We thank reviewers for their thorough review of the manuscript and the constructive suggestions. The comments and suggestions are answered sequentially following the order of reviewers' questions and comments.

**Response for HESS-2020-75 Reviewer #1**
**A new form of the Saint-Venant equations for variable topography**

R1C1
*My main concern is related to the validation methodology. The authors compare their numerical results with analytical solutions (MacDonald benchmarking cases) and with those obtained with HEC-RAS for synthetic cases and a urban creek case. HEC-RAS is widely-accepted model and uses Piezometric gradient version of the Saint-Venant equations in order to obtain stable solutions, avoiding numerical oscillation. As validation strategy, this comparison is correct and valuable. However, it would also be interesting to include the comparison with some of the existing well-balanced models based on the conventional splitting* $\eta = z_0 + h_0$ *and able to deal with discontinuous geometrical source terms. Although this reviewer understands that these models probably are not accessible for the authors, including such comparison for the urban creek case would increase the quality of the discussion. That is only a suggestion.*

Response to R1C1:
**We agree with the idea but we cannot fully implement in the paper. We have added the discussion in Section 5.3 as shown below.**

> *Beginning Line 488: The present scope is limited in that only a single model was modified (SPRNT/SPRNT-RS) and only single model (HEC-RAS) was used as an external control. The validity of the underlying algebraic transformation in the RS method has been demonstrated by these tests; however, it remains to be seen how implementing the RS approach in other models -- particularly well-balanced models -- might alter residual errors, convergence rates, and computational performance. We are interested in collaborating with other researchers who have access to and familiarity with source code of candidate well-balanced models.*

For further clarification -- we appreciate reviewer's suggestion of using well-balanced model to explore how the RS method affects other codes, which we agree would be quite interesting and informative. However, this would not significantly add to the validation of the RS method – which is the focus of the present work (particularly since the key point is that the method itself is a simple algebraic transformation). Nevertheless, we are quite interested in how implementation of the RS method would alter the numerical performance of a well-balanced model. We hypothesize that it would perform better – but this is pure speculation. In any case, such an effort is an extension that is well beyond the present scope of work. Including further modeling would increase the length of the paper and dilute its focus. However, we do think it worth pointing out this limitation more clearly in the discussion, which we have done in the revised text.

R1C2
*Line 26: "...will be designated as 'reference slope',* $S_R$*,..."*

Response to R1C2:
**Agree. The sentence has been corrected.** We appreciate reviewer's carefulness.

R1C3

*Line 63: "...splitting of the Piezometric head to include a body force that is everywhere aligned with a variable $S_0$ is merely creating an unnecessary complexity..."*

*The main advantage of including $S_0$ as a body force is that real disappointing in the topography, as chutes, are included into the forcing terms. Also, from a hydrology viewpoint, $S_0$ provides consistency between kinematic wave solutions (which use $S_0 = S_f$) and the SVE, as the author claim in Section 5.2. Hence this sentence should be explained in detail. Why including $S_0$ " is merely creating an unnecessary complexity"?*

Response to R1C3:

**Agree. We have rewritten the explanation as noted below.**

> *New text beginning Line 59:* From a physics perspective, using $S_0$ to split the Piezometric head is an intuitive way to describe the local interplay of pressure with the bottom slope. Furthermore, $S_0$ has the advantage of readily reducing to a kinematic wave equation where $S_f=S_0$, which has some advantage in multi-purpose codes. However from a numerical modeling perspective, using $S_0$ has a significant limitation based on its smoothness. If the water surface is smooth then non-smooth $S_0(x)$ requires the numerical solver to produce a compensating non-smooth $h_0(x)$, i.e., requiring a "well-balanced" scheme (see §2). If we can discard our (wrong) intuition that the $S_0$ form must somehow "better" represent sharply variable topography – i.e., recognizing the algebraic equivalence of eq. (5) with eqs. (1) and (2) – it follows that splitting of the Piezometric head to include a body force that is everywhere exactly aligned with a sharply varying $S_0$ is (from a numerical perspective) merely creating unnecessary complexity in the governing equation source term that requires compensating complexity in the solution algorithm. In contrast, by requiring $S_R$ to be smooth we can ensure the $h_a$ solution is also smooth for a smoothly-varying free surface.

For further clarification – note that real topography and its forcing is included in *any* Saint-Venant solution (using $S_0$, $S_R$, or the free surface) as long as the physical A(x) and the net piezometric pressure gradients are correctly identified by the governing equations. This is the key point of the algebraic equivalence of eq. (1), (2) and (5). If the equations are algebraically identical, they must represent the exact same physics. Hence S0 is not needed to represent the topography.

