55 groundwater may be the only source of freshwater (Thomas et al., 2020). Fresh groundwater  
56 stored on oceanic islands is mainly from precipitation (usually in the form of a freshwater  
57 lens) and its availability varies due to different factors, e.g., island topography, rainfall  
58 patterns, tides, episodic storms and human activities (White & Falkland, 2010; Storlazzi et al.,  
59 2018). Seawater intrusion is thus an important issue due to its deleterious effect on oceanic  
60 island freshwater storage (e.g., Werner et al., 2017; Lu et al., 2019; Memari et al., 2020).

61 In contrast to coastal aquifers where seawater intrudes into freshwater from one direction  
62 only, seawater intrusion occurs from four directions for narrow strip islands and from all  
63 directions for circular islands. Over the past few decades, seawater intrusion in oceanic  
64 islands has been extensively investigated in field observations (e.g., Röper et al., 2013; Post et  
65 al., 2019), laboratory experiments (e.g., Stoeckl et al., 2015; Bedekar et al., 2019; Memari et  
66 al., 2020), numerical simulations (e.g., Lam, 1974; Gingerich et al., 2017; Liu & Tokunaga,  
67 2019) and analytical solutions (e.g., Fetter, 1972; Ketabchi et al., 2014; Lu et al., 2019).  
68 Among these, analytical solutions are effective tools to assess the extent of seawater intrusion  
69 (i.e., the location of the freshwater-seawater interface), although they cannot incorporate  
70 complex factors (e.g., dispersive mixing and transient oceanic dynamics) (Werner et al.,  
71 2013). The advantages of analytical solutions are that they are computationally efficient, can

72 be used as test cases for numerical models, and can reveal the explicit relationships between  
73 parameters that influence seawater intrusion (e.g., Fetter, 1972; Ketabchi et al., 2014; Liu et  
74 al., 2014; Lu et al., 2019;).

75 Based on the Dupuit-Forchheimer approximation (i.e., ignoring vertical flow) and the  
76 Ghijben-Herzberg equation (Drabbe & Badon Ghijben, 1889, English translation given by  
77 Post (2018); Herzberg, 1901), Fetter (1972) presented analytical solutions describing the  
78 freshwater-seawater interface location and watertable elevation in a circular island. Bailey et  
79 al. (2010) further compared these single-layered analytical solutions with field measurements,  
80 indicating that the analytical solutions perform well in estimating the freshwater-seawater  
81 interface location and watertable elevation. Fetter's solutions formed the foundation for many  
82 subsequent analytical studies on seawater intrusion in island aquifers. Again, for a single  
83 layer, Chesnaux and Allen (2008) and Greskowiak et al. (2013) developed analytical solutions  
84 to predict the steady-state groundwater age distribution in freshwater lenses. In addition, using  
85 single-layered analytical solutions, Morgan and Werner (2014) proposed vulnerability  
86 indicators of freshwater lenses under sea-level rise and recharge change.

87 Since aquifers are usually heterogeneous, the single-layer analytical solutions were  
88 subsequently extended to two-layered island aquifers. Vacher (1988) derived solutions for the  
89 freshwater-seawater interface location and watertable elevation for infinite-strip islands  
90 composed of different layers. Dose et al. (2014) conducted laboratory experiments to validate  
91 and confirm the reliability of analytical solutions proposed by Fetter (1972) and Vacher  
92 (1988). Ketabchi et al. (2014) extended Fetter's analytical solutions to calculate the

93 freshwater-seawater interface location and watertable elevation in two-layered circular islands  
94 subject to sea-level rise. Their results indicated that land-surface inundation caused by sea-  
95 level rise has a considerable impact on fresh groundwater lenses. Recently, Lu et al. (2019)  
96 derived analytical solutions for the freshwater-seawater interface location and watertable  
97 elevation for both strip and circular islands with two adjacent layers, i.e., a less permeable  
98 slice along the shoreline of an island, and a more permeable zone inland.

99 All the abovementioned analytical solutions apply to either strip or circular islands.  
100 According to the classification of sand dunes developed by Stuyfzand (1993; 2017), there are  
101 different island layouts that should be considered, e.g., where the shape of the island is an  
102 annulus segment, instead of a strip or circular disk (Figure 1). Annulus segment-shaped  
103 islands are found in various atolls (i.e., circular chains of islands surrounding a central  
104 lagoon) as found in the Pacific and Indian Oceans (Werner et al., 2017; Duvat, 2019).  
105 Nevertheless, analytical solutions of seawater intrusion are not yet available for annulus  
106 segment aquifers (ASAs). In general, ASAs are conceptually treated as a 2D cross section,  
107 similar to strip islands (e.g., Ayers & Vacher, 1986; Underwood et al., 1992; Bailey et al.,  
108 2009; Werner et al., 2017). Evidently, topography plays an important role in groundwater flow  
109 and hence seawater intrusion (e.g., Zhang et al., 2016; Liu & Tokunaga, 2019). It remains  
110 unclear whether analytical solutions of seawater intrusion for strip islands are appropriate for  
111 ASAs. It is moreover additionally unclear how island geometry affects the freshwater-  
112 seawater interface location and watertable elevation of ASAs.

113 In this study, analytical solutions are derived for steady-state seawater intrusion for ASAs,

114 with a focus on the freshwater-seawater interface location and its corresponding watertable  
115 elevation. After comparing their predictions with experimental data (Memari et al., 2020), the  
116 analytical solutions are employed to investigate the effects of aquifer geometry on the  
117 freshwater-seawater interface location and watertable elevation in ASAs.

## 118 **2. Conceptual Model**

119 Figure 2 shows the conceptual model of an ASA (a slice of an atoll island). The plan  
120 view of the model domain is represented as a sector ( $EFGH$ ) with an angle  $\theta$  (Figure 2a).  
121 **Radial flow only is considered.** The sea ( $EF$ ) and lagoon ( $HG$ ) boundaries are located at  $L +$   
122  $L_0$  [L] and  $L_0$  [L] from the circle center, respectively. Since the longitudinal length is usually  
123 much longer than the lateral length for an atoll island (Werner et al., 2017), seawater intrusion  
124 from the lateral sides ( $EH$  and  $FG$ , Figure 2a) is negligible in comparison to the longitudinal  
125 side, especially for the middle portion of an ASA. Therefore,  $EH$  and  $FG$  are treated as lateral  
126 no-flow boundaries. Note that treating the lateral sides as no-flow boundaries is often used in  
127 studies of freshwater lenses on atoll islands (e.g., Ayers & Vacher, 1986; Underwood et al.,  
128 1992; Bailey et al., 2009; Werner et al., 2017). The **side view** of the model domain is  
129 conceptualized as a rectangle ( $ABCD$ ) along the radial direction with dimensions of  $L$  [L]  
130 (width)  $\times d$  [L] (height) (Figure 2b, c).  $AD$  is the impermeable base while  $BC$  is the land  
131 surface through which aquifer recharge flows.

132 Both the sea and lagoon water levels are set to  $H_s$  [L], which results in an internal no-  
133 flow boundary (water divide, where the slope of the watertable is zero) between the sea and  
134 lagoon (location of the  $z$ -axis in Figure 2b,c). The segment between the sea and the internal

135 no-flow boundary is referred to as Unit 1, whereas the segment between the internal no-flow  
136 and lagoon boundaries is referred to as Unit 2 (Figure 2). The widths of Units 1 and 2 are  $l_1$   
137 [L] and  $l_2$  [L], respectively. In addition, the flow is asymmetrical in Units 1 and 2, with  
138 divergent flow (the aquifer width  $w$  [L] increases along the flow direction) in Unit 1 and  
139 convergent flow ( $w$  decreases along the flow direction) in Unit 2.

