

1 **A new criterion for determining the representative elementary volume of**
2 **translucent porous media and inner contaminant**

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11

12 **ABSTRACT**

13 Representative elementary volume (REV) is essential to measure and quantify the effective
14 parameters of a complex heterogeneous medium. To overcome the limitations of the existing
15 REV estimation criteria, a new REV estimation criterion (χ^i) based on dimensionless range
16 and gradient calculation is proposed in this study to estimate REV of a translucent material
17 based on light transmission techniques. Three sandbox experiments are performed to
18 estimate REVs of porosity, density, tortuosity and perchloroethylene (PCE) plume using
19 multiple REV estimation criteria. In comparison with χ^i , previous REV estimation criteria
20 based on the coefficient of variation (C_v^i), the entropy dimension (DI^i) and the relative
21 gradient error (ε_g^i) are tested in REV quantification of translucent silica and inner PCE
22 plume to achieve their corresponding effects. Results suggest that new criterion (χ^i) can
23 effectively identify the REV in the materials, whereas the coefficient of variation and
24 entropy dimension (DI^i) are not effective. The relative gradient error can make the
25 REV plateau obvious, while random fluctuations make the REV plateau uneasy to
26 identify accurately. Therefore, the new criterion is appropriate for REV estimation of the
27 translucent materials and inner contaminant. Models are built based on Gaussian equation
28 to simulate the distribution of REVs for media properties, which frequency of REV is dense
29 in the middle and sparse on both sides. REV estimation of PCE plume indicates high level
30 of porosity lead to large value of mean and standard deviation for REVs of PCE saturation
31 (S_o) and PCE-water interfacial area (A_{ow}). Fitted equations are derived from distribution of
32 REVs for PCE plume related to d_m (distances from mass center to considered point) and d_l
33 (distances from injection position to considered point). Moreover, relationships between

34 REVs of PCE plume and S_o are fitted using regression analysis. Results suggest a decreasing
35 trend appears for S_o -REV when S_o increases, while A_{ow} -REV increases with increasing of
36 S_o .

37 **Keywords:** new criterion; representative elementary volume (REV); translucent material

38 **1. Introduction**

39 Modelling groundwater and contaminant (such as hazardous irons) transport in
40 subsurface environment is based on the premise that micro-structure of aquifer exist a
41 representative elementary volume (REV) (Wang et al., 2016; Lei and Shi, 2019). REV acts
42 as a micro-scale characteristic, which is important to improve our understanding of
43 materials, inner fluid flow and other processes (Brown and Hsieh, 2000; Costanza-Robinson
44 et al., 2011; Wu et al., 2017). Previous studies suggested that even the Platinum-
45 Nanoparticle-Catalyzed hydrogenation reactions and ion transport through angstrom-scale
46 slits in cell activity existed apparent size effect, implying size effect is a wide characteristic
47 for many process and materials (Bai et al., 2016; Esfandiar et al., 2017). With the help of
48 REV, a porous medium can be treated as continuum medium (Brown and Hsieh, 2000; Kang
49 et al., 2003; Müller and Siegesmund, 2010; Teruel and Rizwan-uddin, 2010; Hendrick et al.,
50 2012; Wang et al., 2012; Ukrainczyk and Koenders, 2014; Kim and Mohanty, 2016;
51 Gilevska et al., 2019). A conceptual representation of “REV curve” (Brown and Hsieh,
52 2000), characterizing porosity (θ) change with measured scale (L) increment, is presented
53 in Fig. 1a. Based on the characteristic of REV curve in Fig. 1a, the REV curve can be divided
54 into three regions. When measured scale (Fig. 1b) is in region I, the porosity fluctuates
55 drastically at small scales. As measured scale size ranging between L_{min} and L_{max} , a flat

56 plateau with constant and steady value is encountered and the property is factored into its
57 average value. Material property in spatial scales less than L_{min} is spatially varied portions
58 with small scale, which can be easily influenced by individual pores in micro-structure such
59 as region I (Fig. 1a). Likewise, material property is allowed drift to new values in spatial
60 scale above L_{max} due to additional morphological structures of large field heterogeneity
61 (region III). As a matter of fact, REV scale of region II can be derived between the small
62 and spatially varied property in region I and large field variability in region III. However,
63 the lower and upper boundaries L_{min} and L_{max} of REV plateau is hard to be identified in
64 reality (Brown and Hsieh, 2000; Costanza-Robinson et al., 2011).

65 As technology advanced and progressed, non-destructive and non-invasive techniques
66 of x-ray and gamma ray micro-tomography were utilized for micro-structure characteristic
67 measurement in materials (Ghilardi, 1993; Brown and Hsieh, 2000; Niemet and Selker,
68 2001; Bob et al., 2008; Al-Raoush and Papadopoulos, 2010; Costanza-Robinson et al., 2011;
69 Al-Raoush, 2012; Borges and Pires, 2012; Fernandes et al., 2012; Rozenbaum and Roscoat,
70 2014; Pereira Nunes et al., 2016; Piccoli et al., 2019). Generally, REV estimation for
71 material properties, inner gas and fluid also was usually implemented by micro visualization
72 and scanning of X-ray and gamma ray in laboratory (Brown and Hsieh, 2000; Razavi et al.,
73 2007; Nordahl and Ringrose, 2008; Al-Raoush and Papadopoulos, 2010; Costanza-
74 Robinson et al., 2011; Rozenbaum and Roscoat, 2014; Borges et al., 2018), while different
75 criteria were utilized to quantify REV (Brown and Hsieh, 2000; Martínez et al., 2007;
76 Nordahl and Ringrose, 2008; Costanza-Robinson et al., 2011). Lower boundary scale L_{min}
77 of REV was identified by means of entropy dimension (DI^*) for eight soil samples (Martínez

78 et al., 2007). Further, REV scale of permeability for ripple laminated sandstone intercalated
79 with mudstone was estimated using the coefficient of variation (C_V^i), which the REV scale
80 is identified by the variability among the ten samples to achieve average REV scale
81 (Nordahl and Ringrose, 2008). As a result, only one REV boundary was identified and
82 not every sample can be estimated effectively (Nordahl and Ringrose, 2008). More
83 interestingly, REV of material property (porosity), moisture saturation and air-water
84 interfacial areas in porous media were estimated by a criterion named the relative gradient
85 error (ϵ_g^i)(Costanza-Robinson et al., 2011). REVs of permeability of translucent material,
86 PCE saturation and PCE-water interfacial area also can be estimated using the relative
87 gradient error (Wu et al., 2017). In summary, the REV estimation was made by multiple
88 kinds of criteria, while the REV identification effects of these criteria were not clear.
89 What's more, these previous criteria estimate REV scale unsatisfactorily that beginning
90 and ending of REV plateau can't be identified simultaneously for translucent porous media
91 based on light transmission technique. Therefore, new criterion which can identify REV
92 plateau accurately is needed.

