Referee Report on "Three-dimensional transient flow to a partially penetrated well with variable discharge in a general three-layer aquifer system" by Qinggao Feng, Xiaola Feng, and Hongbin Zhan

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1 General comments

This paper treats the case of three hydraulically anisotropic aquifer layers of finite thickness where in the middle layer a partially penetrating well is active. There are no restrictions with respect to the aquifer parameters K_{ri} $(= K_{hi})$, K_{zi} $(= K_{vi})$ and S_{si}. In fact, this paper has the same set-up as the paper by Feng *et al.* (2020) in which the well is active in the lower layer or to the paper by Feng *et al.* (2019) for a two layer case in which the well is active in the lower layer. These three papers are based on the assumption made by Neuman (1974) to handle the well by a solution of Hantush (1964) for drawdown in a confined aquifer due to pumping from a well that partially penetrates the aquifer. This assumption is not mentioned in the introduction where the suggestion has been raised that the followed approach is exact, which is not. This simplifying assumption is somewhat hidden in the Appendix A of this paper, namely in requirements (A11) and (A13) where the terms $\partial\widehat{\overline{u}}_D(\lambda,1,p)/\partial z_D$, resp. $\partial\widehat{\overline{u}}_D(\lambda,0,p)/\partial z_D$ are missing.

The main advantage of this approach however is that the authors of this paper and of the papers Feng (2019, 2020) (and for example also the paper by Malama *et al.* (2008)) end up with a Laplace-Hankel transform consisting of an integral with a closed form expression for the integrand. Of course, for every Laplace variable p the Hankel integral has to be repeated because the parameters in the integrand depend on p, but the integrand is a single expression, albeit complicated.

It is remarkable that the authors do not mention the paper by Veling & Maas (2009) [= VeMa] where there is not such a restriction described above with respect to the conditions of the flux at the interface of the layers. Moreover, in the VeMa-paper the solution has been given for an arbitrary number of layers with much freedom with respect to the upper and lower boundary conditions (Dirichlet, Neumann, Robin boundary conditions) and allows for more that one well screen (injection and extraction in the same well bore, even in the same layer, *e.g.*). In VeMa three different strategies are described with respect to the order of transformations. The authors of the paper under review use strategy 5.3 "Integral transform in terms of t and r" in VeMa,.but do not proceed to take into account the influence of the well into the upper and lower layer. In VeMa particle tracking has been applied for a 6-layer aquifer with 3 well screens in the same well bore as an example, among others. In Feng (2020) almost the same authors as the current ones mention VeMa where they only say that their time-dependent extraction function is a generalization with respect to VeMa (which can easily for accounted for in VeMa), but forget to say that VeMa is more general and exact with respect to the conditions along the interface of the layers.

The authors do not specify in which way they found the quite complicated expressions: by hand or by a formula manipulation package? If the authors have used some formula manipulation package, that should be stated clearly. The mathematical problem to be solved for the problem treated by the authors consists of solving three times (for the three Cases A, B and C) a set of 6 equations with 6 unknowns (Appendix A). The numerical approach sounds good with modern tools (de Hoog et al. (1982) and Ogata (2005)).

The authors apply their results in a consistent way for an isotropic system with a fully penetrating well and compare their results extensively with the existing litterature. The authors study also an anisotropic system with a full and a partially (half the thickness of the aquifer) penetrating well. All their results are good understandable and explained.

The overall writing is good and precise, but see below w.r.t. the References.

2 Some remarks

Line 786 and 790: The approximation described above should be mentioned.
Line 657: It is not clear what exactly is meant with the expression "that the middle drawdown of pumped layer is closer to the position of well screen".
Line 864: References. A number of referenced papers are not mentioned in the References.

3 Some minor remarks

Line 223: Table 1: The extra horizontal line is confusing. The variable α_{ri} and α_{zi} are non dimensionless.
Line 775: Eq. (33) should be Eq. (34).

4 Final Remark

Overall, the paper serves as an useful approximation for the specific problem at hand, but compare the remarks made above. Therefore, this reviewer judges that the claims in the paper (Line 324 and Line 679) should be somewhat more modest. It should be interesting to compare their solution to the general solution given by VeMa and to find out under which conditions the conclusions of the authors are still valid. This reviewer suggests that it occurs if the well screen is large compared to the thickness of the layer which implies that the partially penetrating well induces mainly a radial flow.

5 References

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