140 The  $x$ - $z$  coordinate origin is placed at the intersection of the internal no-flow boundary  
141 and impermeable base, with the  $x$ -axis pointing to the circle center and the  $z$ -axis pointing  
142 vertically upward. Further,  $\phi$  [L] is the watertable height,  $h$  [L] is the vertical distance  
143 between the watertable and the interface,  $h_s$  [L] is the vertical distance between the sea level  
144 and the interface, and  $h_c = H_s - h_s$  [L] is the vertical distance from the impermeable base to  
145 the interface for given  $x$  (Figure 2b,c). Constant recharge into the saturated zone,  $N$  [ $LT^{-1}$ ], is  
146 assumed. There are two possibilities for the interface tip (i.e., the location where the  
147 freshwater-seawater interface connects to the  $z$ -axis or the bottom boundary): above the  
148 aquifer bed (Figure 2b) or on the aquifer bed (Figure 2c). The  $x$ -coordinates of the interface  
149 tip in Units 1 and 2 are denoted as  $x_{t1}$  [L] and  $x_{t2}$  [L], respectively (Figure 2c). Note that  $x_{t1} =$   
150  $x_{t2} = 0$  when the interface tip is above the aquifer bed, as in Figure 2b.

151 Consistent with previous studies (e.g., Ketabchi et al., 2014; Lu et al., 2016; 2019), the  
152 following assumptions are made: (1) steady-state flow, (2) sharp freshwater-seawater  
153 interface, (3) homogeneous and isotropic aquifer, (4) negligible unsaturated flow, (5) recharge  
154 is less than the saturated hydraulic conductivity (else overland flow will appear following



155 ponding), and (6) vertical flow in the saturated zone is negligible (Dupuit-Forchheimer  
156 approximation).

### 157 3. Analytical Solutions

158 Groundwater flow in an ASA (Figure 2) can be described as (Paniconi et al., 2003; Troch  
159 et al., 2003),

$$160 \quad -\frac{\partial}{\partial x}(wq) + Nw = \frac{\partial S}{\partial t} \quad (1)$$

161 where  $q$  [ $L^2T^{-1}$ ] is the Darcy flux per unit length along the aquifer,  $x$  [L] represents the  
162 distance from the circle center to the arc,  $S$  [ $L^2$ ] is the total water storage per unit distance  
163 along the aquifer, and  $t$  [T] is time. Equation (1) is the so-called the hillslope-storage  
164 Boussinesq equation and was first proposed by Troch et al. (2003). For a given radial distance  
165  $x$ , this equation assumes that the velocity is the same everywhere on the arc ( $w$ ). Based on this  
166 assumption, the 3D flow problem can be simplified to 1D, making it possible to consider  
167 geometry effects analytically. Paniconi et al. (2003) validated equation (1) by comparing it  
168 with a 3D Richards' equation model and found that predictions of equation (1) matched well  
169 those of the 3D model for nine different geometries. Subsequently, equation (1) was used to  
170 for further analyses (Hilbert et al., 2005, 2007; Hazenberg et al., 2015, 2016; Kong et al.,  
171 2016; Luo et al., 2018). At steady state, equation (1) reduces to,

$$172 \quad -\frac{\partial}{\partial x}(wq) + Nw = 0 \quad (2)$$

173 According to Darcy's law and the Dupuit-Forchheimer approximation, the freshwater  
174 flux in the aquifer segment between the seaward boundary and interface tip can be calculated  
175 as ( $\phi$  is independent of  $z$ ),

$$q = -\int_{h_c}^{\phi} K_s \frac{d\phi}{dx} dz = -K_s (\phi - h_c) \frac{d\phi}{dx} \quad (3)$$

where  $K_s$  [ $LT^{-1}$ ] is the saturated hydraulic conductivity.

### 3.1. Interface Tip above the Aquifer Bed

We first consider the situation where the interface tip is above the aquifer bed (Figure 2b). In Unit 1 where  $w = \theta(L_0 + l_2 - x)$ , substituting equation (3) into equation (2) and then integrating gives,

$$-\frac{1}{2} \left[ (L_0 + l_2 - x)^2 - (L_0 + l_2)^2 \right] N = -(L_0 + l_2 - x) K_s (\phi - h_c) \frac{d\phi}{dx} \quad (4)$$

According to the Ghijben-Herzberg equation, the vertical thickness of the freshwater zone ( $h$ ) in the interface zone is given by,

$$h = \phi - h_c = (1 + \alpha)(\phi - H_s) \quad (5)$$

where  $\alpha = \rho_f / (\rho_s - \rho_f)$  is the dimensionless density difference, and  $\rho_f$  [ $ML^{-3}$ ] and  $\rho_s$  [ $ML^{-3}$ ] are the freshwater and seawater densities, respectively. Substitution of equation (5) into equation (4) yields,

$$-\frac{1}{2} \left[ (L_0 + l_2 - x)^2 - (L_0 + l_2)^2 \right] N = -K_s (L_0 + l_2 - x) (1 + \alpha) (\phi - H_s) \frac{d\phi}{dx} \quad (6)$$

Rearranging equation (6) produces,

$$-\frac{(L_0 + l_2 - x)N}{2} + \frac{N(L_0 + l_2)^2}{2(L_0 + l_2 - x)} = -K_s (1 + \alpha) (\phi - H_s) \frac{d\phi}{dx} \quad (7)$$

Integrating equation (7) leads to,

$$-\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x) - \frac{1}{2} (L_0 + l_2) N x + \frac{1}{4} N x^2 + C_1 = -K_s (1 + \alpha) \frac{(\phi - H_s)^2}{2} \quad (8)$$

where  $C_1$  is the integration constant that is determined by the sea boundary condition (i.e.,

$$x = -l_1, \quad \phi = H_s),$$

196 
$$C_1 = \frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 + l_1) - \frac{1}{2}(L_0 + l_2)l_1 N - \frac{1}{4}l_1^2 N \quad (9)$$

197 The relation between  $h_s$  and  $\phi$  is given by,

198 
$$h_s = \alpha(\phi - H_s) \quad (10)$$

199 Combining equation (8) with equation (10) and eliminating  $\phi$  yields,

200 
$$-\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x) - \frac{1}{2}(L_0 + l_2)Nx + \frac{1}{4}Nx^2 + C_1 = -K_s(1 + \alpha)\frac{h_s^2}{2\alpha^2} \quad (11)$$

201 Equation (11) gives the freshwater-seawater interface location in Unit 1 once  $l_1$  and  $l_2$  are  
202 determined.

