We thank the reviewer for these helpful comments. Reviewer comments are listed below, along with our response to each. In some cases, we describe the proposed revisions to the manuscript (with line numbers), but we recognize that the revised manuscript is requested in a subsequent step.

Comment 1:

This manuscript provides a derivation for expressing the Budyko parameters (n or w, more typically also referred to as omega) explicitly in terms of precipitation, evapo(transpi)ration, and potential ET. The paper argues that this important as past studies could only indirectly infer n or w.

Response 1:

This is an accurate representation, though we note that another outcome of this work is to illustrate how n and w depend on biophysical features specifically through the dependence of $\overline{E_0}$, \overline{P} , and \overline{E} on those same features (e.g., lines 15, 60-63, and 165-170).

Comment 2:

The paper seems technically correct. Being able to explicitly express n and w in terms of precipitation, evapo(transpi)ration, and potential ET can be useful in particular cases, but will not change anything fundamental to the outcome of any study. (Note that the search for factors that determine the catchment-specific parameters of parameterized Budyko curves seems to be largely irrelevant as there is no physical meaning of this parameter that would allow to meaningfully compare this parameter between catchments.)

Response 2:

We thank the reviewer for supporting the technical aspects of the derivation. We wholeheartedly agree with the reviewer on both points, which we address in much more detail in the companion research article to the technical note (Reaver et al., 2020) (hess-2020-584, "Reinterpreting the Budyko Framework", cited on page 3, line 57). We note that the primary aim of this technical note is to analytically invert the parametric Budyko equations and verify that the resulting explicit expressions are correct. A secondary aim is to improve the mechanistic understanding of n and w, however, this theme is more completely developed in the companion article (Reaver et al. 2020).

Given the relatively narrow scope of the technical note, we thus focus on the technical details of the analytical inversion. However, in our response to Comment 3 (below), we do propose several revisions to the abstract, introduction, and discussion sections that aim to better motivate the study and more clearly describe how explicit expressions for *n* and *w* show the direct dependence of *n* and *w* on \overline{P} , $\overline{E_0}$, and \overline{E} , thus illustrating their lack of physical meaning.

Comment 3:

Since the paper seems technically correct, and some people can use it, I propose to publish this with very minor corrections, but I would encourage the authors to better describe what can and cannot be learned from the catchment specific parameter.

Response 3:

We thank the reviewer for the recommendation to publish with very minor corrections and agree that it would be useful to better describe what can and cannot be learned from *n* and *w*. We propose to expand the explanation of our motivation and interpretation with the following edits:

1) Revise the abstract to highlight the direct dependence of dependence of n and w on \overline{P} , $\overline{E_0}$, and \overline{E} and make the point that, for practical applications (e.g., hydrological predictions), the parametric Budyko equations lack utility:

"The non-parametric Budyko framework provides empirical relationships between a catchment's longterm mean evapotranspiration (\overline{E}) and the aridity index, defined as the ratio of mean rainfall depth (\overline{P}) to mean potential evapotranspiration (\overline{E}_0). The parametric Budyko equations attempt to generalize this framework by introducing a catchment-specific parameter (n or w), intended to represent differences in catchment climate and landscape features. Many studies have developed complex regression relationships for the catchment-specific parameter in terms of biophysical features, all of which use known values of \overline{P} , \overline{E}_0 , and \overline{E} to numerically invert the parametric Budyko equations to obtain values of n or w. Critically, the introduction of n or w renders the parametric Budyko equations underdetermined, precluding their use in predicting \overline{E} and severely limiting their practical application. In this study, we analytically invert both forms of the parametric Budyko equations, producing expressions for n and w only in terms of \overline{P} , \overline{E}_0 , and \overline{E} . These expressions allow for n and w to be explicitly expressed in terms of biophysical features through the dependence of \overline{P} , \overline{E}_0 , and \overline{E} on those same features, illustrating explicitly why the parametric Budyko equations cannot be used for predicting \overline{E} ."