93 In this study, a new criterion (χ^i) for REV estimation is proposed to identify the REV
94 scale of the translucent silica and inner contaminant. Three perchloroethylene (PCE)
95 transport experiments are conducted in two dimensional (2D) sandboxes to test the effect of
96 different REV estimation criteria. Translucent silica is selected for associated REV analysis
97 due to its extensive utilization in laboratory experiment of exploring groundwater flow and
98 contaminant migration behavior in micro-structure of a sandy aquifer (Niemet and Selker,
99 2001; Bob et al., 2008; Costanza-Robinson et al., 2011). Moreover, translucent silica is also

100 an important material applied in numerous industries (Bouvry et al., 2016). In laboratory
101 experiments, translucent silica is packed in 2D sandboxes where porosity, density, tortuosity
102 and PCE saturation are derived by light transmission technique (Fig. 1c). Porosity and PCE
103 saturation are selected as the representative variables to explore corresponding REV
104 estimation by different criteria, which is very essential and significant for REV
105 identification. Previous criteria such as the coefficient of variation (C_V^i), entropy
106 dimension (D^i), the relative gradient error (ϵ_g^i) and the new criterion- χ^i are tested in REV
107 estimation. Associated effects are analyzed to achieve the best criterion of effective and
108 appropriate quantification of REV.

109 **2. Experiment procedure and method**

110 2.1 Experiment

111 Three sandboxes (Fig. 2a-c) packed by translucent silica medium are prepared in
112 laboratory to test different criteria of REV quantification. PCE is selected as a typical
113 DNAPL contaminant used in experiments. 2D sandbox is composed by three aluminum
114 interior frames and two glass walls, which thickness is 1.6cm. The dimensions of sandboxes
115 used in Experiment-I are 20 (width) \times 15 (height), and the dimensions of Experiments-II and
116 III are 60 (width) \times 45 (height). F40/50 and F20/30 mesh translucent silica sands are used for
117 background material for Experiments-I and II, while heterogeneous translucent silica with
118 low porosity and permeability are packed in sandbox for Experiment-III. To make the
119 translucent silica fully saturated by water in a flow field close to natural groundwater
120 environment (Erning et al., 2012), water flow at flow velocity of 0.5 m/d is set from left to
121 right in laboratory sandbox experiments (Fig. 2a-c). Water is restricted in a sandbox that the

122 top and bottom layers of the sandbox are packed by F70/80 mesh translucent silica as
123 capillary barriers. Light source is placed behind the sandbox to make light penetrate through
124 translucent media and capture emergent light intensity using a thermoelectrically air-cooled
125 charge-coupled device (CCD) camera (Fig. 1c). Afterward, PCE is injected into sandboxes
126 from the injection needle at constant rate of 0.5 mL /min for three experiments. Detailed
127 experimental conditions are listed in Table 1.

128 2.2 Light transmission technique

129 By means of light transmission technique (Fig. 1c), DNAPL and water saturation can
130 be obtained rapidly and in real-time, which greatly help to explore the mechanism of
131 groundwater flow and contaminant migration in porous media. When light passes through
132 translucent materials of a given thickness, the emergent light intensity after the absorptive
133 and interfacial losses can be expressed as (Niemet and Selker, 2001; Bob et al., 2008; Wu
134 et al., 2017):

$$135 \quad I_T = CI_0(\prod \tau_b)\exp(-\sum \alpha_a d_a) \quad (1)$$

136 where a represents phase number; b represent the number of the interface between phase a
137 and $a+1$; I_0 is the original light intensity; C is a constant of correction for light emission
138 and light observation; τ_b is the transmittance when light penetrate from phase a to $a+1$;
139 α_a is the absorption coefficient when light penetrate in phase a ; d_a is the length of light
140 penetration path in phase a .

141 To derive the porosity, the 2D translucent porous medium should be only saturated
142 by water. Consequently, the emergent light intensity can be expressed as (Niemet and
143 Selker, 2001; Bob et al., 2008; Wu et al., 2017):

144
$$I_s = CI_o \tau_{s,w}^{2k_o} \exp(-\alpha_s k_s d_s) \quad (2)$$

145 where $\tau_{s,w}^{2k_o}$ is the transmittance of solid-water interface; α_s is solid particles absorption
 146 coefficient; d_s is median diameter of the solid particles; k_o is the number of pores across
 147 light penetration path; k_s is the number of solid particles across light penetration path.

148 If we arbitrarily select an infinitesimal element, its area A_o approaches zero ($A_o \rightarrow 0$)
 149 from the 2D translucent porous media (Fig. 1d), and suppose the infinitesimal element
 150 with thickness L_T containing solid particles and pores that can be regarded as lamellar
 151 structure (Fig. 1d), we can obtain the following relationships (Wu et al., 2017):

152
$$\theta A_o L_T = A_o k_o d_o \quad (3)$$

153
$$k_s d_s + k_o d_o = L_T \quad (4)$$

154 where d_o is the median diameter of pores; θ is porosity.

155 Substituting Eq. (3) and Eq. (4) into Eq. (2), the relationship between emergent
 156 light intensity and porosity can be achieved (Wu et al., 2017):

157
$$\ln I_s = \beta + \theta \gamma \quad (5)$$

158 where $\beta = \ln\left(\frac{CI_s}{e^{\alpha_s d_s L_T}}\right)$ and $\gamma = \ln(\tau_{s,w}^{d_o} e^{\alpha_s L_T})$. β and γ can be determined from
 159 experimental data, then porosity can be obtained.

160 The density and tortuosity are derived as (Wu et al., 2018):

161
$$\rho = \theta \rho_w + (1.0 - \theta) \rho_s \quad (6)$$

162
$$\tau = 1 + \frac{\pi - 2}{\sqrt{\frac{\pi}{1 - \theta}}} \quad (7)$$

163 where ρ is the density of translucent porous media; ρ_w is the density of water; ρ_s is the
 164 density of solid particles; τ is tortuosity.

165 The saturation of dense nonaqueous phase liquid (DNAPL) was quantified by light
166 transmission technique based on light pass through translucent materials (Niemet and Selker,
167 2001; Bob et al., 2008):

$$168 \quad S_o = \frac{\ln I_s - \ln I_T}{\ln I_s - \ln I_{oil}} \quad (8)$$

169 where S_o is the saturation of DNAPL; I_s is the light intensity after light penetration through
170 translucent porous when all pores are fully saturated by water; I_{oil} is the light intensity
171 when all pores are fully saturated by DNAPL; I_T is the light intensity after penetration
172 through translucent materials. After quantification of PCE saturation, PCE-water interfacial
173 area (A_{ow}) can be obtained based on the method proposed by Wu et al. (2017), where the
174 unit of A_{ow} is cm^{-1} .