203 Equation (8) applies to Unit 2 by replacing  $C_1$  with  $C_2$ ,

204 
$$-\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x) - \frac{1}{2}(L_0 + l_2)Nx + \frac{1}{4}Nx^2 + C_2 = -K_s(1 + \alpha)\frac{(\phi - H_s)^2}{2} \quad (12)$$

205 where  $C_2$  is chosen to satisfy the lagoon boundary condition ( $x = l_2$ ,  $\phi = H_s$ ),

206 
$$C_2 = \frac{(L_0 + l_2)^2 N}{2} \ln(L_0) + \frac{1}{2}(L_0 + l_2)l_2 N - \frac{1}{4}l_2^2 N \quad (13)$$

207 Combining equations (10) and (12) and eliminating  $\phi$  leads to,

208 
$$-\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x) - \frac{1}{2}(L_0 + l_2)Nx + \frac{1}{4}Nx^2 + C_2 = -K_s(1 + \alpha)\frac{h_s^2}{2\alpha^2} \quad (14)$$

209 Equation (14) gives the freshwater-seawater interface location in Unit 2 once  $l_2$  is  
210 determined. Since the sea level and lagoon water level are the same, an internal no-flow

211 boundary exists between the sea and lagoon, i.e.,

212 
$$x = 0, \quad (h_s)_{unit1} = (h_s)_{unit2} \quad (15)$$

213 where  $(h_s)_{unit1}$  and  $(h_s)_{unit2}$  represent  $h_s$  in Units 1 and 2, respectively.

214 Combining equations (11), (14) and (15) leads to expressions for  $l_1$  and  $l_2$ ,

$$l_1 = L + L_0 - \sqrt{\frac{2LL_0 + L^2}{2\ln(L + L_0) - 2\ln(L_0)}} \quad (16)$$

$$l_2 = \sqrt{\frac{2LL_0 + L^2}{2\ln(L + L_0) - 2\ln(L_0)}} - L_0 \quad (17)$$

As indicated by equations (16) and (17), the internal no-flow boundary between the sea and lagoon only depends on  $L$  and  $L_0$ . For known  $l_1$  and  $l_2$ , equations (11) and (14) can be employed to predict the freshwater-seawater interface location in Units 1 and 2, respectively.

Once the interface location is determined,  $h$  and  $\phi$  are given by,

$$h = \frac{1 + \alpha}{\alpha} h_s \quad (18)$$

$$\phi = \frac{h_s}{\alpha} + H_s \quad (19)$$

### 3.2. Interface Tip on the Aquifer Bed

When the interface tip is on the aquifer bed, the location of the internal no-flow boundary remains the same as for the interface tip above the aquifer bed. The freshwater-seawater interface for Units 1 and 2 can be determined by equations (11) and (14), respectively. Then, from equation (18),  $h$  at the aquifer segment between the sea boundary and the interface tip is determined. To calculate  $h$  for the aquifer segment between the interface tip and the internal no-flow boundary, the  $x$ -coordinate of the interface tip is found. At the interface tip of Unit 1 ( $x = x_{t1}$ ),

$$h_s = H_s \quad (20)$$

$$\phi = \frac{1 + \alpha}{\alpha} H_s \quad (21)$$

With equations (11) and (21),  $x_{t1}$  is given by,

$$-\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x_{t1}) - \frac{1}{2}(L_0 + l_2) N x_{t1} + \frac{1}{4} N x_{t1}^2 = -C_1 - K_s (1 + \alpha) \frac{H_s^2}{2\alpha^2} \quad (22)$$

235 Let,

$$236 \quad a = \frac{1}{4}N \quad (23a)$$

$$237 \quad b = -\frac{1}{2}(L_0 + l_2)N \quad (23b)$$

$$238 \quad c = -\frac{(L_0 + l_2)^2 N}{2} \quad (23c)$$

239 and

$$240 \quad m = -C_1 - K_s(1 + \alpha)\frac{H_s^2}{2\alpha^2} \quad (23d)$$

241 then equation (22) becomes,

$$242 \quad ax_{t1}^2 + bx_{t1} + c \ln(L_0 + l_2 - x_{t1}) = m \quad (24)$$

243 which is solved by a root-finding method.

244 The freshwater discharge for the aquifer segment between the interface tip and the  
245 internal no-flow boundary is calculated as,

$$246 \quad -\frac{1}{2}\left[(L_0 + l_2 - x)^2 - (L_0 + l_2)^2\right]N = -(L_0 + l_2 - x)K_s\phi\frac{d\phi}{dx} \quad (25)$$

247 Repeating the steps from equations (4) to (8) gives,

$$248 \quad -\frac{(L_0 + l_2)^2 N}{2} \ln(L_0 + l_2 - x) - \frac{1}{2}(L_0 + l_2)Nx + \frac{1}{4}Nx^2 + C_3 = -\frac{K_s}{2}\phi^2 \quad (26)$$

249 where  $C_3$  is determined by substituting equation (21) into equation (26). Then, equation (26)

250 can be adopted to calculate  $h$  for the segment between the interface tip and the internal no-

251 flow boundary where  $h = \phi$ .

252 Similarly, the  $x$ -coordinate of the interface tip in Unit 2 ( $x_{t2}$ ) is obtained by substituting

253 equation (20) into equation (14). Then, the watertable ( $h$ ) of the aquifer segment between the

254 interface tip and the internal no-flow boundary for Unit 2 is computed by repeating the steps

255 from equations (22) to (26).

## 256 **4. Results and Discussion**

### 257 **4.1. Validation of the Analytical Solutions**

258 The analytical solutions were validated by comparing their predictions with experimental  
259 data compiled from Memari et al. (2020), who reported experiments carried out using a 15°  
260 radial tank. The tank contained three distinct chambers: internal no-flow boundary condition,  
261 porous medium and constant-head boundary condition (i.e., sea or lagoon). The internal no-  
262 flow and seaward boundaries were respectively located at 10 cm and 55.5 cm from the circle  
263 center, i.e., 45.5 cm from the internal no-flow boundary to the constant-head boundary along  
264 the radial direction. Note that the experimental tank only corresponds to Unit 1 of the radial  
265 aquifer with  $l_1 = 45.5$  cm and  $l_2 = 0$ , so the analytical results were calculated using  
266 equations (11) and (24). The thicknesses of the porous medium and sea level were 28 and 25  
267 cm, respectively, with  $K_s = 1.23 \times 10^{-2}$  m s<sup>-1</sup>. The measured saltwater and freshwater densities  
268 were respectively 1.015 and 0.999 g ml<sup>-1</sup>, leading to  $\alpha = 62$ . Two different recharge events  
269 with constant  $N$ ,  $2.46 \times 10^{-4}$  and  $1.08 \times 10^{-4}$  m s<sup>-1</sup>, were considered in the experiments.

270 Figure 3 shows the comparison between analytical and experimental results of the  
271 freshwater-seawater interface for different recharge events. In general, the analytical solution  
272 predicts the freshwater-seawater interface well for both recharge events, despite there being  
273 some differences between the analytical results and the measurements, particularly in the zone  
274 near the constant-head boundary ( $x = -45$  cm). These deviations are likely due to assumptions  
275 made in the analytical solution, i.e., (i) a sharp freshwater-seawater interface, (ii) ignoring the

276 effect of freshwater discharge, and (iii) neglecting the vertical flow (the Dupuit-Forchheimer  
277 approximation).

#### 278 **4.2. Effects of Aquifer Geometry on Seawater Intrusion**

279 Previous studies showed that boundary conditions play a critical role in estimates of  
280 seawater intrusion (Werner & Simmons, 2009; Lu et al., 2016). Therefore, the internal no-  
281 flow boundary between the sea and lagoon was examined for various ASAs. As indicated by  
282 equations (16) and (17), this internal no-flow boundary depends only on  $L$  and  $L_0$ . The values  
283 of  $l_1$  and  $l_2$  calculated respectively from equations (16) and (17) are shown in Figure 4 for  
284 three typical values of  $L$  (500, 1000 and 2000 m) with  $L_0$  varying from  $10^2$  to  $10^6$  m. In  
285 general, the internal no-flow boundary deviates from the middle of the ASA. When  $L_0$  is less  
286 than  $10^5$  m,  $l_1$  is larger than  $l_2$  for the three different values of  $L$ , indicating an internal no-  
287 flow boundary closer to the lagoon boundary. For example, taking  $L = 2000$  m and  $L_0 = 100$  m  
288 leads to  $l_1 = 1240$  m and  $l_2 = 760$  m, with a deviation of 240 m (12% of 2000 m) from the  
289 middle of the ASA. When  $L_0$  exceeds  $10^5$  m, however, the location of the internal no-flow  
290 boundary can be approximated as being at the middle of the ASA for all considered values of  
291  $L$ . This is in contrast to strip and circular aquifers where the internal no-flow boundary is  
292 always in the middle of aquifers due to symmetry.

293 Since the internal no-flow boundary location between the sea and lagoon deviates from  
294 the middle of the ASA, we expect aquifer geometry to play a significant role in controlling  
295 seawater intrusion. As mentioned previously, ASAs can be convergent (Unit 1) or divergent  
296 aquifers (Unit 2) where the extent of seawater intrusion may be different. However, for strip

297 aquifers, both Units 1 and 2 are rectangular with the same extent of seawater intrusion.  
298 Therefore, three geometries were compared in this study: convergent, rectangular and  
299 divergent (Figure 5). These geometries have been widely examined in hillslope hydrology  
300 regrading to the effects of aquifer geometry on runoff generation (Troch et al., 2003; Kong et  
301 al., 2016; Luo et al., 2018). To present the results more conveniently, we replaced the  $x$ - $z$   
302 coordinate origin at the intersection of the constant-head boundary (sea or lagoon) and the  
303 impermeable base, with the  $x$ -axis pointing horizontally to the internal no-flow boundary and  
304 the  $z$ -axis vertically upward (Figure 5). In addition, the distance between the constant-head  
305 boundary and the internal no-flow boundary (aquifer width) is denoted as  $L^*$  (Figure 5) while  
306 the other parameters remain the same.

307       Following previous studies (e.g., Lu et al., 2016; 2019), different cases were selected to  
308 show the effects of aquifer geometry on seawater intrusion (Cases 1 and 2 in Table 1).  
309 According to Werner et al. (2017), the width of atoll islands generally varies from 100 to 1500  
310 m along the radial direction. In order to focus on the effects of aquifer geometry on seawater  
311 intrusion, the same  $L^*$  and  $L_0$  were assumed for the three aquifers, with  $L^*$  and  $L_0$  equal to  
312 1000 and 200 m, respectively. Note that  $L_0$  is the distance from the circle center to the lagoon  
313 boundary for convergent aquifers, whereas it represents the distance from the circle center to  
314 internal no-flow boundary for divergent aquifers hereafter. The sand characteristics were the  
315 same as in the experiments of Memari et al. (2020). Two recharge events were considered  
316 (Cases 1 and 2, Table 1). The freshwater-seawater interface was calculated using the  
317 analytical solutions for the three different aquifers. Note that the Appendix presents analytical



318 solutions for seawater intrusion in strip aquifers deduced from Lu et al. (2019).

319 Figure 6 shows the freshwater-seawater interface calculated for Cases 1 and 2. As can be  
320 seen, the extent of seawater intrusion is noticeably different for the three aquifer geometries.  
321 For high recharge ( $1 \times 10^{-6} \text{ m s}^{-1}$ ), the interface tip is located at around 500 m for the  
322 divergent aquifer, which is about twice the value of the rectangular aquifer and six times the  
323 value for the convergent aquifer (Figure 6a). When the recharge decreases to  $3 \times 10^{-7} \text{ m s}^{-1}$ ,  
324 the interface tip moves further landward for the three aquifers as expected, but the difference  
325 between results is still great (Figure 6b). The interface tip is displaced above the aquifer bed  
326 for both the rectangular and divergent aquifers, while it remains on the aquifer bed for the  
327 convergent aquifer. Regardless of the recharge rate, the most landward freshwater-seawater  
328 interface occurs in the divergent aquifer and vice versa for the convergent aquifer. This  
329 underlines that aquifer geometry plays a major role in controlling seawater intrusion and  
330 hence it is necessary to account for aquifer geometry in analyses of seawater intrusion.

### 331 **4.3. Sensitivity Analysis**

332 A sensitivity analysis was conducted to investigate to what extent aquifer geometry  
333 affects seawater intrusion. Since we focus on the effects of aquifer geometry on the locations  
334 of the freshwater-seawater interface and watertable, values of  $L_0$  and  $L^*$  were varied, with  
335 other parameters kept constant. When conducting the sensitivity analysis of  $L_0$ ,  $L^*$  was fixed  
336 at 1000 m, which is a typical value for ASAs (Werner et al., 2017). Figure 7 shows the  
337 sensitivity of the locations of the freshwater-seawater interface and watertable to changes in  
338  $L_0$  (Case 3, Table 1). The freshwater-seawater interface and watertable elevation are

339 independent of  $L_0$  for rectangular aquifers (Appendix). However, the freshwater-seawater  
340 interface and watertable elevation differ greatly when varying  $L_0$  for both convergent and  
341 divergent aquifers, highlighting that  $L_0$  plays an important role in affecting seawater intrusion.  
342 Specifically, as  $L_0$  increases, the freshwater-seawater interface moves more landward (larger  
343  $x/L^*$ , Figure 7a) and its corresponding watertable elevation decreases (Figure 7c) for  
344 convergent aquifers. In contrast, for divergent aquifers increasing  $L_0$  moves the freshwater-  
345 seawater interface more seaward (smaller  $x/L^*$ , Figure 7b) and its corresponding watertable  
346 elevation increases (Figure 7d). For a given  $L_0$ , divergent aquifers have the largest extent of  
347 seawater intrusion and the lowest watertable elevation, and conversely for convergent aquifers  
348 (Figure 7).

349       Regardless of the freshwater-seawater interface and watertable elevation, the deviation  
350 between rectangular aquifers and divergent or convergent aquifers is significant when  $L_0$  is  
351 less than 2000 m (Figure 7). For example, the  $x$ -coordinate of the interface tip ( $z = 0$ ) is 262 m  
352 for the rectangular aquifer at  $L_0 = 200$  m, whereas it is 78 (31% of that in the rectangular  
353 aquifer) and 500 m (191% of that in the rectangular aquifer) for the convergent and divergent  
354 aquifers, respectively. As  $L_0$  increases, the deviation between the three aquifers decreases.  
355 When  $L_0 = 2000$  m, the  $x$ -coordinate of the interface tip is 262, 209 (80% of that in the  
356 rectangular aquifer) and 318 m (121% of that in the rectangular aquifer) for the rectangular,  
357 convergent and divergent aquifers, respectively. As  $L_0$  increases to 6000 m, the freshwater-  
358 seawater interface and watertable elevation of both convergent and divergent aquifers tend to  
359 those of rectangular aquifers, i.e., geometry effects decrease with increasing  $L_0$ . These results

360 highlight the critical role played by the shape of aquifers. As a result, ignoring the aquifer  
361 geometry may lead to an inappropriate management strategy for groundwater resources in  
362 atoll islands.