2) Revise the introduction (pages 2-3, lines 44-66) to better contextualize the note within a broader fundamental and conceptual critique of the parametric Budyko framework and catchment-specific parameters:

"The catchment-specific parameter (n or w) has been described as an empirical effective parameter representing the influence of all catchment biophysical features, other than \overline{P} and $\overline{E_0}$, on \overline{E} (Wang et al., 2016a), though this interpretation does not arise from the derivations of Eq. (4) and (6) (Yang et al., 2008) or Eq. (5) and (7) (Zhang et al., 2004). Additionally, the functional forms of the parametric Budyko equations have typically been interpreted as representing the evaporative behavior of individual catchments under different aridity indices (e.g., (Roderick and Farguhar, 2011;Wang and Hejazi, 2011; Yang and Yang, 2011; Wang et al., 2016b; Zhou et al., 2016; Shen et al., 2017; Zhang et al., 2016; Milly et al., 2018)), though this interpretation had not been justified experimentally or observationally. To the contrary, empirical tests of this interpretation strongly suggest that the parametric Budyko equations do not describe the long-term evaporative behavior of catchments, which implies that they are not physically meaningful (Reaver et al., 2020). This means that the values of n and w are not transferable between catchments or across time for individual catchments and thus cannot be related to physical properties in the same manner as the effective parameters in other well-accepted empirical hydrological relationships (e.g., the roughness coefficient in Manning's equation and hydraulic conductivity in Darcy's Law). This renders the parametric Budyko equations under-determined for predicting \overline{E} from only \overline{P} and $\overline{E_0}$ (i.e., one equation with two unknowns, \overline{E} and n or w). The non-transferability and under-determined nature of these equations has been implicitly acknowledged previously in the literature, e.g., (Zhang et al., 2004; Wang et al., 2016a; Greve et al., 2015), where it has been noted that it is not possible to obtain the value of n or w for a specific catchment a priori; one must first estimate \overline{E} , \overline{P} and $\overline{E_0}$ and then invert either Eq. (4), (5), (6), or (7). This lack of predictive ability effectively precludes the practical application of these equations.

Despite this fact, many studies have adopted the "biophysical features" interpretation of n and w and have developed complex regression relationships for the catchment-specific parameter in terms of various

climate and landscape features (Yang et al., 2007;Donohue et al., 2012;Yang et al., 2009;Shao et al., 2012;Li et al., 2013;Xu et al., 2013;Cong et al., 2015;Yang et al., 2016;Zhang et al., 2018;Abatzoglou and Ficklin, 2017;Xing et al., 2018;Zhao et al., 2020;Ning et al., 2020b;Ning et al., 2020a;Li et al., 2020b;Li et al., 2020a;Zhang et al., 2019;Ning et al., 2019;Bai et al., 2019;Ning et al., 2017). In all such studies, known values of \overline{P} , $\overline{E_0}$, and \overline{E} , estimated empirically or via modelling, are used to numerically invert the parametric Budyko equations to obtain values of the catchment-specific parameter, which are then regressed against various biophysical features. The expressions obtained from such endeavours vary significantly between studies, both in their functional forms and what biophysical features are included in the regression, making it difficult to develop a consistent mechanistic understanding of the catchment-specific parameter (Reaver et al., 2020). The intention of these regression expressions is often to substitute them in Eq. (4), (5), (6), or (7) in order to predict \overline{E} . This is a circular process, where an estimate of \overline{E} was used to estimate n or w, which is then used to estimate \overline{E} . In practice, since the inclusion of the parametric Budyko framework adds no new information (Reaver et al., 2020), n and w could be eliminated from this process by fitting the regression models to the already estimated values of \overline{E} directly, bypassing the parametric Budyko framework altogether.