175 Emergent light intensity for three experiments is captured by a thermoelectrically air-
176 cooled charge-coupled device (CCD) camera (Niemet and Selker, 2001; Bob et al., 2008).
177 Every pixel with small scale could be approximated as infinitesimal element in high
178 resolution image to apply light transmission techniques. As consequence, porosity of
179 translucent silica was derived using light transmission technique through Eq. (5). The whole
180 2D translucent silica area was numerically discretized that every cell had the uniform
181 dimensions of $0.015\text{m} \times 0.015\text{m}$. The cuboid window (Fig. 1b) was utilized to quantify the
182 variables (porosity, density, tortuosity, PCE saturation, PCE-water interfacial area) of every
183 cell as measured scale was increased. In detail, the measured cuboid window scale was
184 increased from the center of each cell and associated value of variable was calculated from
185 the high resolution porosity of 2D translucent silica derived by light transmission technique.
186 Observation cells were selected from the discretized cells (Fig. 3b) of which the cells I-1~2,

187 II-1~2 and III-1~2 belong to Experiments-I-III, respectively.

188 To analyze the regularity of REV distribution for PCE plume, the mass center
189 coordinate and the granglia-to-pool ratio (GTP) of PCE plume are quantified for
190 Experiments-I-III. The mass center coordinate and GTP are calculated as:

$$191 \quad X_m = \frac{M_{10}}{M_{00}} \quad (9)$$

$$192 \quad Z_m = \frac{M_{01}}{M_{00}} \quad (10)$$

$$193 \quad GTP = \frac{V_{g\text{anglia}}}{V_{pool}} \quad (11)$$

194 where X_m is x coordinate of mass center for PCE plume; Z_m is z coordinate of mass center
195 for PCE plume; GTP is granglia-to-pool ratio, which equals to the ratio of the $V_{g\text{anglia}}$ to
196 V_{pool} ; $V_{g\text{anglia}}$ is the PCE volume under ganglia state; V_{pool} is the PCE volume under pool
197 state; M_{00} , M_{10} and M_{01} are computed using definition of spatial moment (M_{ij}),
198 $M_{ij} = \int_{x_0}^{x_1} \int_{z_0}^{z_1} \theta(x, z) S_o(x, z, t) x^i z^j dx dz$; x_0 and z_0 are minimum limits of x axis and z axis; x_1
199 and z_1 are maximum limits of x axis and z axis; $\theta(x, z)$ is the porosity at point (x, z); $S_o(x, z, t)$
200 is PCE saturation of point (x, z) at time t.

201 2.3 Criteria of REV quantification

202 The REV is defined as the volume range in which all material characteristics are
203 factored into the average and associated values approach single and constant (Brown
204 and Hsieh, 2000). In the range of REV, the value of one associated property will meet
205 the condition:

$$206 \quad \left. \frac{\partial Y(L_i)}{\partial L} \right|_{L_i=L_o} = 0 \quad (12)$$

207 where the $Y(L_i)$ is the value of an associated property when system scale is L_i ; L_i is the

208 value of system scale; L_o is the scale range of REV, $L_{\min} < L_o < L_{\max}$; L_{\max} is upper
 209 boundary of REV; L_{\min} is lower boundary of REV scale. According to the Eq. (12), when
 210 the measured scale (L_i) reaches REV range, $\frac{\partial Y(L_i)}{\partial L} \rightarrow 0$. As a matter of fact, most
 211 previously used criteria were applied to estimate REV based on this requirement (Brown
 212 and Hsieh, 2000; Martínez et al., 2007; Nordahl and Ringrose, 2008; Costanza-Robinson
 213 et al., 2011).

214 To evaluate the REV of porosity, the coefficient of variation (C_V^i) is utilized to
 215 estimate the variability (Nordahl and Ringrose, 2008):

$$216 \quad C_V^i = \frac{\hat{s}_i}{\bar{\varphi}_i} \quad (13)$$

217 where i is the cuboid window (Fig. 1b) increment number; φ is the measured variable,
 218 such as porosity; \hat{s}_i is the standard deviation of sub-grids' variable in different measured
 219 volume or scale; $\bar{\varphi}_i$ is the arithmetic average of the variable values in the sub-grids.

220 When number of sub-grids (N) is less than 10, a correction is utilized to replace Eq. (13).
 221 According to Nordahl and Ringrose (2008), $0 < C_V^i < 0.5$ is defined as homogeneous
 222 and $C_V^i = 0.5$ can be used as criterion to identify the REV scale.

223 Similarly, for porosity of translucent silica, entropy dimension (DI^i) is utilized for
 224 REV analysis and estimation (Martínez et al., 2007), which is defined as:

$$225 \quad DI^i \approx \frac{\sum_{j=1}^{m(i)} \mu_j(L_e) \log \mu_j(L_e)}{\log L_e} \quad (14)$$

226 where, L_e is the scale of sub-grid; “ \approx ” indicates the asymptotic equivalence as $L_e \rightarrow 0$
 227 (Martínez et al., 2007); j is the ordinal number of sub-grid in measured cuboid window
 228 (Fig. 1b) of increment number i ; $m(i)$ is the total number of sub-grids in measured cuboid
 229 window (Fig. 1b) of increment number i ; $\mu_j(\varepsilon)$ is the proportion of the variable of sub-

230 grid j in the whole measured cuboid window i . The right hand side of Eq. (14) is the
 231 simplification of Shannon entropy of all sub-grids, in which DI^i can be considered as the
 232 average of logarithmic values of the variable distribution weighted by $\mu_j(L_\varepsilon)$ to quantify
 233 the degree of medium heterogeneity. Using Eq. (14), a series of values of DI^i ($i=1,2,3\dots$)
 234 are obtained for each measured cuboid window (Fig. 1b) of increment number i . For
 235 estimation of the REV in a porous medium, the relative increment of entropy dimension
 236 and associated criterion of REV identification are respectively expressed as:

$$237 \quad RI^i = \left| \frac{DI^j - DI^{j-1}}{DI^{j-1}} \right| \times 100 \quad (15)$$

$$238 \quad RI^i \leq 0.2CV_{DI} \quad (16)$$

239 where CV_{DI} is the coefficient of variation of DI^i series ($i=1,2,3\dots$), which is calculated
 240 through $CV_{DI} = (\hat{s}_{DI}/\overline{DI}) \times 100$; \overline{DI} is the mean value of the DI^i series; \hat{s}_{DI} is the
 241 standard deviation of the DI^i series.

242 To achieve the REV for multiple system variables, such as porosity, moisture saturation
 243 and air-water interfacial areas in an unsaturated porous medium, a criterion named the
 244 relative gradient error was applied (Costanza-Robinson et al., 2011):

$$245 \quad \varepsilon_g^i = \left| \frac{\varphi^{i+1} - \varphi^{i-1}}{\varphi^{i+1} + \varphi^{i-1}} \right| \frac{1}{\Delta L} \quad (17)$$

246 where ε_g^i is relative gradient error; ΔL is the measured cuboid window size increment
 247 length for REV estimation. Usually, ε_g^i less than 0.2 (Costanza-Robinson et al., 2011) is
 248 utilized to identify a REV sizes.

249 According to the requirement in Eq. (12), a new criterion based on the required
 250 condition of REV is proposed to estimate the REV range of the translucent silica in this
 251 study:

252
$$\chi^i = \frac{|\delta^{i+1} - \delta^{i-1}|}{\delta^i \Delta L} \quad (18)$$

253 where δ^i is the dimensionless range, $\delta^i = \frac{\varphi(L_i)_{max} - \varphi(L_i)_{min}}{\overline{\varphi(L_i)}}$; $\varphi(L_i)_{max}$ is the
 254 maximum value of the variable on the volume scale L_i ; $\varphi(L_i)_{min}$ is the minimum value
 255 of the variable on the volume scale L_i ; $\overline{\varphi(L_i)}$ is the mean value of the variable on the
 256 volume scale L_i . Brown and Hsieh (2000) suggested $\delta^i = \frac{\varphi(L_i)_{max} - \varphi(L_i)_{min}}{\overline{\varphi(L_i)}} \ll 1$ can
 257 be used for REV estimation. In fact, the calculated value of δ^i mostly is less than 1, while
 258 $\delta^i \ll 1$ is hard to be used to identify the REV scale for realistic materials, such as the
 259 translucent silica used in this study. The value limit of χ^i used for REV estimation also is
 260 explored in this study.

261 In this study, criteria for the coefficient of variation (C_v^i), entropy dimension (DI^i),
 262 the relative gradient error (ε_g^i) and the new criterion (χ^i) are all applied to REV estimation
 263 for porosity and PCE saturation. Corresponding REV plateau identification effects are
 264 compared to select the best criterion for REV quantification.

265 **3. Results and discussion**

266 3.1 REV identification effect of different criteria

267 3.1.1 The coefficient of variation

268 Emergent light intensity distributions of translucent silica for three experiments, which
 269 had been fully saturated by water, were obtained by a thermoelectrically air-cooled CCD
 270 camera (Niemet and Selker, 2001; Bob et al., 2008). The porosity, density, tortuosity and
 271 PCE saturation for three experiments are derived by light transmission technique as shown
 272 in Figs. 3a and b. The PCE spreads from the injecting point shaped like a drop of water at
 273 $t=5$ min (Fig. 3b). In 2D sandboxes for three experiments, PCE plume infiltrates in

274 translucent silica sands and reaches the bottom after $t=80$ min.

275 Porosity and PCE saturation variation curves of all observation cells with increasing
276 measured cuboid window scale were shown in Figs. 4a and b. However, for all observation
277 cells from translucent silica, the REV plateaus were not obvious to be objectively judged
278 visually, which made REV plateaus hard to identify effectively by original variation curves
279 for porosity and PCE saturation (Figs. 4a and b).

280 To make the REV plateau more explicit, different criteria of REV quantification are
281 utilized. The coefficient of variation (C_V^i) of porosity and PCE saturation fluctuating with
282 increasing of measured cuboid window size is shown in Fig. 4. The measured cuboid
283 window scale is limited to the dimensions of cells in discretization of 2D translucent
284 silica. The observation cells show various characteristics of variation tendency for the
285 coefficient of variation (C_V^i). The θ and S_o variation curves of the coefficient of variation
286 (C_V^i) for all observation cells do not reach stable values as those shown in Figs. 4a and b,
287 the beginning of the REV flat plateau is not easy to identify, the coefficient of variation (C_V^i)
288 is not suitable for REV estimation. According to the heterogeneity definition by Corbett and
289 Jensen (1992), the heterogeneity of materials is defined by C_V^i magnitude that $0 <$
290 $C_V^i < 0.5$ is classed as homogeneous medium, $0.5 < C_V^i < 1.0$ is classed as
291 heterogeneous medium and $1.0 < C_V^i$ is classed as strong heterogeneous medium. For
292 the coefficient of variation (C_V^i) magnitude in Figs. 4a and b, the C_V^i values are all below
293 0.5 that the criterion $C_V^i = 0.5$ is unable to identify the REV scale for translucent silica.

294 3.1.2 Entropy dimension

295 Entropy dimension (DI^i) is utilized by Martínez et al. (2007) for multifractal analysis
296 of a porous medium porosity and REV estimation. In this study, entropy dimension (DI^i) is
297 tested to avoid unclear REV plateau in porosity curves. The entropy dimension (DI^i) of
298 porosity is calculated by Eq. (14). Variation curves of entropy dimension (DI^i) for all
299 observation cells (Fig. 2a) are presented in Fig. 4. The curves of entropy dimension (DI^i) of
300 porosity and PCE saturation generally result in the increasing trend curves which makes
301 REV estimates become very difficult and invalid. Entropy dimension (DI^i) was quickly
302 increased with increasing of measured cuboid window size. Compared to the coefficient of
303 variation (C_V^i) of porosity and PCE saturation, entropy dimension (DI^i) increased step by
304 step without opposite fluctuation tendency in the variation curves as the length scale of
305 measured cuboid window increased simultaneously. In general, REV plateau in region II
306 (Fig. 1a) of porosity is not obvious for the entropy dimension (DI^i) curves of all observation
307 cells from three experiments, which suggests REV scales is uneasy to identify for
308 translucent silica using entropy dimension (DI^i) by light transmission technique.

309 3.1.3 The relative gradient error

310 The relative gradient error (ε_g^i) of porosity and PCE saturation is calculated by Eq.
311 (17). The variation of ε_g^i at different measured cuboid window scales is shown in Fig. 4
312 for all observation cells in the 2D translucent silica. For all ε_g^i curves at observation
313 cells from experiments, the REV plateaus in region II (Fig. 1a) are more clear than the
314 variation curves based on the criteria of C_V^i and DI^i . Apparently, erratic variations of

315 the relative gradient error (ε_g^i) at small measured cuboid window scales are observed for
316 all ε_g^i curves as the characteristic of REV region I in Fig. 1a. When measured cuboid
317 window scale further increases for all observation cells, the variability and magnitude of
318 the relative gradient error (ε_g^i) decrease well and factor into average, which can be
319 identified as REV plateau in region II (Fig. 1a). The relative gradient error (ε_g^i) makes the
320 REV plateau quantification convenient for all observation cells. At the measured cuboid
321 window size above the REV plateau, ε_g^i curves result in large variability for observation
322 cells I-1~2. These findings suggest that that the relative gradient error (ε_g^i) can make the
323 REV plateau more obvious, which greatly contribute to convenient and applicable REV
324 quantification for translucent silica by light transmission technique. However, random
325 fluctuations exist in ε_g^i curves visually, which make the REV plateau uneasy to identify
326 accurately.

327 3.1.4 The new criterion (χ^i)

328 χ^i of porosity and PCE saturation changing with measured cuboid window size is
329 shown in Fig. 4. Like the region I (Fig. 1a), erratic and random fluctuations appear at
330 small measured cuboid window sizes and χ^i increases with the increase of the measured
331 cuboid window size. When measured scale increases, the values of χ^i for all observation
332 cells appear fast reduction and rapidly tend to steady, which exhibit the characteristic of
333 REV plateau as measured scale reaches region II. The χ^i for observation cells restores the
334 erratic variation state of increasing trend after measured cuboid window size exceeding the
335 REV plateau. As shown in the variation curves of χ^i for all observation cells, the beginning
336 of the REV flat plateaus can be identified easily, suggesting χ^i is more convenient and

337 reliable than other methods for REV estimation. All observation cells show similar variation
 338 curves of χ^i that low value intervals are quite apparent, indicating that χ^i is very effective to
 339 make the REV plateau obvious for translucent silica used in this study. Using the criterion
 340 of χ^i , the REV plateau of region II is flat, which is easily identified, compared with other
 341 criteria for observation cells (Figs. 4a and b).

342 3.2 REVs of material properties

343 Based on the REV plateau identifications using the coefficient of variation
 344 (C_V^i), entropy dimension (DI^i), the relative gradient error (ϵ_g^i) and the proposed new
 345 criterion χ^i in Figs. 4a and b, the new criterion χ^i appears to be the most appropriate criterion
 346 for REV plateau identification. Even though the relative gradient error (ϵ_g^i) can also make
 347 REV plateau obvious, but various random fluctuations weaken the method and reduce the
 348 associated accuracy. Therefore, REVs of porosity, density, tortuosity and PCE plume are
 349 estimated using the new criterion χ^i in the following study.

350 In fact, large number of discretized cells in the 2D translucent silica for three
 351 experiments are quantified using the new criterion χ^i , which is convenient to examine the
 352 regularities for REV sizes and related factors. Using the new criterion χ^i , the REV estimation
 353 is conducted based on Eq. (18). Fig. 5a shows minimum REV sizes of porosity, density and
 354 tortuosity quantified by χ^i for all cells of three experiments. Associated statistical analysis
 355 for REVs is illustrated in Fig. 5b, where circular points represent frequency and triangular
 356 points represent cumulative frequency. Frequency of REVs is dense in the middle and sparse
 357 on both sides, so the distribution of REVs can be fitted by Gaussian equation:

$$358 \quad F = \omega + \frac{1}{\sqrt{2\pi\epsilon}} e^{-\frac{(REV-v)^2}{2\epsilon^2}} \quad (19)$$

359 where F is the frequency of REV; ω , ϵ and ν are fitted parameters of the model.

360 After regression analysis, the derived models for REV frequency are listed in Table 2.

361 The coefficients of determination (R^2) of models for REVs of porosity and density for three

362 experiments all exceed 0.85. R^2 for REV of tortuosity for three experiments exceeds 0.76.

363 Moreover, the computed cumulative frequency based on models fit cumulative frequency

364 from experimental results well in Fig. 5b.

365 The minimum REV size frequency of porosity appears a peak between 4.0 mm and 5.0

366 mm for Experiment-I. As minimum REV size of porosity increases, corresponding

367 frequency continuously decreases. Further, smooth convex shape of cumulative frequency

368 is observed for minimum REV size of porosity (Fig. 5b). Most minimum REV sizes of

369 translucent silica distributed in 0.0-7.0mm. For density of translucent silica sand, associated

370 REV frequency appears high values between 2.0~3.0 mm. For the REV sizes of tortuosity,

371 minimum REV sizes distribute in 0.0~6.0 mm. Compared with Experiment-I (F40/50 mesh

372 translucent silica sand), the frequency of REV for Experiment-II (F20/30 mesh translucent

373 silica sand with larger porosity) shows flat shape and has larger value of standard deviation,

374 especially for REV of porosity. Fig. 5b shows that translucent silica with larger porosity will

375 achieve border distribution of minimum REV sizes distribution compared to translucent

376 silica with relative lower porosity. Moreover, the frequency of REV of porosity and

377 permeability for Experiment-III (background material is F20/30 mesh translucent silica sand

378 with larger porosity, five lenses with lower porosity is packed in sandbox to create

379 heterogeneity) is similar to the frequency of REV for Experiment-II. However, the

380 frequency of τ -REV for Experiment-III is different from the frequency of τ -REV for

381 Experiment-II under homogeneous condition. The mean REV sizes of porosity, density and
382 tortuosity for Experiment-I are 4.35 mm, 2.89 mm and 3.65 mm, respectively. All mean
383 REV sizes of these variables for Experiment-II are larger than REVs of Experiments-I,
384 which corresponding mean REV sizes are 8.05 mm, 2.97 mm and 4.30 mm. These results
385 suggest translucent porous media with higher porosity lead to larger values of mean and
386 standard deviation for REV sizes.

387 3.3 REVs of S_o and A_{ow} for PCE plume

388 Based on the new criterion χ^i and light transmission technique, the real-time
389 distributions of S_o -REV and A_{ow} -REV for PCE plume can be obtained over the entire
390 experimental period. The minimum REV sizes of S_o and A_{ow} obtained using new criterion
391 χ^i are shown in Figs. 6a and b. When PCE migrates in sandbox, the REV of PCE plume is
392 changed over time (Fig. 6). The REVs of PCE plume for Experiment-I mostly are lower
393 than the REVs of PCE plume for Experiments-II and III. Moreover, when heterogeneous
394 porous media is packed in sandbox, the REV distribution of Experiment-III become more
395 heterogeneous compared with REV distribution of Experiments-II under homogeneous
396 condition. Based on REV distributions of PCE plume for three experiments, statistical
397 analysis is conducted to explore the regularity of REV distribution for PCE plume.

398 The mass center coordinate of PCE plume, GTP and plume area are shown in Fig. 7a.
399 The values of X_m , Z_m and GTP for Experiment-II and III are higher than the X_m , Z_m and
400 GTP of Experiment-I (lower porosity). Moreover, the plume area of Experiment-II is larger
401 than the plume of Experiment-I. When packed material is heterogeneous, the plume area of
402 PCE is increased further for Experiment-III. Besides, the mean and standard deviation of

403 REVs of PCE plume during 0~1523 min are derived by statistical analysis (Fig. 7a).
404 Compared with REVs of PCE plume for Experiment-I, Experiment-II (F20/30 mesh
405 translucent silica sand with higher porosity) has larger value of mean and standard deviation
406 of REVs. The mean value of A_{ow} -REV for Experiment-III is much higher than A_{ow} -REV
407 for Experiments-I and II.

408 The average value of REVs (\overline{REV}) and associated distance (d_m) from mass center to
409 corresponding cells contained in PCE plume at $t=1523$ min are presented in Fig. 7b.
410 Regression analysis is performed for average REVs of PCE plume and d_m , where fitted
411 models and associated R^2 for Experiments-I-III are listed in Table 3. Simultaneously, the
412 fitted equations between \overline{REV} and d_l (the distance from injection point to cell contained in
413 PCE plume) also are derived by regression analysis. From the results in Fig. 7a, \overline{REV} of
414 S_o and A_{ow} appear a peak and then decrease with increasing of d_m and d_l for Experiment-I.
415 \overline{REV} of S_o and A_{ow} for Experiment-I all firstly increase and then decrease with the
416 increasing of d_m and d_l . However, \overline{REV} of S_o presents apparent decreasing tendency as d_m
417 and d_l increase for Experiment-II, and \overline{REV} of A_{ow} just slightly increase first and then
418 decrease for Experiment-II. In addition, the value of A_{ow} -REV mostly is higher than the
419 value of S_o -REV for three experiments. Compared with the R^2 of the fitted relationship
420 between average REVs of PCE plume and d_m , d_l for Experiments-I and II, the values of R^2
421 achieved by Experiment-III are much lower (Table 3).

422 Besides, the relationship between REVs and PCE saturation are fitted by regression
423 analysis, where fitted equation and R^2 for three experiments are listed in Table 4 and Fig.
424 7b. With increasing of PCE saturation, REV of S_o appear decline trend for three experiments.

425 However, REV of A_{OW} increases when S_o increases for all three experiments (Fig. 7b). On
426 the other hand, REV of S_o for Experiment-II is higher than corresponding REV for
427 Experiment-I, while Experiments-I and II have similar values of A_{OW} -REV (Fig. 7b).
428 Moreover, REVs of S_o and A_{OW} for Experiment-III are higher than REVs of S_o and A_{OW} for
429 Experiments-I and II. These results suggest higher porosity will lead to high value of S_o -
430 REV and the relationship between REVs of PCE plume and d_m , d_f . S_o -REV and A_{OW} -REV
431 are increased under heterogeneous condition.

432 **4. Conclusions**

433 In this study, a new criterion χ^i is proposed to identify the REVs of translucent porous
434 media and inner contaminant transformation based on previous criteria. The REV plateaus
435 of observation cells selected from three experiments of PCE transport are hard to judge
436 visually from the porosity and PCE saturation curves. From the REV identification effects
437 of different criteria, the REV flat plateau is difficult to identify by the coefficient of variation
438 (C_V^i) and entropy dimension (DI^i), indicating the coefficient of variation (C_V^i) and entropy
439 dimension (DI^i) are not suitable for REV estimation of translucent porous media. The
440 relative gradient error (ε_g^i) can make REV plateaus of all kinds of translucent silica
441 explicit in variation curves, but random fluctuations weaken REV plateau identification. In
442 comparison with these previous criteria, the beginning and ending of the REV flat plateaus
443 could be easily and directly identified in the curves based on the new criterion χ^i , suggesting
444 the new criterion χ^i is more convenient and effective for REV estimation. In this study, REVs
445 of porosity, density, tortuosity, and PCE plume are estimated using the new criterion χ^i .

446 Statistical results of minimum REV scales quantified by new criterion χ^i reveal

447 cumulative frequencies of porosity, density and tortuosity all have smooth convex shapes.
448 Models based on Gaussian equation are built for the distribution of REV sizes of porosity,
449 density and tortuosity, which porous media with larger porosity leads to larger values of
450 mean and standard deviation for REV sizes of media properties. For REV sizes of PCE plume,
451 result suggested larger porosity lead to larger value of mean and standard deviation.
452 Regression analysis is performed to study the regularity for distribution of REV sizes, where
453 fitted relationship between REV sizes and d_m , d_l are derived for PCE plume. \overline{REV} of S_o and
454 A_{ow} firstly increase and then decrease with the increasing of d_m and d_l for Experiment-I
455 whose sandbox packed by translucent porous media with relatively lower porosity. However,
456 \overline{REV} of S_o and A_{ow} directly decrease with the increment of d_m and d_l when porosity became
457 larger for Experiment-II. The values of R^2 of the fitted relationship between average REV sizes
458 of PCE plume and d_m , d_l for Experiment-III are much lower under heterogeneous condition.
459 Significantly, REV size of S_o presented decreasing trend as S_o increases, while increasing
460 tendency appeared for REV size of A_{ow} . Through regression analysis, the fitted equations
461 between REV sizes of PCE plume and PCE saturation are derived for three experiments.
462 Implications of these finding are essential for quantitative investigation of scale effect of
463 porous media and contaminant transformation. The fluid migration and transform in porous
464 media can be simulated accurately according to the REV estimation results using light
465 transmission technique and the appropriate criterion χ^1 .

466 **Code and data availability**

467 The codes and data for this paper are available by contacting the corresponding author at

468 jfwu@nju.edu.cn.

469 **Author contributions**

470 Ming Wu: Conceptualization, Methodology, Writing;

471 Jianfeng Wu: Conceptualization, Methodology, Writing;

472 Jichun Wu: Conceptualization;

473 Bill X. Hu: Conceptualization, Writing.

474 **Declaration of interests**

475 The authors declare that they have no known competing financial interests or personal

476 relationships that could have appeared to influence the work reported in this paper.

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586

587 **Table 1.** Experimental conditions

Experiment	I	II	III
Sandbox dimensions (cm)	20×15	60×45	60×45
Background translucent silica sand	F40/50	F20/30	F20/30
Medium condition	Homogeneity	Homogeneity	Heterogeneity
Median grain diameter (mm)	0.36	0.72	0.72
Permeability (m ²)	4.25×10 ⁻¹¹	1.35×10 ⁻¹⁰	1.35×10 ⁻¹⁰
V _{PCE} (ml)	9	32	40
Injection rate (ml/min)	0.5	0.5	0.5

588

589

590 **Table 2.** The fitted equations of frequency for REV's of porosity, density and tortuosity

Experiment	I	II	III
θ-REV	ω	-2.11×10^{-4}	7.63×10^{-4}
	ε	1.73	3.18
	v	4.35	6.50
	R ²	0.938	0.924
ρ-REV	ω	-6.51×10^{-4}	1.51×10^{-3}
	ε	1.08	2.40
	v	2.89	3.70
	R ²	0.967	0.859
τ-REV	ω	-3.36×10^{-4}	1.29×10^{-3}
	ε	1.39	1.20
	v	3.65	1.05
	R ²	0.769	0.875

591 *θ represents porosity, ρ represents density, τ represents tortuosity; ω, ε and v are fitted parameters
 592 of the model

593

594

595 **Table 3.** The fitted equations between average value of REV and d_I , d_m

Experiment	d_m	d_I
I	S _o -REV $\overline{REV} = -1.67 \times 10^{-3}d_m^2 + 0.193d_m + 2.72$ (R ² =0.807)	$\overline{REV} = -1.97 \times 10^{-3}d_I^2 + 0.245d_I + 1.12$ (R ² =0.832)
	A _{ow} -REV $\overline{REV} = -6.10 \times 10^{-4}d_m^2 + 5.82 \times 10^{-2}d_m + 7.20$ (R ² =0.401)	$\overline{REV} = -1.47 \times 10^{-3}d_I^2 + 0.205d_I + 1.84$ (R ² =0.733)
II	S _o -REV $\overline{REV} = -4.08 \times 10^{-5}d_m^2 + 1.50 \times 10^{-2}d_m + 7.54$ (R ² =0.655)	$\overline{REV} = -3.94 \times 10^{-5}d_I^2 + 7.80 \times 10^{-3}d_I + 8.50$ (R ² =0.327)
	A _{ow} -REV $\overline{REV} = -1.92 \times 10^{-5}d_m^2 + 4.47 \times 10^{-3}d_m + 9.46$ (R ² =0.616)	$\overline{REV} = -1.92 \times 10^{-5}d_I^2 + 4.47 \times 10^{-3}d_I + 9.46$ (R ² =0.616)
III	S _o -REV $\overline{REV} = -6.06 \times 10^{-6}d_m^2 + 2.27 \times 10^{-3}d_m + 7.76$ (R ² =0.153)	$\overline{REV} = 1.69 \times 10^{-5}d_I^2 - 1.21 \times 10^{-2}d_I + 9.62$ (R ² =0.236)
	A _{ow} -REV $\overline{REV} = -8.71 \times 10^{-6}d_m^2 + 5.66 \times 10^{-3}d_m + 11.5$ (R ² =0.115)	$\overline{REV} = -1.50 \times 10^{-5}d_I^2 + 7.88 \times 10^{-3}d_I + 11.4$ (R ² =0.150)

596 * \overline{REV} is the average value of REV size, d_m is the distance from mass center of PCE plume
597 to the cell contained in PCE plume, d_I is the distance from injection point to the cell
598 contained in PCE plume

599

600

601 **Table 4.** The fitted relationship between REV and S_o

Experiment	S_o -REV	A_{ow} -REV
I	$\text{REV} = -2.13 \times \ln S_o + 0.876$ $(R^2=0.466)$	$\text{REV} = 2.27e^{2.70 \times S_o}$ $(R^2=0.366)$
II	$\text{REV} = -0.961 \times \ln S_o + 1.09$ $(R^2=0.415)$	$\text{REV} = 1.70e^{3.30 \times S_o}$ $(R^2=0.500)$
III	$\text{REV} = -1.40 \times \ln S_o + 3.96$ $(R^2=0.538)$	$\text{REV} = 2.05e^{3.22 \times S_o}$ $(R^2=0.573)$

602

603

604 **Figure Captions**

605 **Figure 1.** (a) Variable changes as measured scale (L) increment in conceptual curve
606 (Costanza-Robinson et al., 2011); (b) Scale effect and the cuboid image window
607 geometry; (c) System Device for acquisition of parameters (porosity and density,
608 etc.) of translucent material; (d) The infinitesimal selected from translucent porous
609 media packed in 2D sandbox.

610 **Figure 2.** (a) The system sandbox equipment of Experiment-I; (b) The system sandbox
611 equipment of Experiment-II; (c) The system sandbox equipment of Experiment-III

612 **Figure 3.** (a) The emergent light intensity, porosity, permeability and tortuosity of 2D
613 translucent silica sand for Experiments-I-III; (b) The PCE saturation of
614 Experiments-I-III during 0~1523 min and observation cells

615 **Figure 4.** (a) The change of porosity (θ), associated coefficient of variation (C_V^i), entropy
616 dimension (DI^i), the relative gradient error (ε_g^i), and new criterion- χ^i for observation
617 cells as cuboid window scale (L) increases; (b) The change of PCE saturation (S_o),
618 associated C_V^i , DI^i , ε_g^i , and χ^i for observation cells as cuboid window scale (L)
619 increases

620 **Figure 5.** (a) The distributions of minimum REV sizes of porosity, sand density and
621 tortuosity for Experiments-I-III; (b) The frequency of minimum REV sizes of
622 Experiments and fitted models

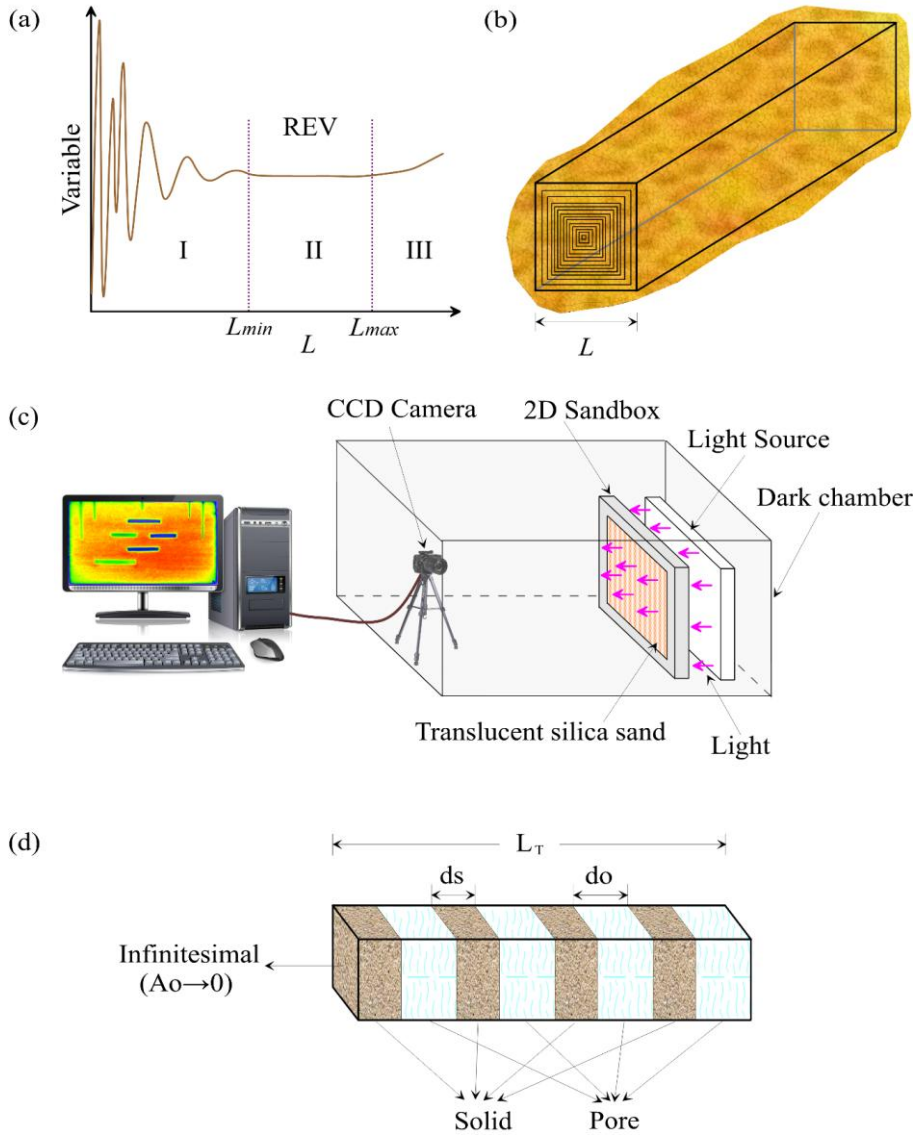
623 **Figure 6.** (a) The distributions of S_o -REV sizes during 0~1523 min for Experiments-I-III;
624 (b) The distributions of A_{ow} -REV sizes during 0~1523 min for Experiments-I-III

625 **Figure 7.** (a) The mass center coordinate of PCE plume, GTP, plume area and the mean,
626 standard deviation of S_o -REV and A_{ow} -REV during 0~1523 min; (b) The change of

627 average REV size as the distance d_i , d_m increases and fitted relationship between
628 REV sizes and S_o for Experiments-I and II

