Dear Professor Veling:

Upon the recommendation, we have carefully replied our manuscript HESS-2020-586 entitled "Three-dimensional transient flow to a partially penetrated well with variable discharge in a general three-layer aquifer system" after considering all your comments. The following is the point-by-point reply to all the comments.

1 General comments

This paper treats the case of three hydraulically anisotropic aquifer layers of finite thickness where in the middle layer a partially penetrating well is active. There are no restrictions with respect to the aquifer parameters K_{ri} (= K_{hi}), K_{zi} (= K_{vi}) and S_{si}. In fact, this paper has the same set-up as the paper by Feng et al. (2020) in which the well is active in the lower layer or to the paper by Feng et al. (2019) for a two layer case in which the well is active in the lower layer. These three papers are based on the assumption made by Neuman (1974) to handle the well by a solution of Hantush (1964) for drawdown in a confined aquifer due to pumping from a well that partially penetrates the aquifer. This assumption is not mentioned in the introduction where the suggestion has been raised that the followed approach is exact, which is not. This simplifying assumption is somewhat hidden in the Appendix A of this paper, namely in requirements (A11) and (A13) where the terms $\partial\hat{\bar{u}}(\lambda,1,p)/\partial z_D$ resp. $\partial\hat{\bar{u}}(\lambda,0,p)/\partial z_D$ are missing.

Reply: *Thank Dr. Veling for the detailed comments. When deriving the solution, we have used the same method adopted by Neuman (1974), as clarified in Appendix A. As suggested by Dr. Veling, we have revised the text in the introduction to address this concern.*

The main advantage of this approach however is that the authors of this paper and of the papers Feng (2019, 2020) (and for example also the paper by Malama et al. (2008)) end up with a Laplace-Hankel transform consisting of an integral with a closed form expression for the integrand. Of course, for every Laplace variable *p* the Hankel integral has to be repeated because the parameters in the integrand depend on *p*, but the integrand is a single expression, albeit complicated.

It is remarkable that the authors do not mention the paper by Veling & Maas (2009) [= VeMa] where there is not such a restriction described above with respect to the conditions of the flux at the interface of the layers. Moreover, in the VeMa-paper the solution has been given for an arbitrary number of layers with much freedom with respect to the upper and lower boundary conditions (Dirichlet, Neumann, Robin boundary conditions) and allows for more that one well screen (injection and extraction in the same well bore, even in the same layer, e.g.). In VeMa three different strategies are described with respect to the order of transformations. The authors of the paper under review use strategy 5.3 "Integral transform in terms of *t* and *r*" in VeMa,.but do not proceed to take into account the influence of the well into the upper and lower layer. In VeMa particle tracking has been applied for a 6-layer aquifer with 3 well screens in the same wellbore as an example, among others. In Feng (2020) almost the same authors as the current ones mention VeMa where they only say that their time-dependent extraction function is a generalization with respect to VeMa (which can easily for accounted for in VeMa), but forget to say that VeMa is more general and exact with respect to the conditions along the interface of the layers.

Reply: *We have revised the text to include a detailed analysis of the Veling & Maas (2009) [= VeMa] paper, and have also clarified the difference of the work of Veling & Maas (2009) and this study. The section 5.3 "Integral transform in terms of t and r" in VeMa has given the expression of hydraulic head (Eq. 40) using the Laplace transform to t, the Hankel transform to r and the generalized Fourier transform to z. The semi-analytical solution of VeMa did not formulate the closed-form expression for their solution in the z direction. In addition, one must calculate the eigenvalues* λ_m^2 *for every value of the Laplace transform parameters p and Hankel transform parameters* α, *and a matrix must be constructed to find the values of the eigenvalues of the eigenfunction. The solution became an simpler expression with the restrictive hypothesis of* $K_{h,i}/S_{si} = \rho$. *One can see that our present solutions does not apply any integral transform with respect to z in the process of the derivation, so it does not involve the problems of eigenvalue or eigenfunction. The section 5.3 "Integral transform in terms of t and r" in VeMa also provided another way used by Lenoach et al. (2004) to obtain the semi-analytical solution in a matrix form for a multi-layer aquifer system with no-flux boundary at the top and bottom (our solution for Case 2). However, the semi-analytical solutions with the other two types of boundary conditions (the prescribed head at top and bottom boundary (our solution for Case 1), or prescribed head at top and zero flux at bottom (our solution for Case 3)) used in our present study have not been considered by VeMa.*