Given the large number of studies seeking to relate the catchment-specific parameter to biophysical features, it seems that the process of numerically inverting the parametric Budyko equations, coupled with the assumption that they are empirically valid and physically meaningful, obscures their under-determined nature and the complete dependence of n and w on \overline{P} , $\overline{E_0}$, and \overline{E} . In this study, we analytically invert both forms of the parametric Budyko equations. The resulting expressions give n and w only in terms of \overline{P} , $\overline{E_0}$, and \overline{E} , illustrating that if n and w depend on any biophysical features, it is due directly to the dependence of \overline{P} , $\overline{E_0}$, or \overline{E} on those same features. Notably, there has not been an analytical derivation illustrating how n and w relate to biophysical features, though the importance of doing so has been noted many times (Zhang et al., 2004;Yang et al., 2008;Donohue et al., 2012;Xu et al., 2013;Greve et al., 2015;Wang et al., 2016a;Zhang et al., 2018). The expressions we develop here for n and w satisfy this need, providing a general expression for the dependence of n and w on any possible biophysical features through the dependence of \overline{P} , $\overline{E_0}$, and \overline{E} on those same features.

3) Revise the Discussion and Conclusions section (pages 9, lines 165-170) to summarize interpretations regarding the utility of catchment-specific parameters and the overall parametric approach:

"Notably, the explicit analytical expression for n and w from Eq. (14) illustrates that the value of the catchment-specific parameter is only determined by \overline{E} , $\overline{E_0}$, and \overline{P} . Therefore, if n or w depend on biophysical features, it is directly due to the dependence of \overline{E} , $\overline{E_0}$, or \overline{P} on those features. In short, this means that Eq. (14) is the general solution for how n and w depend on biophysical features. By substituting \overline{E} , $\overline{E_0}$, or \overline{P} as functions of specific biophysical features into Eq. (14), one obtains the expression for n and w as a function of those features. Eq. (14) thus fulfills the literature-identified need of an analytical expression for n and w in terms of biophysical features. The main implication of the direct dependence of n and w on \overline{E} , $\overline{E_0}$, and \overline{P} is that \overline{E} , $\overline{E_0}$, and \overline{P} must always be estimated prior to obtaining a value of n or w, meaning the parametric Budyko equations are unable to independently predict \overline{E} from $\overline{E_0}$ and \overline{P} . Since the prediction of \overline{E} and its possible temporal evolution are the primary applications of the Budyko framework, the practical utilities of the parametric Budyko equations are severely limited."

Comment 4:

Line 25: To my knowledge, Schreiber (1904) did not use the concept of PET, and this has only been falsely attributed to Schreiber in later publications. It might be worth checking.

Response 4:

We thank the reviewer for calling this to our attention. We revisited the text of Schreiber (1904) to assess whether the concept of potential evapotranspiration was utilized within the manuscript. While the concept of potential evapotranspiration is not explicitly stated, Schreiber (1904) has a functionally equivalent constant "k" in its place. He refers to "k" as the limiting value that the difference between mean annual precipitation and runoff ($\overline{P} - \overline{Q}$, referred to as "die Rückstandshöhe" or the catchment's residue/hold-back height) approaches as precipitation becomes large (i.e., $\overline{P} \to \infty$). Quoting the specific passage:

Je größer x [der jährichen Niederschlagschöhe] wird, um so kleiner wird $\frac{k}{x'}$ so daß man für sehr große x

y = x - k

[die jähriche Abflußhöhe] setzen kann. Heiraus ergibt sich sofort die physikalische Bedeutung des Exponenten k als die Größe, der sich die Differenz zwischen Niederschlag und Abfluß [y] um so mehr nähert, je größer der Niederschlag selbst wird. Dieses Verhältnis scheint mir in der Natur des Problemes begründet zu sein. Die Differenz

z = x - y

kann man als die Rückstandshöhe bezeichen. Schreiber (1904), page 3.

In our current language, the constant k would be the mean annual value of evapotranspiration under energy-limited conditions, i.e., the mean annual potential evapotranspiration, $\overline{E_0}$. However, while constant k is functionally equivalent to $\overline{E_0}$, Schreiber (1904) does not discuss or specify how the water that does not become discharge is being "held back" (i.e., does not discuss it as being evaporated) and therefore does not explicitly introduce the concept of potential evapotranspiration. We propose the following edits to the manuscript to more accurately reflect the contribution of Schreiber (1904):

1) Add the following sentences immediately following Eq. (3) to clarify Schreiber's contribution:

"However, we note that Eq. (2) was originally introduced by Schreiber (1904) with a constant "K" in place of $\overline{E_0}$. While "K" was functionally equivalent to $\overline{E_0}$ in its implementation, its physical interpretation in relation to catchment hydrology was only partially developed by Schreiber (1904). Subsequent investigations by Ol'Dekop (1911) ascribed the concept of maximum possible evaporation (i.e., potential evapotranspiration) to "K", as detailed in Andréassian et al. (2016)."