629 Fig. 1

630

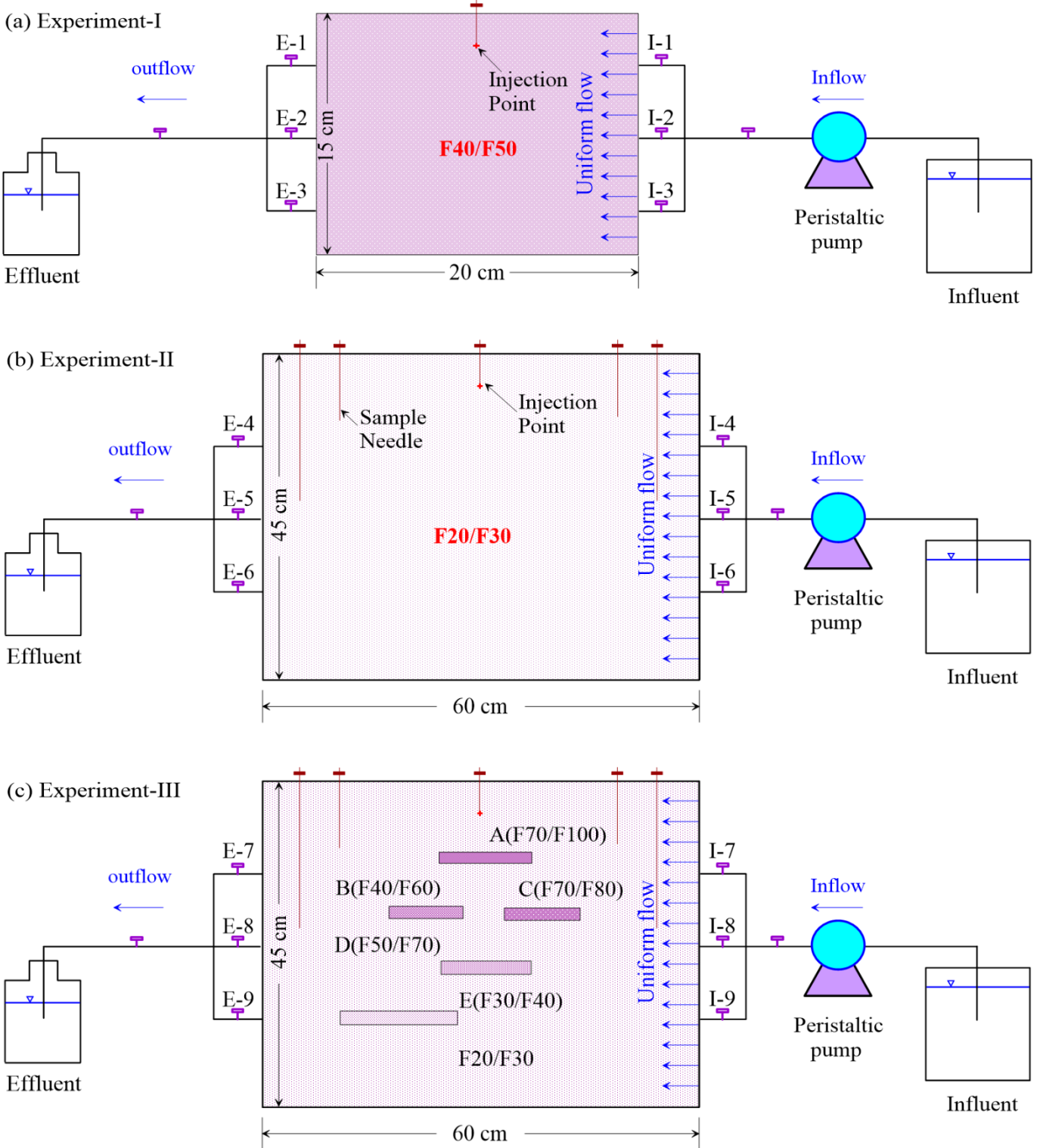


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633 **Fig. 2**

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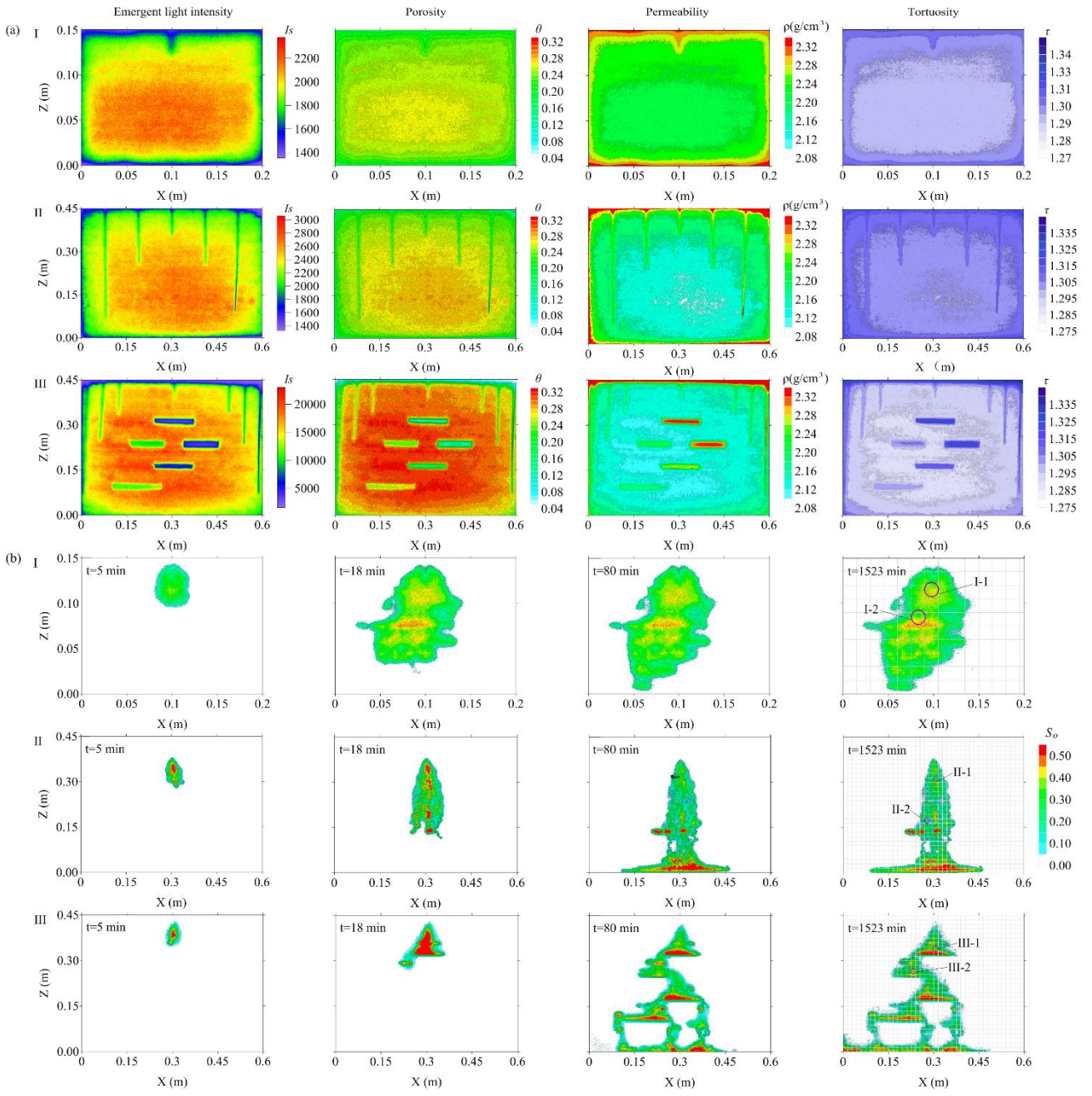


Fig. 4