The authors do not specify in which way they found the quite complicated expressions: by hand or by a formula manipulation package? If the authors have used some formula manipulation package, that should be stated clearly. The mathematical problem to be solved for the problem treated by the authors consists of solving three times (for the three Cases odnA, B and C) a set of 6 equations with 6 unknowns (Appendix A). The numerical approach sounds good with modern tools (de Hoog et al. (1982) and Ogata (2005)).

Reply: *We have revised the text to accommodate this suggestion. Briefly speaking, we have solved the 6 equations with 6 undetermined by using Maple.*

The authors apply their results in a consistent way for an isotropic system with a fully penetrating well and compare their results extensively with the existing litterature. The authors study also an anisotropic system with a full and a partially (half the thickness of the aquifer) penetrating well. All their results are good understandable and explained.

The overall writing is good and precise, but see below w.r.t. the References

Reply: *Thanks.*

Some remarks

Line 786 and 790: The approximation described above should be mentioned.

Reply: *We have revised the text in revised manuscript to address this concern.*

Line 657: It is not clear what exactly is meant with the expression "that the middle drawdown of pumped layer is closer to the position of well screen".

Reply: *It has been rewritten as "the drawdown at the middle part of pumped layer is closer to the position of well screen."*

Line 864: References. A number of referenced papers are not mentioned in the References.

Reply: *We have carefully checked the Reference to ensure all papers are listed.*

Some minor remarks

Line 223: Table 1: The extra horizontal line is confusing. The variable αri and αzi are non dimensionless.

Reply: *Revised.*

Line 775: Eq. (33) should be Eq. (34).

Reply: *Revised*.

Final Remark

Overall, the paper serves as a useful approximation for the specific problem at hand, but compare the remarks made above. Therefore, this reviewer judges that the claims in the paper (Line 324 and Line 679) should be somewhat more modest.

Reply: *Revised.*

It should be interesting to compare their solution to the general solution given by VeMa and to find out under which conditions the conclusions of the authors are still valid.

Reply: *The general solution shown in Eq. (40) is given by VeMa based on the application of the Laplace transform to t, the Hankel transform to r and the generalized Fourier transform to z. Our general solution is obtained by only using the Laplace and Hankel transform. The strategy for obtaining these two solutions has some similar features, but the final semi-analytical expression is different and the method to obtain the time-domain is also different. More importantly, we have verified our solution with comparison of available studies and numerical solution (Done by Feng et al.2020). We did not find similar comparison of analytical and numerical works in the paper of VeMa, which focused primarily on the "strategy" and "application" for particle tracking. In contrast, our study mostly aims to provide a relatively simple and direct general solution, to compare with the verified available studies, and to explore the drawdown behavior for the three-layer aquifer system induced by a variable discharge (exponentially declining pumping rate) with different top and bottom boundaries. After some careful considerations, we think the study of comparation presented in this paper is sufficient to support the conclusions and findings, so the comparison with the solution of VeMa is not conducted. However, it may be an interesting exercise to conduct a comparative investigation about different approaches, including this study, the study of VeMa, and other studies involving* *multi-layer systems in the future to identify the advantages and disadvantages of those different approaches.*

This reviewer suggests that it occurs if the well screen is large compared to the thickness of the layer which implies that the partially penetrating well induces mainly a radial flow.

Reply: *Thanks for your suggestion. The content as suggested has been added in the revised manuscript.*

On behalf of the authors
Sincerely Yours,
Hongbin Zhan

References cited

Feng Q., Luo Y., Zhan H., 2020. Three-dimensional response to a partially penetration well pumping in a general anisotropic three-layer aquifer system, Journal of Hydrology, 2020, 585, 124850.

Lenoach, B., Ramakrishnan, T.S., Thambynayagam, R.K.M., (2004) Transient flow of a compressible fluid in a connected layered permeable medium. Trans Porous Media 57:153–169

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