2) Modify "Equations (2) and (3) were selected..." (page 2, lines 34-36) to "The functional forms of Eq. (2) and (3) were selected...".

Comment 5:

L32: Gentine et al. (2012) removed all catchments with Mediterranean and snowy climates, which are (for example in that same dataset) much less accurately following Budyko (see multiple MOPEX studies

on climate seasonality effects on E/P and Q/P). Therefore I am not sure it's really appropriate to cite Gentine to support this statement...

Response 5:

We thank the reviewer for calling this to our attention. Gentine et al. (2012) describe their methodology for excluding catchments as follows:

"We exclude those basins with missing data; records shorter than 50 years; significant topographical gradients, i.e., elevation changes greater than 1000m or slope steeper than 15 percent, since these basins are likely to span distinct climate regimes and associated impacts on the hydrologic cycle; important anthropogenic modifications (e.g., irrigation, reservoirs) based on the estimates of Wang and Hejazi [2011]. A total of 77 [out of 431] basins was removed from the analysis." Gentine et al. (2012), page 2.

Thus, from our reading, Gentine et al. (2012) do not exclude Mediterranean (i.e., hot dry summers and cool wet winters) nor snowy climates. In fact, several catchments with Mediterranean climates were specifically included in the analysis, labeled as "Out-of-phase climates" by Gentine et al. (2012), and the authors specifically note that phase differences did not cause significant departures from the non-parametric Budyko curve:

"The seasonality between rainfall and potential evaporation does not alter the fit of the basins to the Budyko curve. Noticeably, no systematic biases are present for summer or winter-dominated rainfall regimes (Figure 1b)." Gentine et al. (2012), pages 2-3.

Even though Mediterranean and perennial snowy climates were not explicitly excluded from Gentine et al. (2012), we acknowledge the reviewer's point that catchments with such climates may be underrepresented in the sample—and therefore the ~10% error referenced by Gentine et al. (2012) may be too low. As a check, we computed the distribution of absolute percent errors from the non-parametric Budyko curve for all MOPEX catchments (Schaake et al., 2006) with sufficient data to calculate $\overline{E_0}$, \overline{P} , and \overline{E} (428 out 438 total catchments). Plotting these catchments in Budyko space (Figure 1), we see more spread around the Budyko curve than found in Gentine et al. (2012), however the mean error is 10.3% (Figure 2), closely aligned with both Gentine et al. (2012) and Budyko and Zubenok (1961).

We thus chose to retain the Gentine et al. (2012) reference, but propose the following edits (page 2, lines 31-32) to acknowledge that the 10% uncertainty number refers to the mean of the error distribution:

"The geometric mean of Eq. (2) and (3) have been shown to predict $\frac{\overline{E}}{\overline{P}}$ with a mean uncertainty of ~10% (Budyko and Zubenok, 1961;Gentine et al., 2012) for ungauged basins if \overline{P} and $\overline{E_0}$ are known."

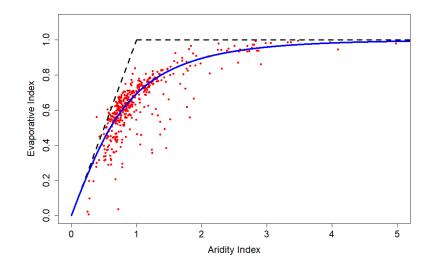


Figure 1: 428 MOPEX catchments (red dots) compared to the non-parametric Budyko equation (blue curve) plotted in Budyko space.

