Author Response to the referee comments on "Citizen rain gauge improves hourly radar rainfall bias correction using a two-step Kalman filter"

by Punpim Puttaraksa Mapiam [1], Monton Methaprayun [1], Thom Adrianus Bogaard [2], Gerrit Schoups[2], Marie-Claire Ten Veldhuis[2]*

[1]*Department of Water Resources Engineering, Kasetsart University, PO Box 1032, Bangkok, 10900, Thailand*

[2]*Department of Water Management, Delft University of Technology, PO Box 5048, 2600 GA Delft, The Netherlands*

Responses to referee #1

RC: Referee comment

AR: Author response

General comments

RC: I found the topic of the manuscript highly interesting, as it exists a strong need to explore the potential of lower quality, but high density crowd-sourced observations. To this end, the authors suggest to use a Kalman filter, a well-established approach for data fusion in presence of uncertainty.

Following the invitation from the editor, I focused my attention on the methods of Section 3. In there, I could notice sound technical foundations and good consideration of previous work in the field. Despite these positive notes, I believe the clarity of the text in this Section can be improved, in particular by providing more details on the original contribution, while reducing some of the details from the basic algorithm that can be found in past references.

My understanding is that the methodology presented in Section 3.2 mostly follows the one already introduced in previous studies, particularly in Chomchean et al (2006), from which the authors seemed to have adopted the basic algorithm and notation. Therefore, it could be possible to move some of the mathematical details in the Appendix, and replace them with a higher-level summary of the algorithm together with a clear literature reference, so that the reader can more easily find the original formulations if needed.

In turn, this would allow to better highlight the original contribution of this study, which is the application of the algorithm to two sets of rain gauges of contrasting quality. I thus encourage the authors to consider some refactoring of Section 3. In particular, I found Section 3.2.2 a bit difficult to follow. I would suggest to first list more clearly the parameters needing estimation, and then explain the actual estimation methods alongside the underlying assumptions.
I have also listed below a number of minor comments and suggestions to the authors.

AR: Thanks for your valuable comments. To provide more clarity on the original contribution of the methodology in this study, we more explicitly mentioned the contributions of the study in Abstract, Introduction and Conclusions and reorganized Section 3.2 by stating the contributions directly at the beginning of the section, i.e. stating that "our approach extends a previously used Kalman filter radar bias model by including two different types of rainfall observations (data from traditional and from citizen rain gauges) and by using a likelihood-based method for parameter estimation."

Furthermore, we refactored and clarified Section 3.2 as follows: instead of first describing the one-update/one-observation KF, followed by the two-update case, we focused each subsection on the two updates/observations (i.e. the contribution of our work). Namely:
-subsection 3.2.1 describes how radar bias is modelled using a Kalman filter model with two observations, contrasting it with previous KF bias models used in the literature.
-subsection 3.2.2 describes how observations are assimilated into the model, resulting in a KF filter with two updates instead of one. The two updates are now explicitly distinguished (also in response to reviewer 2), and this subsection has been streamlined so that it is easier to follow.
-subsection 3.2.3 describes how model parameters are estimated by maximizing marginal likelihood. Also this subsection was slightly rearranged to improve readability.

Minor comments

RC (1): Lines 210-211
The formulation "Since [...] . However […]. " is unclear, please consider rephrasing.
AR: Since we refactored Section 3.2.1 as described above; these sentences were removed from the revised manuscript.

RC (2): Line 242
"comprising an updating (prediction) step" -> "comprising a time updating (prediction) step"
AR: Since we refactored Section 3.2.1 as described above; these sentences were removed from the revised manuscript.

RC (3): Equation 10
Why using O_t here to denote the observation at time t, while in Eq. 6 y_t was used?
AR: In the revised manuscript, O_t was replaced with y_t and z_t to represent observed logarithmic mean field bias at hour t gained from the TMD and citizen rain gauge networks, respectively. Additionally, two steps of measurement updating using the y_t and z_t datasets were specified as presented in Eq (11) and Eq (14) of the modified manuscript.

RC (4): Line 272
Please check "and the a priori estimate error variance" -> "and the a posteriori estimate error variance"
AR: This mistake was corrected according to the suggestion.

RC (5): Equation 12

Without a measurement update, I would have expected to see this equation expressed simply as the posterior equal to the prior. Is there a particular reason for repeating the equations from the time update step?

AR: In the revised manuscript, we removed Eq.12 and added some sentences between lines 282 and 284 to explain how to calculate the CKF without a measurement update as shown below.

"If there is no observation data available at any time t, $K_{y,t}$ is zero (mathematically, a missing observation is equivalent to an observation with infinite variance $\sigma^2_{M_{y,t}}$ in Eq. 10), and the previous equations reduce to not performing any update , i.e. the posterior mean and variance are equal to the prior mean and variance."

RC (6): Equation 13

Why is the posterior estimate error variance P_t in absence of a measurement step not simply equal to Eq. 8?

AR: Thanks for pointing this out, we realize the equation in the previous manuscript is incorrect. If there is no observation available at any time t, the posterior mean and variance are equal to the prior mean and variance. In the revised manuscript, we removed Eq.13 and added some sentences between lines 282 and 284 to simply explain how to calculate the CKF without a measurement update as shown below.

"If there is no observation data available at any time t, $K_{y,t}$ is zero (mathematically, a missing observation is equivalent to an observation with infinite variance $\sigma^2_{M_{y,t}}$ in Eq. 10), and the previous equations reduce to not performing any update , i.e. the posterior mean and variance are equal to the prior mean and variance."

RC (7): Equation 14

The formula computes the mean of a log-normal distribution based on its parameters. I believe it would help the reader to remind this alongside the reference.

AR: For clearer explanation, we rewrote the sentences between line 299 and 301 in the revised manuscript as described below.

"Since the logarithmic mean field radar rainfall bias β_t is assumed to be normally distributed, the mean field bias (B_t) is log-normal distributed with posterior mean at time *t* obtained from the posterior mean and variance of β_t (Smith and Krajewski, 1991):"

RC (8): Lines 295-297

The citizen rain gauge data are available only at the end of the day, meaning that their application cannot be in real time. Isn't this an important limitation since the Kalman filter is typically designed for real-time applications? Is this aspect worth mentioning and discussing, as for example in the conclusions?

AR: We appreciate this comment and agree with the referee to indicate the limitation of the modified KF approach in the conclusion of the updated manuscript between lines 551 and 555 as described below.

"Note that citizen rain gauge data are available only at the end of the day and consequently, the modified two-step Kalman filter as used in this study has restrictions in real-time applications. However, the here proposed method has great potential when creating high quality historical radar-rainfall time series for climatological studies and in post-event analysis. Moreover, near real-time assessment could be achievable if the citizen rain gauge data are collected at sub-daily time scale."

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[2]*Department of Water Management, Delft University of Technology, PO Box 5048, 2600 GA Delft, The Netherlands*

Responses to referee #2

RC: Referee comment
AR: Author response

1. General comments

RC (1): Thank you for this manuscript. Using citizen rain gauge data is an interesting approach and the authors analyzed the gathered data carefully and with a lot of different techniques. There seems to be a benefit in using these Citizen rain gauge data, so from my point of view, this study should be published.

I had some difficulties to understand where all the errors are coming from and how these errors are calculated. There are often some unreferenced assumptions on the error characteristics. Sometimes I would have wished to have a bit more insight into the error distributions of the different measurements.

There is also nothing mentioned on radar calibration errors. Most weather radars are not well calibrated and this obviously influences the estimation of R via the Z - R relationship. Is this measurement bias automatically canceled out by calculating an individual Z - R relationship?

AR: For clearer explanation on radar rainfall estimation errors, we inserted more information in the revised manuscript between lines 31 and 35 as described below.

However, a weather radar provides an indirect measurement of backscattered electromagnetic waves called radar reflectivity data (Z) *"and quantitative estimation of radar rainfall data (R) is acknowledged as a complex process. Various sources of errors affect radar rainfall estimates, mainly errors in reflectivity measurements and reflectivity-rainfall conversion (Jordan et al., 2000). Correction of these two sources of error is a crucial procedure to increase the accuracy of radar rainfall estimates. Ground-truthing by rain gauge data is required to calibrate the Z-R relationship ($Z=AR^b$). The calibrated parameter A in the Z-R relationship will include any errors in the radar system caused by the electrical calibration of the radar (Seed et al., 2002)."*

RC (2): Regarding the KF and its mathematical formulation: I guess that this has been done properly, at least from my understanding. However, Kalman filters are especially great for multi-dimensional data sets with observation vectors being larger than 1. So I'm therefore not sure if averaging everything together and calculating one bias value is the best approach. But at least it is an approach that shows some nice results.

AR: A few previous studies apply the Kalman filter for radar mean bias adjustment (Smith and Krajewski, 1991; Chumchean et al., 2006; Shi et al., 2018). Using the mean-field bias observed from only one rain gauge network and combining it with different techniques for assessing the observation noise variance at hour t ($\sigma^2_{M_t}$) is one of the inspiring tasks they tried to solve. In this study, we added one more observation dataset, while accounting for different error characteristics between the two datasets, to improve the measurement update procedures.

RC (3): There are still quite some language issues and after a couple of page I got tired to correct them all. So there should be some polishing done on that.

AR: English writing has been thoroughly checked in the revised manuscript.

2 Detailed comments

RC (1): Line 12: *of Sattahip radar station and gauge rainfall* → of Sattahip radar station, gauge rainfall...

AR: This mistake was corrected in the revised manuscript.

RC (2): Line 27: *accuracy of flash flood estimates and warning.* → accuracy of flash flood estimates and warnings.

AR: This mistake was corrected in the revised manuscript.

RC (3): Line 28: *However, weather radar provides indirect measurement of backscattered electromagnetic waves called radar reflectivity data (Z).* → However, a weather radar provides an indirect measurement of backscattered electromagnetic waves called radar reflectivity data (Z).

AR: This mistake was corrected in the revised manuscript.

RC (4): Line 30: *ground-truthing by rain gauge data is required to calibrate the Z-R relationship (Z = ARb)* → I'm not happy with this sentence. It's not a priori clear that just the Z-R relation needs to be calibrated. Biases between the radar rainfall intensity can also stem from radar calibration errors, i.e., from a bias in Z. Most radar operators do not alter the Z-R relationship at all, but just adjust the radar rainfall vs gauge rainfall bias (Sideris, 2014).

AR: Thank you for pointing this out, we added more information and rephrased the sentence for clearer explanation in the revised manuscript between lines 31 and 35 as explained below.

However, a weather radar provides an indirect measurement of backscattered electromagnetic waves called radar reflectivity data (Z) *"and quantitative estimation of radar rainfall data (R) is acknowledged as a complex process. Various sources of errors affect radar rainfall estimates, mainly errors in reflectivity measurements and reflectivity-rainfall conversion (Jordan et al., 2000). Correction of these two sources of error is a crucial procedure to increase the accuracy of radar rainfall estimates. Ground-truthing by rain gauge data is required to calibrate the Z-R relationship ($Z=AR^b$). The calibrated parameter A in the Z-R relationship will include any errors in the radar system caused by the electrical calibration of the radar (Seed et al., 2002)."*

RC (5): Line 60: *variances affecting the mean field bias estimate*.

AR: This mistake was corrected in the revised manuscript.

RC (6): Line 76: *However, these methods are not usually designed for real-time* → However, these methods are usually not designed for real-time

AR: This mistake was corrected in the revised manuscript.

RC (7): Figure 1: I can't see the location of the radar in this map. It's indicated in the legend but not on the map (or hardly visible).

AR: Figure 1 was updated according to the comments in the revised manuscript as shown below.

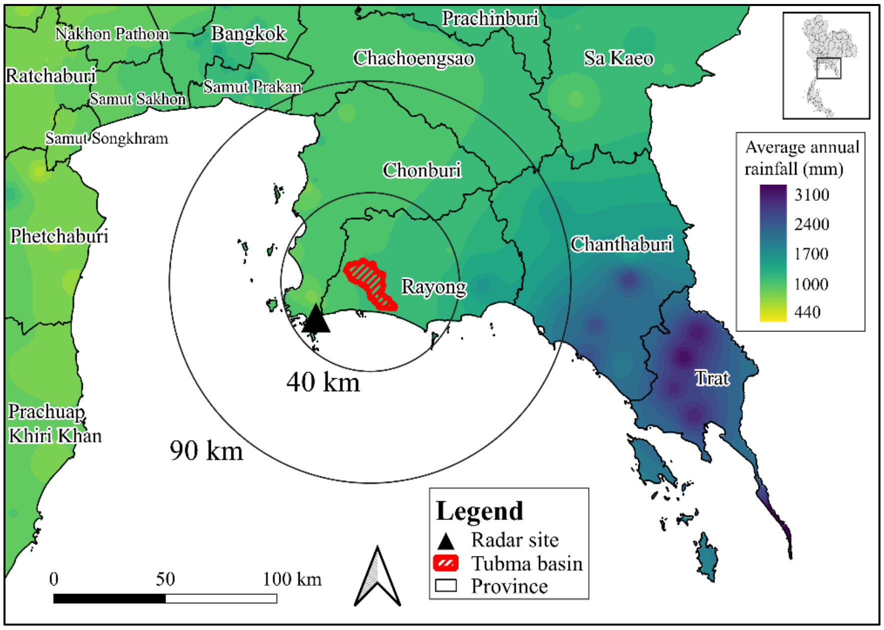

Figure 1: Climatological spatial rainfall distribution in and around the Tubma basin calculated from 30-year average annual rainfall data of 311 daily rain gauge network by using IDS method.

RC (8): Line 107: *The Tubma basin is covered within the range of Sattahip radar station.* → The Tubma basin is located within the coverage of the? Sattahip radar station.

AR: This sentence was adjusted in the revised manuscript.

RC (9): Line 107: *a beam width of 1.0°* → a half power beam width of 1.0°

AR: This mistake was corrected in the revised manuscript.

RC (10): Line 109: *The radar reflectivity product is in a Cartesian* → The radar reflectivity product is provided in a Cartesian

AR: This sentence was adjusted in the revised manuscript.

RC (11): Line 110: *The Sattahip radar provides the CAPPI reflectivity data derived from the 2.5-km constant altitude plan position indicator (CAPPI).* → CAPPI derived from CAPPI: that's not a meaningful sentence.

AR: This sentence was corrected as appeared below.

"The Sattahip radar provides the reflectivity data derived from the 2.5-km constant altitude plan position indicator (CAPPI)"

RC (12-1): Line 114: *Additionally, the noise and hail effects were eliminated by setting reflectivity values below 15 dBZ to zero, and reflectivity values greater than 53 dBZ to 53 dBZ.* → I'm not sure if this is allowed. Could you give some reasoning on this method or a reference?

AR: Hail is a type of precipitation in the form of ice with a diameter of at least 5 mm (Rinehart, 1991). Reflectivity from dry hail has a lower reflectivity than wet hail of the same size. The apparent reflectivity of hail particles further increases when the hail melts and becomes coated with a film of water leading to an over-estimation of rainfall rates. Fulton et al. (1998) suggested that measured reflectivity that are greater than 53 dBZ are limited to 53 dBZ in radar rainfall estimation to mitigate false interpretation caused by hail. This reference was inserted in the revised manuscript at line 123.

RC (12-2): If I use a Marshall-Palmer relation (Z = 200 $R^{1.6}$ then 100 mm of rain already give 55 dBZ. 100 mm are not unrealistic in the tropics, even higher values are possible. But probably Marshall-Palmer is not suitable in the tropics?

AR: The Z-R parameters significantly change by raindrop size distribution, hence there is no universal relationship connecting these parameters (Doviak and Zrnic, 1992). Marshall and Palmer was calibrated based on stratiform rainfall characteristics in Ottawa, Canada, the A and b parameters are then unrealistic to directly apply in different areas. In this study, the optimal relationship was $Z=251R^{1.5}$, as described in section 2.2.2. Applying the calibrated relationship to the maximum 53 dBZ produces 86 mm/h of rainfall, while Marshall and Palmer (1948) give 75 mm/h. In the other area with heavier rainfall characteristics, the Z-R parameters typically differ for converting the same reflectivity to higher rainfall rate. For example, the relation $Z=300R^{1.4}$ (Fulton et al., 1998) appropriate for storm characteristic in United States can be used to convert 53 dBZ of reflectivity to 103 mm/h of rainfall.

RC (12-3): And clipping below 15 dBZ might be ok, but reflectivity values below this value or not necessarily due to noise, especially not receiver noise. It might need a specification of what you mean by 'noise'.

AR: Noise is any undesired electrical disturbance or spurious signal which enters the radar receiver. Noise power is composed of signals originating at various sources such as emission from space (cosmic noise), radiation from electrical sources near the radar antenna, and internally generated noise. Variations in these noise sources occur, often at random intervals and are challenging to control. To avoid accumulation of the power of noise, measured radar reflectivity below 15 dBZ was set to zero (Fulton et al., 1998 and Doviak and Zrnic, 1984).

To clarify noise and hail issues, we rephrased the sentences between lines 121 and 125 in the revised manuscript as describe below.

"*Additionally, the noise power caused by various sources such as emission from space (cosmic noise), radiation from electrical sources near the radar antenna, and internally generated noise were eliminated by setting reflectivity values below 15 dBZ to zero (Doviak and Zrnic, 1984). While Fulton et al. (1998) suggested that measured reflectivity that are greater than 53 dBZ are limited to 53 dBZ in radar rainfall estimation to mitigate false interpretation caused by hail.*"

RC (13): Line 137: *radar rainfall accumulation (mm/h)* → Accumulation is not in mm/h

AR: In the revised manuscript, the description of $R_{i,t}$ was changed from "radar rainfall accumulation (mm/h)" to "the hourly radar rainfall accumulation (mm).

RC (14): Line 141: *was validated against a second, independent dataset. Results found that a locally calibrated ZR relationship that was used in this* → I don't understand this sentence. What is the *Z - R* relation of the climatological dataset? What is the *Z - R* relation of the locally calibrated dataset?

AR: To clarify this point, this sentence was rewritten in the revised manuscript between lines 149 and 151 as appeared below.

"Results found that the optimal climatological Z-R relationship for the Sattahip radar used in this study is $Z=251R^{1.5}$*. This relation is appropriate for both the calibration and validation datasets with the MAE of 1.36 mm and 1.47 mm, respectively."*

RC (15): Line 146: *These 15-min rain gauges*: → These rain gauge data have a temporal resolution of 15 minutes

AR: This sentence was adjusted in the revised manuscript.

RC (16): Line 148: *double mass curves method*: → What is this?

AR: Double mass curves is a traditional method to be used for checking the consistency of gauge rainfall data. The cumulative data of a single station are plotted against the mean accumulated rainfall of adjacent gauges in the area. If the slope of the double mass curve tends to be straight with a single slope, it can ensure the reliability of rainfall data at that considered rain station.

RC (17): Line 155: *based on spatial decorrelation analysis in the process.* → based on spatial decorrelation analysis for this? process.

AR: This sentence was adjusted in the revised manuscript.

RC (18): Line 180: RMSE does not tell you much about the bias between the TMD and the citizen gauge. Bias needs to be given as well.

AR: According to the comment, the bias between the TMD and the citizen gauge was calculated as 1.04 and specified in the revised manuscript at line 190 and in the supplementary between lines 7 and 15.

RC (19): Line 188: *First, daily citizen...* → From line 126 onward you were using 'Firstly, Secondly...'.

AR: This line First was changed to Firstly as the suggestion.

RC (20): Line 206: I'm not sure if 'noise' is the right word here. KF accounts for measurement errors or uncertainties, but not for noise.

AR: The word 'noise' was replaced with uncertainties in the updated manuscript.

RC (21): Line 208: *different uncertainty characteristics, i.e., hourly...*

AR: This sentence was adjusted in the revised manuscript.

RC (22): Line 210: *Since the MFB (G/R ratio) is assumed to follow a log-normal distribution.* → Has this sentence a relationship to the previous sentence? And what is an ordinary KF scheme.

AR: Since we refactored Section 3.2.1 to describes how radar bias is modelled using a Kalman filter model with two observations, contrasting it with previous KF bias models used in the literature. This sentence was removed from the revised manuscript.

RC (23): Line 300: *downscaled hourly citizen rain gauge data were used to back-calculate the hourly citizen rain gauges data* → I don't understand this.

AR: This mistake was corrected in the revised manuscript between lines 312 and 313 as follows.

“The TMD and downscaled hourly citizen rain gauge data were together used to conduct two steps of a second measurement update in the CKF process for all hourly time-steps of day i.”

RC (24): Figure 5: This is Kalman filter bias corrected rain accumulation data from gauges?

AR: Figure 5 is not the Kalman filter bias, but it shows the different hourly rainfall distribution patterns at each day generated from four downscaling techniques. The rainfall distribution patterns were presented as the cumulative fraction of daily rainfall at an hourly scale. Daily citizen rain gauge data were afterwards multiplied by the corresponding hourly fraction factor to obtain the hourly downscaled citizen rain gauge data to be used as input for the KF bias correction.

RC (25): Line 420: *%-ile* → percentile

AR: All the words ‘*%-ile’ were replaced with percentile in the revised manuscript.*

RC (26): Line 508: *obviously appear steady light rainfall accumulation* → ?

AR: This sentence was rephased in the revised manuscript as described below.

“Figure 10 (c)…. obviously appear a steady gradient of the mass curve reflecting light rainfall accumulation, ..”

RC (27): Figure 9: *Rainfall Depth* → probably not the right annotation.

AR: Figure 9: Rainfall Depth was changed to Hourly Rainfall (mm) in the revised manuscript.

3 Kalman filter

I guess I have understood your KF approach, but there are some questions left:

RC (28) y_t and z_t are your log-transformed bias observations from the TMD and the citizen gauge network, respectively, corresponding to O_t in Eq. 10?

AR: Yes, you are correct. We therefore updated the equations relating to y_t and z_t as shown in Eq (11) and Eq (14) of the modified manuscript for the first and second measurement updating, respectively.

RC (29) Figure 4: In the second KF step you take the variance estimates *Pt* from the previous KF step as a priori variances?

AR: Yes, it is correct. Confusion caused by the symbols was fixed in the revised manuscript.

RC (30) You calculate the Kalman gain K_t in the second step with these *Pt* variances. From my point of view, this Kalman gain should be named differently, since K_t is already used for the first step.

AR: We agree with the referee to separate symbols differently to distinguish the two updates. Please have a look at section 3.2.2 in the revised manuscript between lines 260 and 303 for new changes as appeared below.

"3.2.2 Kalman filter with two observations: data assimilation

Having defined the model, we describe how observations are assimilated into the model, resulting in adjustments of the estimated bias. While the regular Kalman filter has two steps (prediction and update), assimilating two observations at each time step involves three steps, i.e. a prediction followed by two updates. Figure 3 (b) shows a visual depiction of the prediction and update steps as probabilistic information propagating along the edges of the factor graph. .

1) Time update step (prediction)

The first step for each hour *t* computes the prior mean $\hat{\beta}_t^{\,-}$ and variance $P_t^{\,-}$ of β_t :

$$\hat{\beta}_t^- = r_1 \hat{\beta}_{t-1} \tag{8}$$

$$P_t^- = r_1^2 P_{t-1} + \sigma_W^2 \tag{9}$$

where P_{t-1} is the posterior variance of β_{t-1}. For the first time step 0 (t = 0), we assume $\beta_0 = 0$ (climatological logarithmic mean field bias) and $P_0 = (1-r_1^2)\,\sigma_\beta^2$ (represents stationary process variance) (Smith and Krajewski, 1991; Chumchean et al., 2006).

2) The first measurement update step (1st correction)

The first update merges the bias prediction from step 1 with noisy observation y_t (with measurement error variance $\sigma^2_{M_{y,t}}$), resulting in posterior mean $\hat{\beta}_{y,t}$ and variance $P_{y,t}$ of β_t given by the following Kalman update equations:

$$K_{y,t} = P_t^- \left(P_t^- + \sigma_{M_{y,t}}^2\right)^{-1} \tag{10}$$

$$\hat{\beta}_{y,t} = \hat{\beta}_t^- + K_{y,t}(y_t - \hat{\beta}_t^-) \tag{11}$$

$$P_{y,t} = (1 - K_{y,t})P_t^- \tag{12}$$

where $K_{y,t}$ is the Kalman gain for assimilating observation y_t. If there is no observation available at time t, $K_{y,t}$ is zero (mathematically, a missing observation is equivalent to an observation with infinite variance $\sigma_{M_{y,t}}^2$ in Eq. 10), and the previous equations reduce to not performing any update, i.e. the posterior mean and variance are equal to the prior mean and variance.

3) The second measurement update step (2nd correction)

The second update is done using the posterior values from the 1st correction ($\hat{\beta}_{y,t}$ and $P_{y,t}$) as the prior values. The resulting Kalman gain and posterior mean and variance are given by:

$$K_{z,t} = P_{y,t}\left(P_{y,t} + \sigma_{M_{z,t}}^2\right)^{-1} \tag{13}$$

$$\hat{\beta}_{z,t} = \hat{\beta}_{y,t} + K_{z,t}(z_t - \hat{\beta}_{y,t}) \tag{14}$$

$$P_{z,t} = (1 - K_{z,t})P_{y,t} \tag{15}$$

If there is no observation available at time t, Kalman gain $K_{z,t}$ is zero and these equations result in no update being applied, i.e. posterior mean and variance are the same as after the first update.

Since the logarithmic mean field radar rainfall bias β_t is assumed to be normally distributed, the mean field bias (B_t) is lognormally distributed with posterior mean at time t obtained from the posterior mean and variance of β_t (Smith and Krajewski, 1991):

$$\hat{B}_t = 10^{(\hat{\beta}_{z,t}+0.5P_{z,t})} \quad (16)”$$

RC (31) Likewise, the equation (2) in the 2nd KF step in Figure 4 should be rewritten, since the value on the left side of the equation ($\hat{\beta}_t$) is an update of the bias value on the right side.

AR: This mistake in Figure 4 was corrected according to the suggestion as shown below. The mentioned equation was changed to $\hat{\beta}_{z,t} = \hat{\beta}_{y,t} + K_{z,t}(z_t - \hat{\beta}_{y,t})$

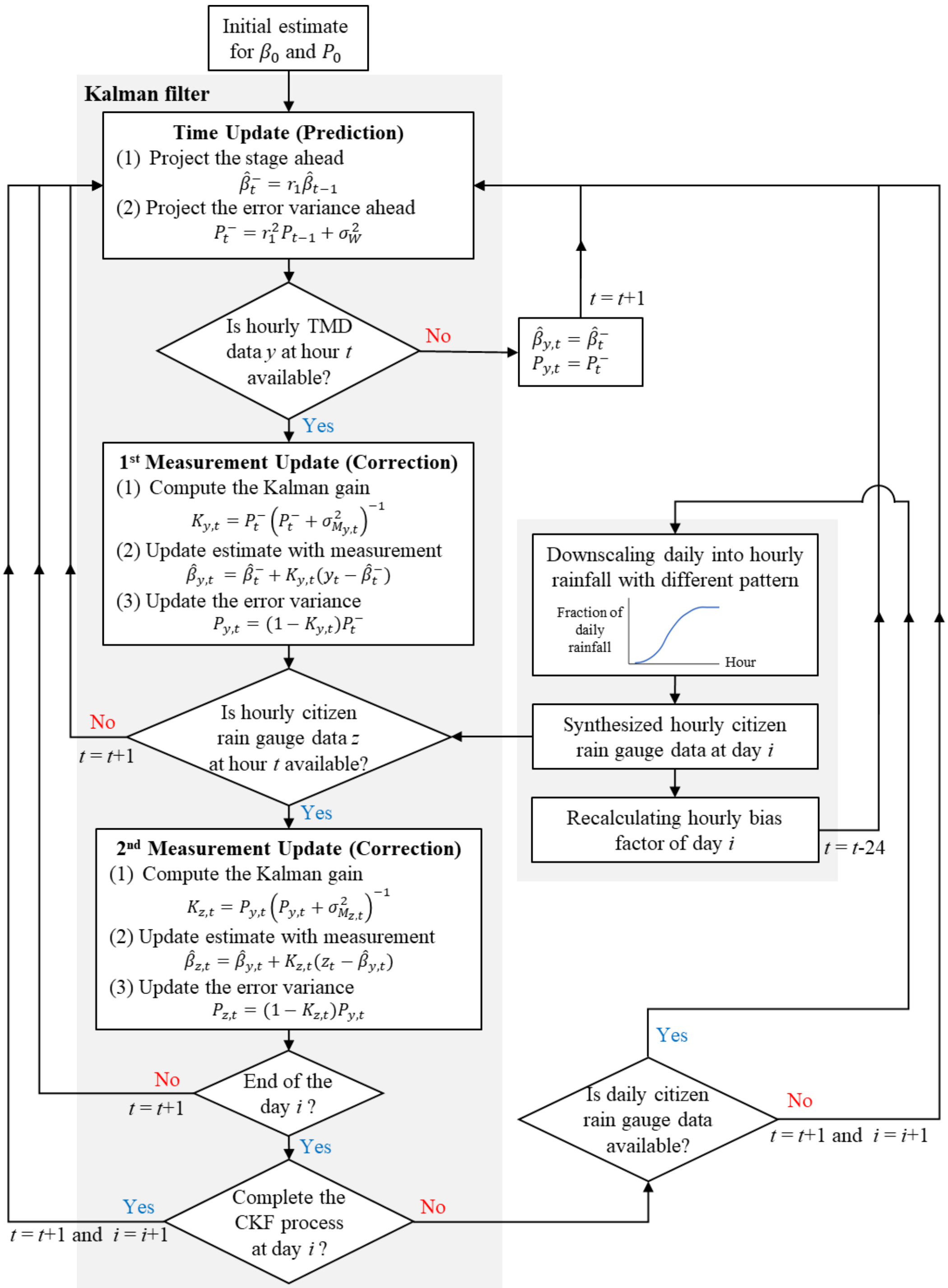

Figure 4: A diagram of the procedure of Kalman filter combined with the citizen rain gauge data (CKF)

RC (32) Same is true for Eq. 3 in Figure 4, 2nd step: P_t is an update of P_t on the right side of the equation.

AR: This mistake in Figure 4 was corrected according to the suggestion. The mentioned equation was changed to. $P_{z,t} = (1 - K_{z,t})P_{y,t}$

RC (33) I'm somehow irritated by the usage of these capital letters *K* and *P*, which, at least for me, would represent matrices. In KF theory, *P* then represents the error covariance matrix, but since you are using variances only it is somehow strange to go from $\sigma^2_{Mz,t}$ to Pt. Probably just an unimportant detail.

AR: Yes, in this paper the Kalman gain and variances P are scalar numbers instead of matrices. To make this clear to the reader, we avoid the usual bold/upright notation for these variables as would be common if they were actually matrices.

RC (34) Line 229 onwards: Error estimation of $\sigma^2_{Mz,t}$ and $\sigma^2_{My,t}$: I did not fully understand this. σ^2_{Ot} is the (spatial?) variance of all stations? And why is Equation 19 necessary if you calculate $\sigma^2_{Mz,t}$ and $\sigma^2_{My,t}$ by the spatial variance of the individual stations?

AR: In the revised manuscript, we refactored and clarified Section 3.2 to provide more clarity on the original contribution of the methodology in this study. We consider two types of observations, y_t and z_t, (O_t was replaced with y_t and z_t) of the unknown bias at time *t*, derived from the TMD and citizen rain gauges. Each observation is modelled as a random sample from a normal distribution conditioned on the underlying unknown bias with distinct measurement error variances $\sigma^2_{M_{y,t}}$ and $\sigma^2_{M_{z,t}}$.

$$y_t = \beta_t + M_{y,t}; \quad M_{y,t} \sim \mathcal{N}(0, \sigma^2_{M_{y,t}}) \tag{6}$$

$$z_t = \beta_t + M_{z,t}; \quad M_{z,t} \sim \mathcal{N}(0, \sigma^2_{M_{z,t}}) \tag{7}$$

where $\sigma^2_{M_{y,t}}$ and $\sigma^2_{M_{z,t}}$ are time-varying measurement error variances for the TMD and citizen rain gauges, respectively. We estimate $\sigma^2_{M_{y,t}}$ and $\sigma^2_{M_{z,t}}$ using formulas for the variance of the mean bias across individual TMD and citizen rain gauges:

$$\sigma^2_{M_{y,t}} = \frac{\sigma^2_{y,t}}{n_{y,t}} \tag{17}$$

$$\sigma^2_{M_{z,t}} = \frac{\sigma^2_{z_t}}{n_{z,t}} \tag{18}$$

where $\sigma_{y,t}^2$ and $\sigma_{z,t}^2$ are variances that quantify spatial variability of radar bias at time t at TMD and citizen rain gauge locations, respectively, and $n_{y,t}$ and $n_{z,t}$ are the corresponding number of observations at hour t from the two networks.

Reference

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