363 The sensitivity of the freshwater-seawater interface and watertable elevation to  $L^*$  was  
364 investigated by varying  $L^*$  from 600 to 1600 m while fixing  $L_0$  to 200 m (Case 4, Table 1). As  
365 shown in Figure 8, contrary to the results for varying  $L_0$ , in this case the freshwater-seawater  
366 interface and watertable elevation in all three topographies are related to  $L^*$ . Again, the extent  
367 of seawater intrusion is greatest in divergent aquifers and least in convergent aquifers for  
368 given  $L^*$ . When  $L^*$  increases, the freshwater-seawater interface moves seaward and the  
369 watertable elevation increases, regardless of aquifer geometry, i.e., the seawater intrusion  
370 decreases (Figures 8a-c). This is because the total freshwater flux increases with increasing  
371  $L^*$ , leading to a higher hydraulic gradient and hence less seawater intrusion (Figures 8d-f).  
372 Moreover, an increase in  $L^*$  reduces the differences in the seawater intrusion distance among  
373 the three geometries, i.e., the effects of aquifer geometry on seawater intrusion are more  
374 significant at small  $L^*$ . However, even at the maximum  $L^*$  considered (1600 m), the deviation  
375 between three aquifers remains significant: The  $x$ -coordinate of the interface tip is about 148  
376 m for the rectangular aquifer, whereas it is about 32 (22% of that in the rectangular aquifer)  
377 and 278 m (188% of that in the rectangular aquifer) for the convergent and divergent aquifers,  
378 respectively. Both  $L_0$  and  $L^*$  can greatly impact seawater intrusion estimates for divergent and  
379 convergent aquifers, highlighting the necessity to include geometry effects in analytical  
380 solutions of seawater intrusion.

## 5. Conclusions

Based on the Ghijben-Herzberg and hillslope-storage Boussinesq equations, we derived analytical solutions of steady-state seawater intrusion for ASAs, with a focus on the freshwater-seawater interface and its corresponding watertable elevation as affected by recharge. After comparing with experimental data of Memari et al. (2020), the analytical solutions were employed to examine the effects of aquifer geometry on seawater intrusion in island aquifers. Three different shapes of island aquifer were compared: convergent, rectangular and divergent. The results lead to the following conclusions:

- The presented analytical solutions perform well in predicting the experimental freshwater-seawater interface, suggesting that these analytical solutions can predict seawater intrusion reasonably in different aquifer geometries.
- Island geometry plays a significant role in affecting the freshwater-seawater interface and watertable elevation. Other factors being equal, the extent of seawater intrusion is greatest in divergent aquifers, and conversely least in convergent aquifers. In contrast, the watertable elevation is lowest in divergent aquifers and highest in convergent aquifers.
- The effects of aquifer geometry on seawater intrusion are dependent on the aquifer width and distance from the circle center to the internal no-flow boundary (Figures 7 and 8). A larger aquifer width and distance from the circle center to the inner arc (the lagoon boundary for convergent aquifers while the internal no-flow boundary for divergent aquifers) weakens the role played by aquifer geometry and hence lead to a smaller deviation of the extent of seawater intrusion between the three topographies.

402 Real island aquifers are expected to exhibit more complexity than considered here, e.g.,  
403 that will have more complex shapes and are subjected to transient flow conditions caused by  
404 tides, waves and groundwater pumping (Mantoglou et al. 2003; Pool & Carrera., 2011;  
405 Werner et al., 2013). In addition, since the experimental scale of Memari et al. (2020) is  
406 necessarily small, future experiments and field data are needed to further validate and  
407 facilitate the analytical solutions. Despite this, the new analytical solutions, validated against  
408 experiments, can be used as a tool for rapid estimation of seawater intrusion in ASAs once  
409 known island geometry and corresponding soil properties are given.

410 **Appendix: Analytical Solutions for Rectangular Aquifers**

411 For rectangular aquifers, the seawater intrusion in Unit 1 is identical to that in Unit 2  
 412 because of symmetry. With the interface tip on the aquifer bed, analytical solutions for the  
 413 freshwater-seawater interface ( $h_s$ ), watertable elevation ( $h$ ), and  $x$ -coordinate of the interface  
 414 tip in Unit 2 ( $x_{t2}$ ) can be respectively written as (Lu et al., 2019),

$$415 \quad h_s = \alpha \sqrt{\frac{N}{(1+\alpha)K_s} \left( \frac{L^2}{4} - x^2 \right)} \quad (\text{A1})$$

$$416 \quad h = \begin{cases} \sqrt{\frac{N}{K_s} (x_{t2}^2 - x^2) + \left( \frac{H_s}{\alpha} + H_s \right)} & 0 \leq x \leq x_{t2} \\ \sqrt{\frac{N}{(1+\alpha)K_s} \left( \frac{L^2}{4} - x^2 \right) + H_s} & x_{t2} < x \leq \frac{L}{2} \end{cases} \quad (\text{A2})$$

$$417 \quad x_{t2} = \sqrt{\frac{L^2}{4} - \frac{(1+\alpha)K_s}{N} \left( \frac{H_s^2}{\alpha^2} \right)} \quad (\text{A3})$$

418 When the interface tip is above the aquifer bed, the analytical solution for the freshwater-  
 419 seawater interface location and watertable elevation in Unit 2 are the same as equations (A1)  
 420 and (A2), respectively.

421 **Code/Data availability**

422 Experimental data used in this study are compiled from Memari et al. (2020).

423 **Author contributions**

424 All authors contributed to the design of the research. ZL carried out data collation,  
425 developed the analytical solutions and prepared the manuscript with contributions from all  
426 co-authors. All authors contributed to the interpretation of the results and provided feedback.



427 **Competing interests**

428 The authors declare that they have no conflicts of interest.

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**Table 1.** List of parameters use in different simulations.

	No.	$L^*$ (m)	$L_0$ (m)	$H_s$ (m)	$d$ (m)	$\alpha$ (-)	$K_s$ (m s <sup>-1</sup> )	$N$ (m s <sup>-1</sup> )
Cases	1	1000	200	38	45	40	$1.23 \times 10^{-2}$	$1 \times 10^{-6}$
	2	1000	200	38	45	40	$1.23 \times 10^{-2}$	$3 \times 10^{-7}$
	3	1000	†	38	45	40	$1.23 \times 10^{-2}$	$1 \times 10^{-6}$
	4	†	200	38	45	40	$1.23 \times 10^{-2}$	$1 \times 10^{-6}$

†The parameter is varied: The range of  $L_0$  is from 200 to 6000 m, whereas the range of  $L^*$  is

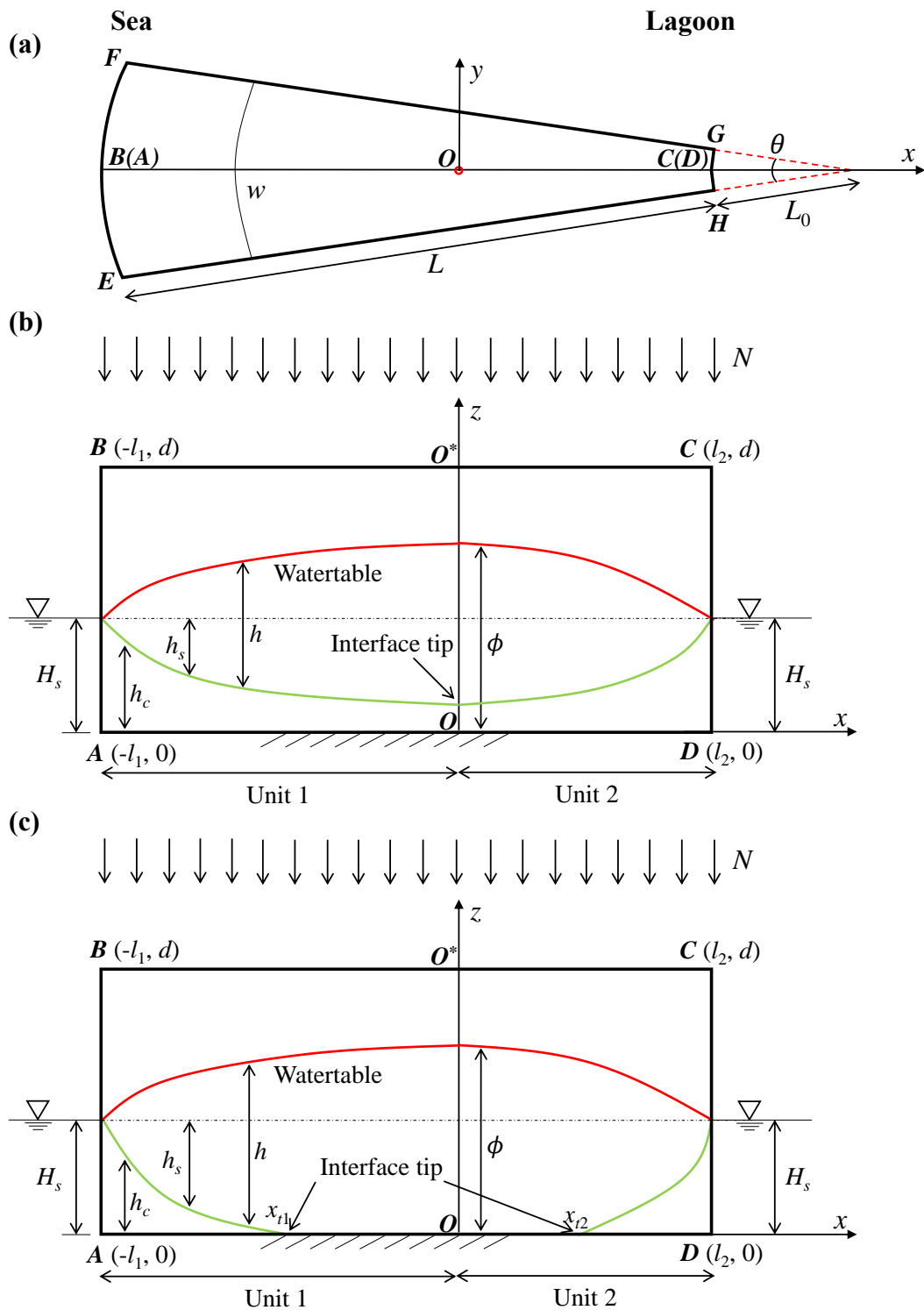
from 600 to 1600 m.



587

588 **Figure 1.** Island with an annulus segment in the Namu Atoll, Marshall Islands (© Google

589 Earth).



590

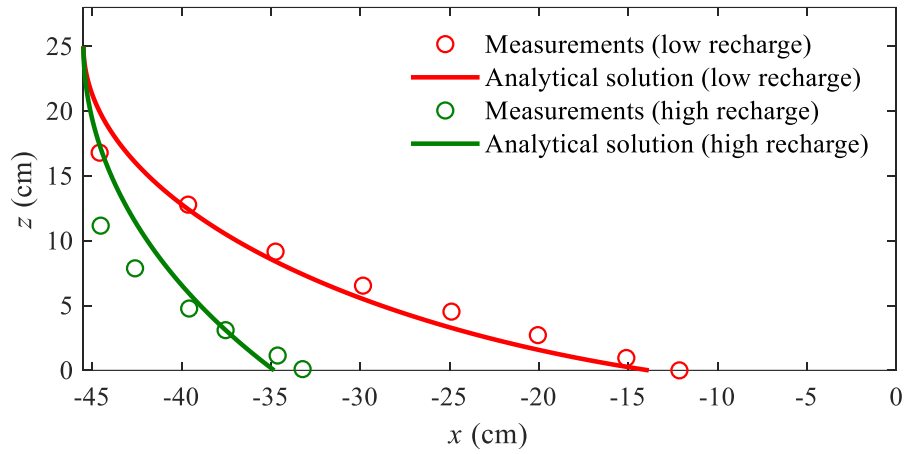
591 **Figure 2.** Conceptual model of an annulus segment aquifer (a slice of an atoll island). (a) Plan

592 view and (b, c) side view with the saltwater interface tip (b) above the aquifer bed (single

593 location) and (c) on the aquifer bed (two locations). In (a), the sea boundary is on  $EF$  and the

594 atoll lagoon boundary is on  $HG$ ; In (b) and (c),  $AD$  is the impermeable base and  $OO^*$  is the

595 internal no-flow boundary.

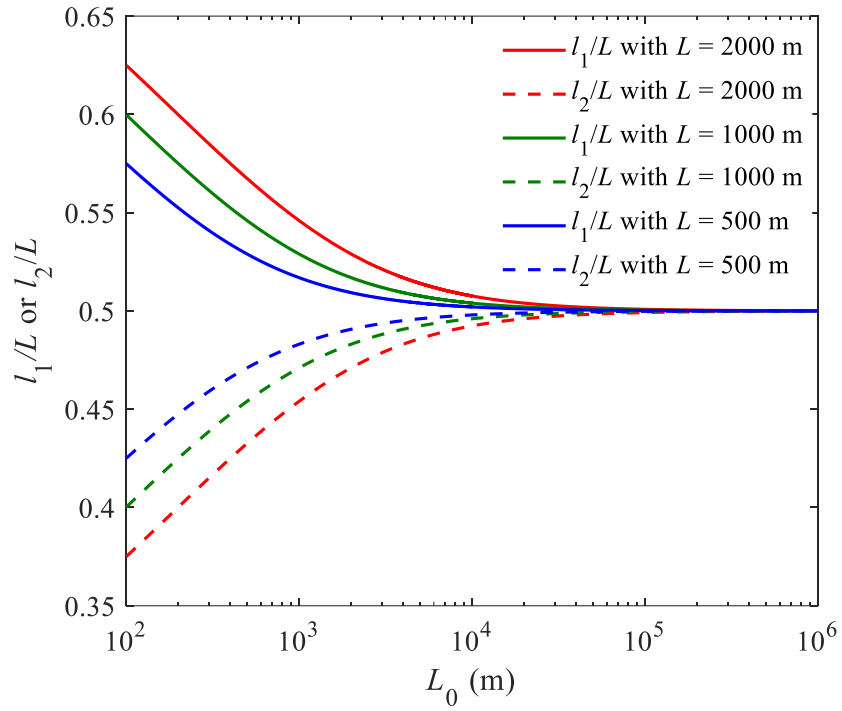


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597 **Figure 3.** Comparison between analytical and experimental (data compiled from Memari et

598 al., 2020) results for the freshwater-seawater interface location for different recharge events.

599 Note that the left and right sides are the sea and internal no-flow boundaries, respectively.

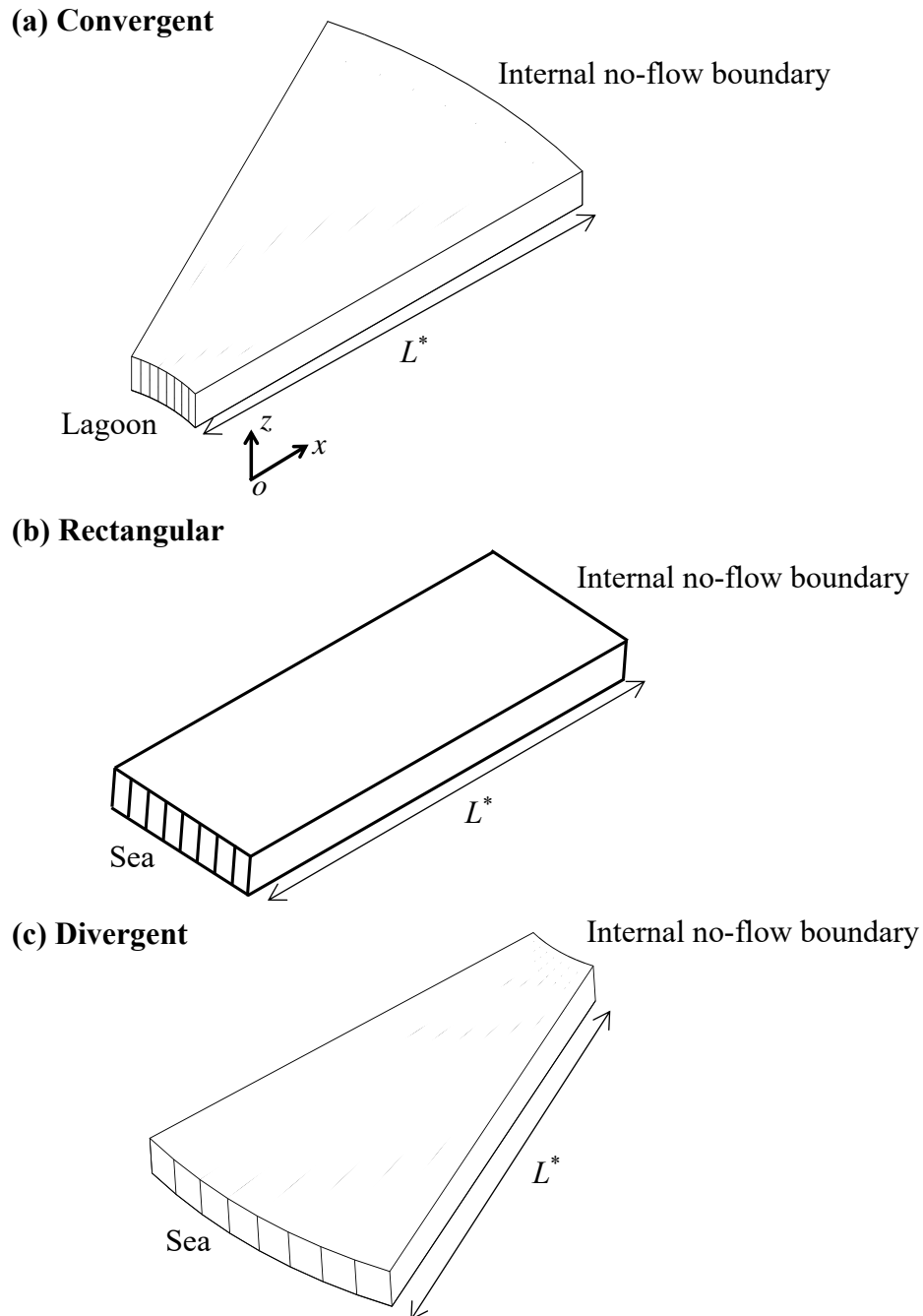


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**Figure 4.** Widths of Unit 1 and Unit 2 versus  $L_0$  for aquifers with different total width  $L$ .



