Dear Professor Neuman:

Upon the recommendation, we have carefully replied our manuscript HESS-2020-586 entitled "Three-dimensional transient flow to a partially penetrated well with variable discharge in a general three-layer aquifer system" after considering all your comments. The following is the point-by-point reply to all the comments.

1. This manuscript provides a mathematical solution to Hankel and Laplace transformed equations describing flow to a line sink of variable strength partially penetrating a hydraulically anisotropic aquifer of finite thickness, confined above and below by anisotropic aquitards of finite thickness, all three layers extending horizontally to infinity. Horizontal boundaries at the top and bottom this three-layer system are assigned prescribed head (Case 1), zero flux (Case 2), or prescribed head at top and zero flux at bottom (Case 3). The solutions are transformed back into real spacetime coordinates numerically. Although the authors consider their model to represent a general three-layer aquifer system, there are two indications that the top and bottom layers must be aquitards: 1) Pumping takes place only from the middle layer, suggesting that the top and bottom layers are not productive, and 2) top and bottom boundary conditions are those of the classical Hantush-Jacob "leaky aquifer" model, which the authors consistently compare with theirs. This comparison leads the authors to conclude that theirs is a more general model because it allows for partial penetration, anisotropy, multidirectional flow and variable pumping rate.

Reply: *Thank Dr. Neuman for his comments. In fact, Dr. Neuman and his co-authors have made tremendous contributions for developing the well flow theories since 1960 and our work here is inspired by their works and it represents our effort of pushing the work one step further to advance the analytical theories of well flow by relaxing some of the strict assumptions employed in present theoretical framework. Our responses are as follows.*

Firstly, pumping taking place from the middle layer is simply an example of illustration of the methodology. As a matter of fact, the pumping can take place in either layer, and the mathematical modeling can be applied in a similar fashion based on the procedures documented in this investigation.

Secondly, pumping taking place from the middle layer does not necessarily mean that the top and bottom layers are aquitards. For instance, it could be that the middle layer is more productive than the upper and lower layers whose permeabilities are less than (but not several orders of magnitude smaller than) the permeability of the middle layer. For the upper and lower layers to be classified as aquitards, their permeabilities must be at least two, three, or even more orders of magnitude smaller than their counterpart of the middle layer. This study, however, does not impose such a strict constrain on the contrasts of permeabilities of different layers. Furthermore, it is entirely possible that the well may be installed in the middle layer (as a partially penetrating well) for a variety of reasons even though the upper and lower layers are also permeable and productive.

Thirdly, we agree with Dr. Neuman that the selection of the top and bottom boundary conditions follows the same line as what has been done by the traditional Hantush-Jacob "leaky aquifer" model. However, such a selection does not imply that the upper and lower layers must be *aquitards. For example, if there is an open surface body above the upper layer, the top boundary of the upper layer can be described using a prescribed-head or the first-kind of boundary condition, or if there is an extremely permeable aquifer above the less permeable (but definitely not an aquitard) upper layer and the permeability contrasts are more than two or three orders of magnitude, sometimes we may also approximate the top boundary of the upper layer as a prescribed-head boundary, provided that any flux exchange between the upper layer and the extremely permeable aquifer above the upper layer will not affect the hydraulic head in that extremely permeable aquifer noticeably. As another example, if a horizontal fracture exists above the upper layer or below the lower layer, and such a horizontal fracture extends sufficiently far from the domain of interest and is connected with a surface water body, the upper or lower boundary may also be described as a prescribed head boundary. As a third example, if an intact impermeable rock is underneath the lower layer, then the bottom boundary of the lower layer can be described using a prescribed zero flux boundary condition.*

In summary, the choice of the top and bottom boundary condition does not necessarily imply that the upper and lower layer must be aquitards. One reason of using the classical Hantush-Jacob "leaky aquifer" types of boundary conditions here is for the sake of model comparison, because our model should be degenerated to the classical Hantush-Jacob "leaky aquifer" model when the upper and lower layers are indeed aquitards.

We do agree with the comment that we cannot simply limit to the classical Hantush-Jacob "leaky aquifer" types of boundary conditions. Thus, in the future, we need to investigate what other possible boundary conditions should be considered for a general three-layer aquifer system. We also feel that it is necessary to validate the model and the choices of boundary conditions using controlled laboratory experiments and field pumping tests as well in the future.

2. In reality, the proposed solution is severely limited by the replacement of aquifers above and below the two aquitards with artificially imposed boundary conditions and by treating the top and bottom layers as aquitards rather than, potentially, productive aquifer layers. The authors forget to mention that analytical solutions exist for more realistic multiaquifer systems with aquifers above and below their aquitards (replaced in their model by artificial Hantush-Jacob boundary conditions); see Neuman (1968), Neuman and Witherspoon (1969) and Li and Neuman (2007). Though it is true that these models restrict flow in aquifers to horizontal, flow in aquitards to vertical, and flow to a fully penetrating well, those restrictions have been demonstrated by Neuman (1968) and Neuman and Witherspoon (1969) to be much less severe than implied by the authors.

Reply: *The available models (Neuman,1968, Neuman and Witherspoon,1969 and Li and Neuman, 2007) restricted flow in aquifers to horizontal, flow in aquitards to vertical, and flow to a fully penetrating well. Apart from these, another important assumption used in those models is that mass exchange between two adjacent aquifers can be treated as a volumetric sink/source incorporated into the governing equations of flow in each individual layer, which cannot reflect the actual leakage process which occurs only at the interface of different layers and is better treated as an interface phenomenon as done in this investigation. This study has relaxed such restrictions or assumptions, and it represents the advancement of existing models to a physically based more general setting.*

3. The authors likewise forget that any analytical expression for constant pumping rate is easily generalized to the case of variable pumping rate through the temporal superposition of elementary expressions; this has been done routinely for years in software packages such as Aqtesolv.

Reply: *We acknowledge that one can obtain the analytical solution of variable pumping rate case by using the principle of superposition based on the solution of constant pumping rate case. But it is much needed to provide a simple and more direct (semi-)analytical solution (if possible) with variable discharge, as this is commonly encountered in real-world pumping tests according to our field experiences.*

4. In summary, I find the manuscript to be somewhat misleading in its claim of providing a useful general solution to well flow in a multilayer aquifer system, and the proposed solution to be at best of marginal interest to hydrologists.

Reply: *We have clearly clarified the applications and limitations of our solution in this study. The authors believe that this study is a valuable contribution to the subsurface hydrology by advancing the present well hydraulics theory one step further and it will be valuable to the hydrological community to keep pushing the boundary of the present stage of knowledge on the subject.*

On behalf of the authors
Sincerely Yours,
Hongbin Zhan