R1C4

*Line 97: "...Unfortunately, many water resources models do not use well-balanced schemes, and those that do are often computationally intensive and therefore impractical for simulating regional-to-continental scale river networks or stormwater systems for megacities..." This sentence is misleading. Maybe can be reworded.*

Response to R1C4:

**Agree: We have rewritten the explanation as noted below.**

> *Beginning Line 102: Although well-balanced schemes are relatively robust in handling the discontinuous boundary conditions, they have not been extensively applied in water resources models to simulate the regional-to-continental scale river networks or stormwater systems for megacities. The rapidly varying and discontinuous S0 in natural systems can significantly increase the difficulty and computational burden of obtaining a well-balanced method (Schippa and Pavan, 2008). Hence, when a large-scale open-channel model develops oscillations and/or instabilities,*

*practitioners may resort to the traditional approach of removing cross-sections105or smoothing bathymetry to mitigate oscillatory or unstable solution behavior (Tayfur et al., 1993).*

R1C5

*Line 127: "...even when ∂A/∂x is non-smooth..."*

*Line 189: "...Note that in extreme cases of geometric discontinuity the values of n, $P_w$ and A in eq. (9) can cause a non-Lipschitz source term; however, most solution methods are relatively robust to such discontinuities as they are in the coefficient of the solution variable rather than an additive source term..."*

*Integration of friction source terms has been in main issue in numerical models during decades, especially when wet-dry fronts are involved. This led to a wide range of proposed solutions, from the implicit computation of the friction term to limiting its value for ensuring the positivity of the water depth solution. At least this should be mentioned in the text including some references.*

Response to R1C5:

**Agree: We have added the text below to the Background.**

*Beginning line 114: That numerical instabilities are often caused by non-smooth source terms is not a new observation. A wide variety of numerical schemes have been developed to address this issue, including (e.g.) extensive work on wetting/drying (Liang and Marche, 2009; Song et al., 2011), positivity-preserving methods for coupled models (Singh et al., 2015) and implicit schemes that address stiffness of the nonlinear friction term (Xia and Liang, 2018). The literature in this area is vast – particularly if both 1D and 2D models are considered. For the present purposes we focus on only one part of the source term, S0, whose non-smoothness has previously been treated as a problem to be handled rather than as a problem that can be directly eliminated in the governing equations. Existing well-balanced schemes (see reviews noted above) seek to compensate for non-smoothness of all parts of the source term in the structure of the numerical discretization. Arguably, if the slope term is guaranteed smooth then a well-balanced scheme should be simpler to create.*

**We have also revised the Methods with the following clarification**

Beginning line 208: Note that in extreme cases of geometric discontinuity the combined values of n, $P_w$ and A in eq. (9) can cause a non-Lipschitz friction term; thus, the RS method cannot guarantee that the entire source term is smooth. Numerical solution methods are usually robust to discontinuities in n and $P_w$ as they are coefficients of the solution variables {A,Q}. More subtle problems might arise due to discontinuities developed in the $Q^2A^{-10/3}$ ratio in eq. (9); countering incipient instabilities from this term requires other numerical strategies (e.g., Xia and Liang, 2018)

Note that Line 127 (now line 142) is in the Background whereas Line 189 (now line 208) is in the Methods. We chose to add the bulk of the revised text as a new discussion in the Background to address past work beyond the bottom slope. Note that a full review of this issue would be a major undertaking and is beyond the present scope of validating that we have a new method that works. Just as a matter of interest -- at last count Greenberg & Leroux 1996 paper on well-balanced methods has been cited 443 times!

R1C6

*Section 3.3 Generating a smooth $S_R(x)$: How the points of the real thalweg are selected to construct the reference profile $z_R$? Are there any optimization method to select them?*

Response to R1C6:

**This question points out some clarifications needed. We have added the text noted below to the Methods section**.

> *Beginning Line 242: It follows that there is some (limited) choice in the selection of the subset of $z_0(x)$ used as the spline knots, with different sets producing slightly different $\{z_R(x), h_R(x)\}$ over the domain. Each set is algebraically identical to the underling geometry so the generated solutions should be identical within truncation error. Implications of the method chosen for generating $z_R(x)$ are discussed in §5.3. Further details and test cases are provided in Yu et al. (2019b)*

**Additional clarifications provided in the Discussion section:**

> *Beginning Line 503: In the present work, the profile of $z_R$ in the Waller Creek case is generated by the cubic B-spline technique, which is controlled by the number of "knots" and their spacing. In general, the distance between knots must be longer than the spacing between cross-sections so that the generated $S_R$ is smooth at the model's discretization scale. It is not clear that a mathematical "optimum" for selection of knots will necessarily exist, but there are likely (unknown) practical limits on knot selection spacing for "adequate" smoothness of $z_R(x)$. Our results indicate that approximating cubic B-splines are adequate for producing smooth $z_R$ for the tested geometries, and the solutions are robust to the selection of $z_R$ as long as $S_R$ is smooth (Yu et al., 2019b). However, it is likely there are limitations to applying the RS method in large-scale river network simulation that will make it difficult to use a simple globally-applied knot spacing. Such networks might consist of $10^4$ to $10^5$ reaches spanning wide geographical regions with a variety of topology and inconsistent data availability. Some reaches may have well-defined cross-sections at close spacing, other reaches might be poorly documented (Hodges, 2013). Thus, it seems likely that a method for automatically generating approximating splines (or some other form of smoothing) would be useful, but such an advance arguably requires a method for quantitatively evaluating the "goodness" of a particular set of $z_R(x)$, which remains an open question. We speculate that simple window filtering techniques may be adequate for river databases such as NHDplus, but further investigation and examination are needed to better understand the interplay between the smoothing scales and the numerical solution using the RS method for large networks.*

R1C7

*Figures 16 and 18: Line colors for the bed profile and the WSL are changed. Maybe it can be more appropriated that the bed and WSL lines have the same color for each model.*

Response to R1C7:

**Agree. The colors of the lines in Figure 16 and 18 have been made consistent.**