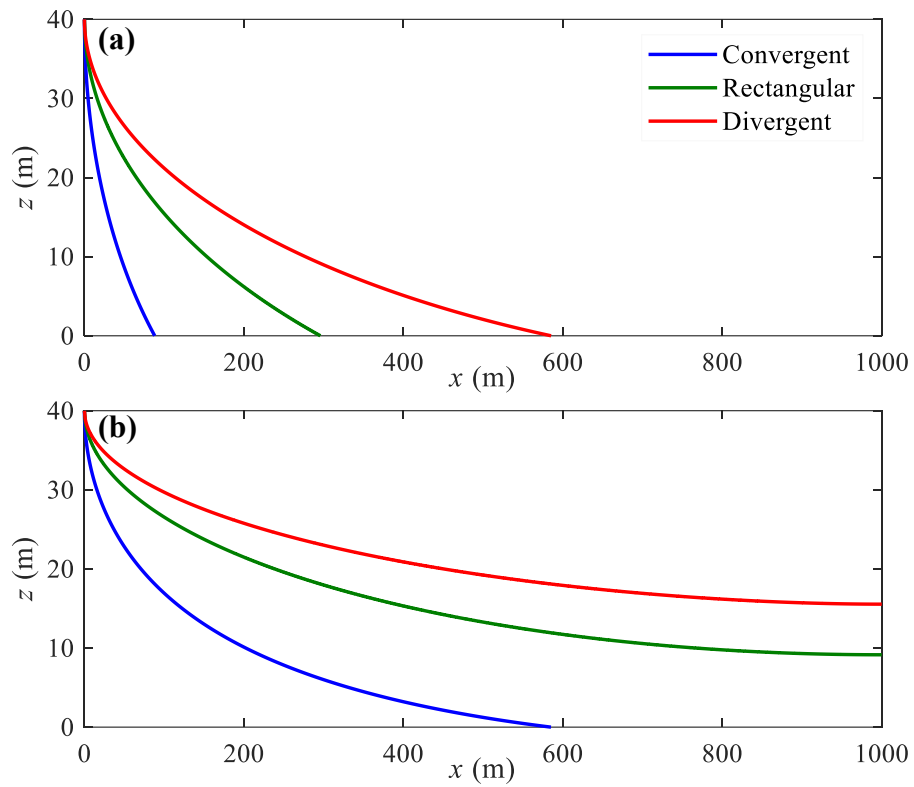


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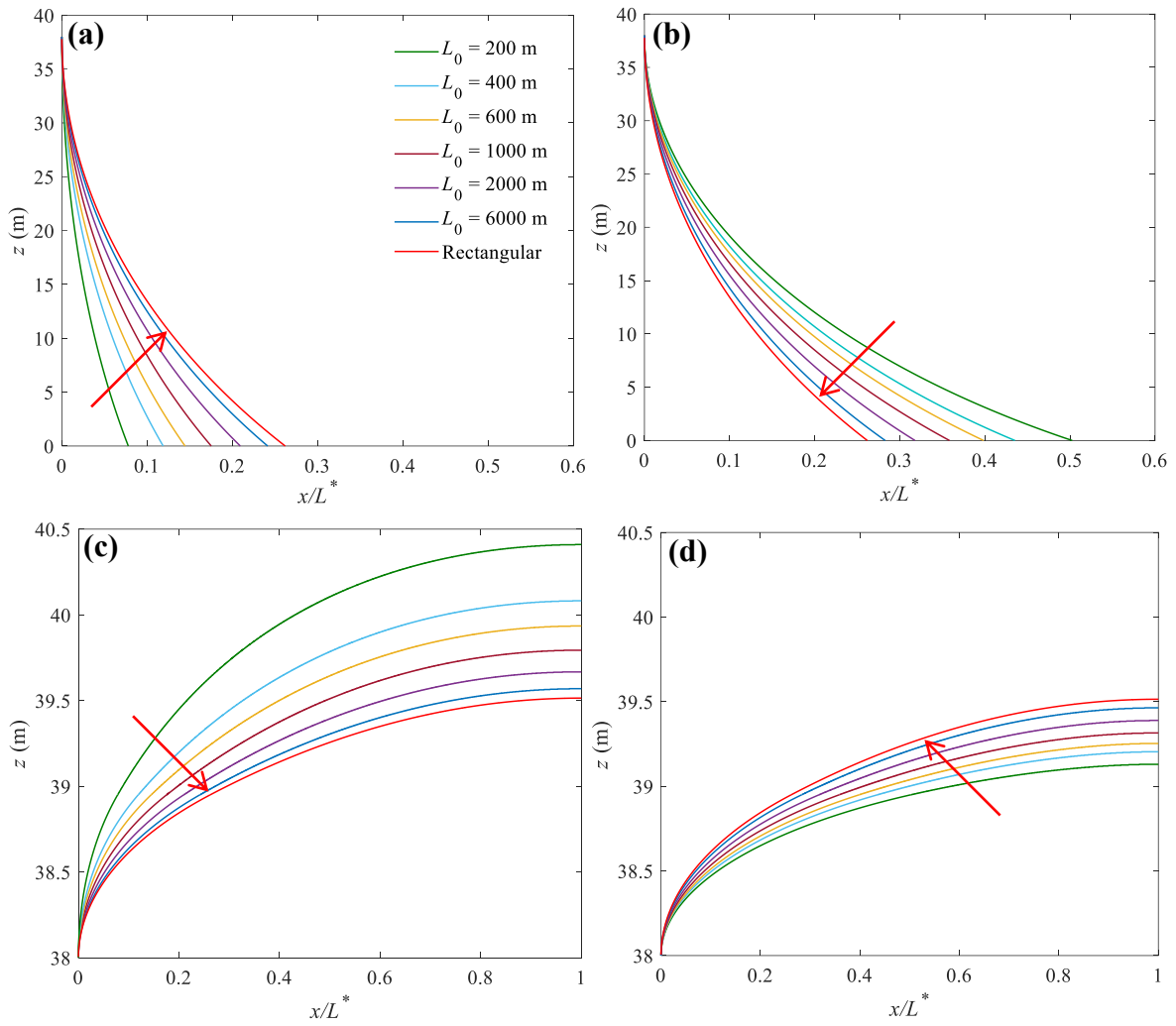
605 **Figure 5.** Three-dimensional view of (a) convergent (smaller side facing the lagoon), (b)  
 606 rectangular and (c) divergent aquifers (larger side facing the sea) compared in this study.  $L^*$   
 607 represents the distance from the sea/lagoon to the internal no-flow boundary, i.e.,  $l_1$  or  $l_2$  in  
 608 Figure 2. The internal no-flow boundary corresponds to the  $z$ -axis in Figure 2.



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611 **Figure 6.** Freshwater-seawater interface predicted by analytical solutions for three different  
 612 aquifers with (a) high and (b) low recharge (Cases 1 and 2 in Table 1). Note that  $x = 1000$  m is  
 613 the internal no-flow boundary in Figure 5.



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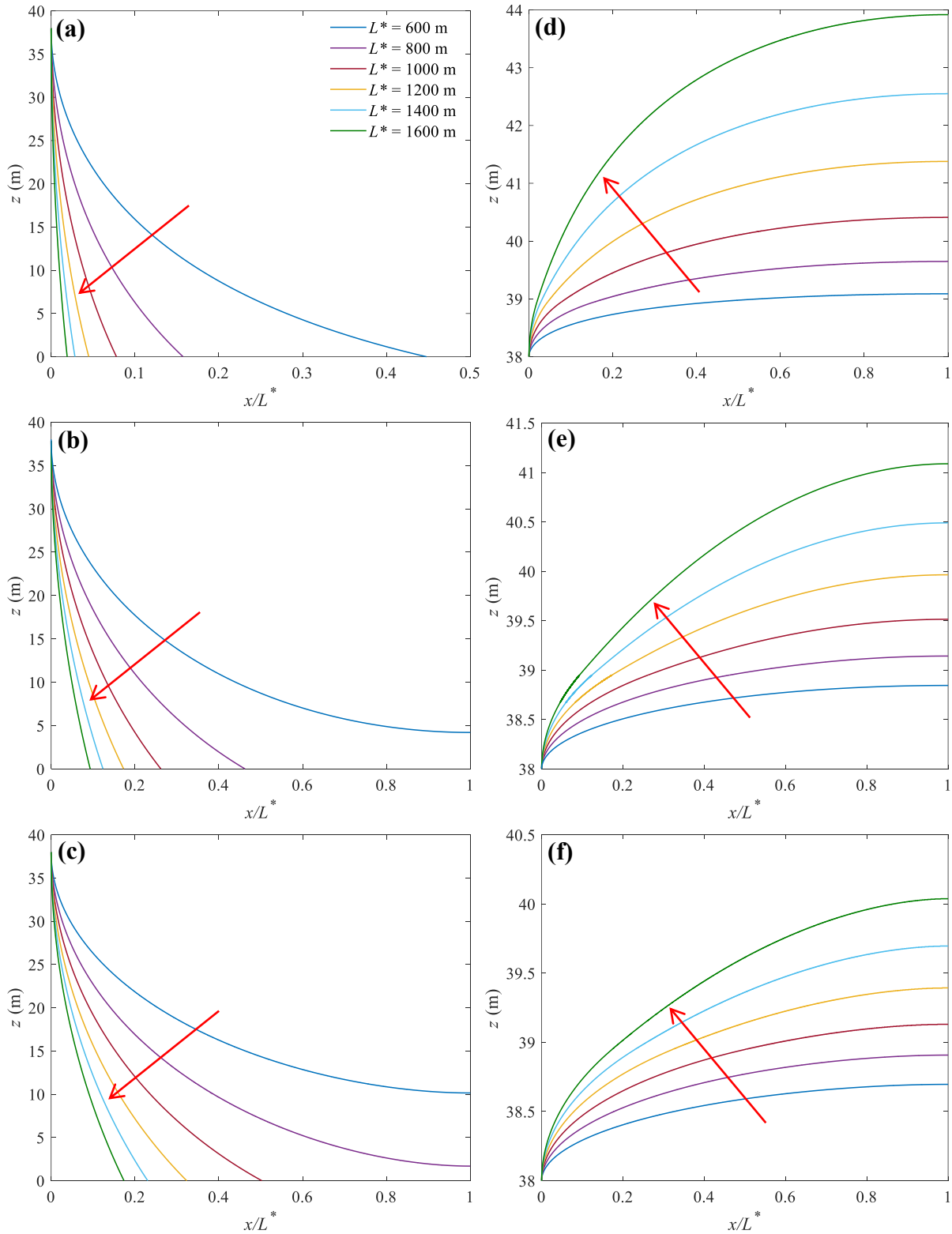
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**Figure 7.** Sensitivity of (a, b) the locations of the freshwater-seawater interface and (c, d) watertable to  $L_0$  for convergent (left panel) and divergent (right panel) aquifers. The arrow in each plot shows the direction of increasing  $L_0$  (values given in (a), used to produce the different curves). Note that predictions for rectangular aquifers are independent of  $L_0$ .



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**Figure 8.** Sensitivity of (a-c) the locations of the freshwater-seawater interface and (d-f) watertable to  $L^*$  for convergent (a, d), rectangular (b, e) and divergent (c, f) aquifers. The arrow in each plot points to the increase of  $L^*$  values used to construct each curve (values

626 indicated in (a)).