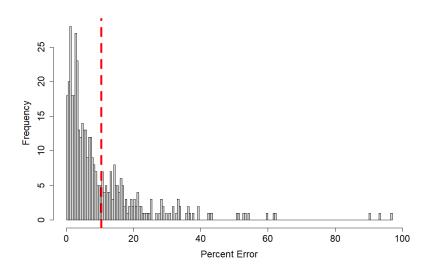


Figure 2: Histogram of the absolute percent errors between the 428 MOPEX catchments and the nonparametric Budyko equation. The mean percent error (vertical dashed red line) is 10.3%.

References:

Abatzoglou, J. T., and Ficklin, D. L.: Climatic and physiographic controls of spatial variability in surface water balance over the contiguous United States using the Budyko relationship, Water Resources Research, 53, 7630-7643, 2017.

Andréassian, V., Mander, Ü., and Pae, T.: The Budyko hypothesis before Budyko: The hydrological legacy of Evald Oldekop, Journal of Hydrology, 535, 386-391, 10.1016/j.jhydrol.2016.02.002, 2016.

Bai, P., Liu, X., Zhang, D., and Liu, C.: Estimation of the Budyko model parameter for small basins in China, Hydrological Processes, 34, 125-138, 10.1002/hyp.13577, 2019.

Budyko, M., and Zubenok, L.: The determination of evaporation from the land surface, Izv. Akad. Nauk SSSR Ser. Geogr, 6, 3-17, 1961.

Cong, Z., Zhang, X., Li, D., Yang, H., and Yang, D.: Understanding hydrological trends by combining the Budyko hypothesis and a stochastic soil moisture model, Hydrological Sciences Journal, 60, 145-155, 10.1080/02626667.2013.866710, 2015.

Donohue, R. J., Roderick, M. L., and McVicar, T. R.: Roots, storms and soil pores: Incorporating key ecohydrological processes into Budyko's hydrological model, Journal of Hydrology, 436-437, 35-50, 10.1016/j.jhydrol.2012.02.033, 2012.

Gentine, P., D'Odorico, P., Lintner, B. R., Sivandran, G., and Salvucci, G.: Interdependence of climate, soil, and vegetation as constrained by the Budyko curve, Geophysical Research Letters, 39, n/a-n/a, 10.1029/2012gl053492, 2012.

Greve, P., Gudmundsson, L., Orlowsky, B., and Seneviratne, S. I.: Introducing a probabilistic Budyko framework, Geophysical Research Letters, 42, 2261-2269, 10.1002/2015gl063449, 2015.

Li, D., Pan, M., Cong, Z., Zhang, L., and Wood, E.: Vegetation control on water and energy balance within the Budyko framework, Water Resources Research, 49, 969-976, 10.1002/wrcr.20107, 2013.

Li, S., Zhang, L., Du, Y., Zhuang, Y., and Yan, C.: Anthropogenic Impacts on Streamflow-Compensated Climate Change Effect in the Hanjiang River Basin, China, Journal of Hydrologic Engineering, 25, 10.1061/(asce)he.1943-5584.0001876, 2020a.

Li, T., Xia, J., She, D., Cheng, L., Zou, L., and Liu, B.: Quantifying the Impacts of Climate Change and Vegetation Variation on Actual Evapotranspiration Based on the Budyko Hypothesis in North and South Panjiang Basin, China, Water, 12, 10.3390/w12020508, 2020b.

Milly, P. C. D., Kam, J., and Dunne, K. A.: On the Sensitivity of Annual Streamflow to Air Temperature, Water Resources Research, 54, 2624-2641, doi:10.1002/2017WR021970, 2018.

Ning, T., Li, Z., and Liu, W.: Vegetation dynamics and climate seasonality jointly control the interannual catchment water balance in the Loess Plateau under the Budyko framework, Hydrology and Earth System Sciences, 21, 1515-1526, 10.5194/hess-21-1515-2017, 2017.

Ning, T., Zhou, S., Chang, F., Shen, H., Li, Z., and Liu, W.: Interaction of vegetation, climate and topography on evapotranspiration modelling at different time scales within the Budyko framework, