Page 4, Line 98-100 in the manuscript (HESS-2021-188), **the derivation from Eqs. (9) and (10) to (11)** (cited form Zhang et al. (2021)):

Dean (1979) developed a model for the damping of incident wave height (H_0) by coastal plants, based on empirical estimates of fluid drag forces acting on vertical, rigid cylinders:

$$K_v = \frac{H}{H_0} = \frac{1}{1+\alpha\prime X} = \frac{1}{1+\alpha x} = F(x), \qquad \text{(S1)}$$

in which

$$\alpha' = \frac{C_D d}{6\pi h} N H_0 \; (0 \le x = X/L \le 1), \qquad \text{(S2)}$$

where K_v (-) is the scaled wave height, L (m) is the length of vegetation area, α ($= \alpha' L$) (-) is the scaled damping factor, and x (-) is the scaled distance through the vegetation field.

On the other hand, Kobayashi et al. (1993) linearized the horizontal drag force as a function of fluid particle velocity. The local wave height was assumed to decay exponentially with propagation through a vegetation bed according to the following form:

$$\frac{H}{H_0} = \exp(-k'X) = \exp(-kx) = G(x), \qquad \text{(S3)}$$

where k' (m^{-1}) is an exponential damping factor, indicating a slighter decrease in a lower value. k ($= k'L$) (-) is the scaled exponential damping factor.

Based on reliable calibration methods, these two expressions appear to be linked. This can be demonstrated mathematically using Taylor series expansion. When the scaled distance x equals half, the following equations can be derived:

$$F(x) = \frac{2}{\alpha+2} - \frac{4\alpha}{(\alpha+2)^2}(x - 1/2) + \frac{8\alpha^2}{(\alpha+2)^3}(x - 1/2)^2 - \frac{16\alpha^3}{(\alpha+2)^4}(x - 1/2)^3 + R_1(x) \qquad \text{(S4)}$$

and

$$G(x) = \frac{1}{\mathrm{e}^{k/2}} - \frac{k}{\mathrm{e}^{k/2}}(x - 1/2) + \frac{k^2}{2\mathrm{e}^{k/2}}(x - 1/2)^2 - \frac{k^3}{6\mathrm{e}^{k/2}}(x - 1/2)^3 + R_2(x), \quad \text{(S5)}$$

where $R_1(x)$ and $R_2(x)$ are the residual terms. To analyze the importance of each term in Eq. (S4), the first four terms are represented as:

$$f_1 = \frac{2}{\alpha+2}, \quad \text{(S6)}$$

$$f_2 = -\frac{4\alpha}{(\alpha+2)^2}(x - 1/2), \quad \text{(S7)}$$

$$f_3 = \frac{8\alpha^2}{(\alpha+2)^3}(x - 1/2)^2, \quad \text{(S8)}$$

$$f_4 = -\frac{16\alpha^3}{(\alpha+2)^4}(x - 1/2)^3. \quad \text{(S9)}$$

In these equations α is larger than zero due to wave attenuation. Since x is in the range of zero to unit, Eq. (S7) can obtain its largest value when x equals zero, and in this case,

$$f_{2,\max} = \frac{2\alpha}{(\alpha+2)^2}. \quad \text{(S10)}$$

Similarly, Eq. (S8) has the largest value when x equals zero or unity:

$$f_{3,\max} = \frac{2\alpha^2}{(\alpha+2)^3}. \quad \text{(S11)}$$

And Eq. (S9) can obtain the largest value in the case of $x = 0$:

$$f_{4,\max} = \frac{2\alpha^3}{(\alpha+2)^4}. \quad \text{(S12)}$$

To evaluate the relative magnitudes of the different terms of Eq. (S4), Fig. S1 presents the factors. The result demonstrates that the first two terms play the most significant roles.

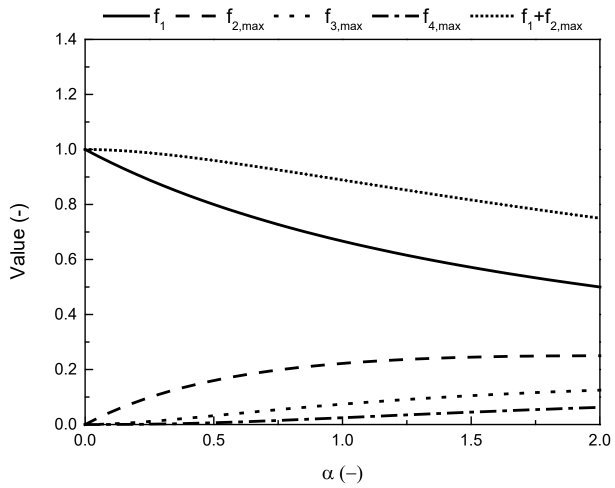

Fig. S1. Comparison between the factors in Eq. (S4) as a function of the damping factor α.

Similarly, the importance of each term in Eq. (S5) is analyzed and the following expressions are obtained:

$g_1 = \frac{1}{e^{k/2}}$, (S13)

$g_{2,\max} = \frac{k}{e^{k/2}}$, (S14)

$g_{3,\max} = \frac{k^2}{8e^{k/2}}$, (S15)

$g_{4,\max} = \frac{k^3}{48e^{k/2}}$. (S16)

Fig. S2 shows the comparison between these terms as a function of the exponential damping factor. Based on experience, the value of k is always in the range of zero to two. Under this circumstance, it is obvious that the first two terms are the key ingredients to Eq. (S5) and the lower the value of k, which means the slower the wave attenuates, the more important the first term.

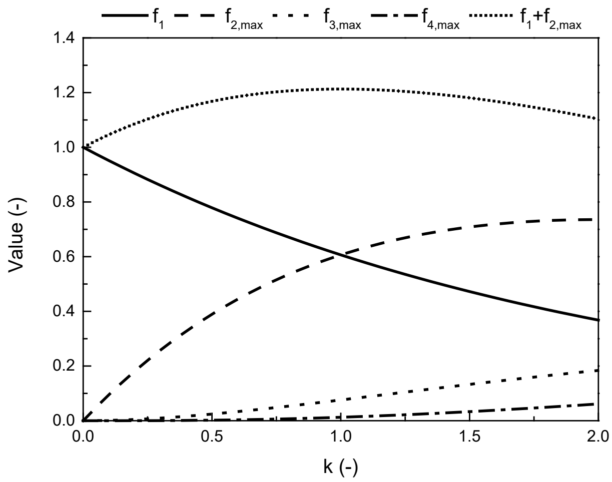

Fig. S2. Comparison between the factors in Eq. (S5) as a function of the exponential damping factor k.

Consider only the first two terms, equating the first and second terms of Eqs. (S4)–(S5), from which the x-dependent part can be eliminated, leads to two equations which results in the following proportionality:

$$\alpha = \frac{2k}{2-k}. \qquad \text{(S17)}$$