We thank the reviewer for these helpful comments. Reviewer comments are listed below, along with our response to each. In some cases, we describe the proposed revisions to the manuscript (with line numbers) or refer to proposed revisions described in our responses to the other reviewers, but we recognize that the revised manuscript is requested in a subsequent step.

Comment 1:

This manuscript is an attempt to investigate the mechanistic understanding of the catchment-specific parameter in parametric Budyko equations. It is an interesting topic in the hydrological studies.

Response 1:

We appreciate the reviewer's interest in our manuscript but stress that the primary aim of this technical note is to present the technical details of analytically inverting the parametric Budyko equations. We strongly agree that improving our mechanistic understanding of the catchment-specific parameter is important and address this concept in several places in the text (page 1, lines 15-17; page 3, lines 55-62; and page 9, lines 165-168), but we believe that the technical aspects of the analytical inversion and resulting explicit expressions for *n* and *w* are novel additions to the literature on their own. We note that the companion research article to this technical note, (Reaver et al., 2020) (hess-2020-584, Reinterpreting the Budyko Framework, referenced on page 3, line 57 of the technical note), provides a much more in-depth analysis and critique of the catchment-specific parameter and its use in the parametric Budyko framework.

Comment 2:

However, I have some major concerns on this manuscript. I did not see how this manuscript has made a further step towards explaining the longterm water balance at the catchment scale. By obtaining the mathematical inversion of the catchment-specific parameter from Budyko equations is not helpful in understanding the possible hydrological processes that remain hidden when the non-parametric or parametric Budyko representation is chosen.

Response 2:

As noted above, the primary objective of this technical note is the analytical inversion of the parametric Budyko equations and resulting explicit expressions for *n* and *w* (not explaining the long-term catchment water balance). Regarding the utility of the inversion, we illustrate that the dependence of *n* and *w* on any "hidden" biophysical processes must be through the dependence of $\bar{P}$, $\overline{E_0}$, and $\bar{E}$ on those same features and processes, thus advancing our understanding of the parametric Budyko equations (and limitations to their utility). While the companion research article (Reaver et al., 2020) explicitly states that the parametric Budyko equations cannot be productively used to understand the long-term water balance (nor to identify potential "hidden" hydrological processes), this point could be more clearly articulated in this technical note. We thus propose revisions to the abstract, introduction, and discussion and conclusions sections to better motivate the study and summarize our interpretations regarding the utility of catchment-specific parameters and the overall parametric approach. These proposed revisions are given in our response to **Reviewer 1, Comment 3.**

Comment 3:

As mentioned in this paper, what climatic and physiographic features and how they control the long-term water balance are important for explaining the Budyko curve. To achieve this goal, one approach is to express the parameter of parsimonious Budyko equation (e.g., n or ω) in terms of biophysical

features in a way that could be applied to different catchments; the other approach is to explicitly represent the features in the model.

Response 3:

The question of why the Budyko curve emerges from the aggregate behavior of many catchments across a wide range of aridity indices is not the topic of this manuscript. Instead, we are focused on inverting the parametric Budyko equations, while secondarily commenting on how the derived expressions for *n* and *w* provide a general relationship between the catchment-specific parameters and biophysical features through the dependence of $\overline{E_0}$, $\bar{P}$, and $\bar{E}$ on those same features. As we emphasized in our response to **Reviewer 2, Comment 2** above, understanding how *n* and *w* depend on biophysical features is tangential to understanding or explaining the Budyko curve. The reviewer also suggests that *n* or *w* should be expressed in terms of biophysical features in a way that can be applied to other catchments. Our explicit expressions (Equation 14, page 4, line 85) do exactly that through the direct dependence of $\overline{E_0}$, $\bar{P}$, and $\bar{E}$ on biophysical features. However, we again note that, if the dependence of $\overline{E_0}$, $\bar{P}$, and $\bar{E}$ on biophysical features are known (which they must be in order to calculate n or w), then the long-term water balance could be understood directly (the reviewer's suggested second approach) without the need for the parametric Budyko framework. Both of these topics are explored in detail in the companion research article to this technical note (Reaver et al., 2020) (hess-2020-584, Reinterpreting the Budyko Framework, cited on page 3, line 57).

Comment 4:

In addition, this paper treats n or ω as a function of long-term P, E0, and E, and E actually is treated as a function of P, E0. Does it mean that the value of n or ω is only dependent on the long-term climate? It seems conflict with the existing studies that found the short-term climate variations and catchment features (those could not be explained by the long-term climate) also have impacts on the long-term water balance.

Response 4:

The reviewer is correct that we derive *n* and *w* solely as functions of long-term of $\overline{E_0}$, $\bar{P}$, and $\bar{E}$. However, it is not correct that we treat $\bar{E}$ only as a function of $\overline{E_0}$ and $\bar{P}$. This confusion may be due to the introduction of Equation 1 (page 1, line 24), which states that non-parametric Budyko equations are functions of the aridity index (i.e., functions of only $\overline{E_0}$ and $\bar{P}$). Critically, this only applies to the non-parametric Budyko equations. When the parametric Budyko equations were formally derived (Yang et al., 2008;Zhang et al., 2004), the starting point of the derivation was the relaxation of the requirement that $\bar{E} = f_0(\overline{E_0}, \bar{P})$ to the implicit relationship $\bar{E} = f_1(\overline{E_0}, \bar{P}, \bar{E})$ (i.e., $\bar{E}$ depends on $\bar{E}$). In both derivations, this relaxation eventually leads to the introduction of an arbitrary constant, which eventually became *n* and *w* in the parametric Budyko equations. By inspecting the parametric Budyko equations (Equations 4, 5, 6, and 7, page 2 , lines 36-43), it is clear that *n* and *w* are functions of $\overline{E_0}$, $\bar{P}$, and $\bar{E}$; our technical note only gives the explicit form of those functions (Equation 14, page 4, line 85).

In short, *n* and *w* are only dependent on $\overline{E_0}$, $\bar{P}$, and $\bar{E}$. Short-term climate variations and catchment features are likely to impact $\bar{E}$ (i.e., the long term water balance) and would also impact *n* and *w* through their dependence on $\bar{E}$. Therefore, our results are not in conflict with existing studies that have found catchment properties other than $\overline{E_0}$ and $\bar{P}$ to impact the long term water balance. To make this clearer we propose to revise our description of the parametric Budyko equation development (page 2, lines 33-35) to explicitly mention its implicit nature:

"In an attempt to generalize the non-parametric Budyko framework and explain deviations in $\frac{\bar{E}}{\bar{P}}$ *from the central tendency of the empirically observed catchment clustering pattern, the implicit relationship,* $\bar{E} = f_1(\overline{E_0}, \bar{P}, \bar{E})$*, was proposed, resulting in two parametric Budyko equations (Turc, 1953; Choudhury, 1999; Mezentsev, 1955; Yang et al., 2008),…"*

References:

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