Review for hess-2020-75
A new form of the Saint-Venant equations for variable topography

This work presents a interesting reformulation of the Saint-Venant equations in order to allow the inclusion of smoother geometrical source terms but maintaining a realistic representation of the arbitrary geometry of natural rivers and creeks. The proposed transformation splits the Piezometric gradient $\partial\eta/\partial x$ into a reference body force in the bottom slope direction S_R and a hydrostatic head gradient $\partial h_a/\partial x$, ensuring that S_R is Lipschitz continuous. The limit case for the proposed formulation is the widespread splitting form of the Saint-Venant equations, in which S_R and h_a agree with the thalweg slope S_0 and the maximum water depth h_0 at each section. The authors propose this simple algebraic transformation in order to avoid oscillatory solutions, or even unstable behavior, which the conventional splitting technique can cause in most of the numerical schemes when S_0 is Lipschitz discontinuous.

This work is original and well written. The tests carried out to demonstrate the applicability of the proposed technique are suitable and the discussion clear and well structured. From my point of view, I can see any important weakness in the mathematical approach and the discussion. Only some minor corrections must be included before this work can be considered for publication.

My main concern is related to the validation methodology. The authors compare their numerical results with analytical solutions (MacDonald benchmarking cases) and with those obtained with HEC-RAS for synthetic cases and a urban creek case. HEC-RAS is widely-accepted model and uses Piezometric gradient version of the Saint-Venant equations in order to obtain stable solutions, avoiding numerical oscillation. As validation strategy, this comparison is correct and valuable. However, it would also be interesting to include the comparison with some of the existing well-balanced models based on the conventional splitting $\eta = z_0 + h_0$ and able to deal with discontinuous geometrical source terms. Although this reviewer understands that these models probably are not accessible for the authors, including such comparison for the urban creek case would increase the quality of the discussion. That is only a suggestion.

Other minor corrections:

Line 26: "...will be designated as 'reference slope', S_R,..."

Line 63: "...splitting of the Piezometric head to include a body force that is everywhere aligned with a variable S_0 is merely creating an unnecessary complexity..."

The main advantage of including S_0 as a body force is that real disappointing in the topography, as chutes, are included into the forcing terms. Also, from a hydrology viewpoint, S_0 provides consistency between kinematic wave solutions (which use $S_0 = S_f$) and the SVE, as the author claim in Section 5.2. Hence this sentence should be explained in detail. Why including S_0 "is merely creating an unnecessary complexity"?

Line 97: "...Unfortunately, many water resources models do not use well-balanced schemes, and those that do are often computationally intensive and therefore impractical for simulating regional-to-continental scale river networks or stormwater systems for megacities..."

This sentence is misleading. Maybe can be reworded.

Line 127: "...even when $\partial A/\partial x$ is non-smooth..."

Line 189: "...Note that in extreme cases of geometric discontinuity the values of n, Pw and A in eq. (9) can cause a non-Lipschitz source term; however, most solution methods are relatively robust to such discontinuities as they are in the coefficient of the solution variable rather than an additive source term..."

Integration of friction source terms has been in main issue in numerical models during decades, specially when wet-dry fronts are involved. This led to a wide range of proposed solutions, from the implicit computation of the friction term to limiting its value for ensuring the positivity of the water depth solution. At least this should be mentioned in the text including some references.

Section 3.3 Generating a smooth $S_R(x)$: How the points of the real thalweg are selected to construct the reference profile z_R? Are there any optimization method to select them?

Figures 16 and 18: Line colors for the bed profile and the WSL are changed. Maybe it can be more appropriated that the bed and WSL lines have the same color for each model.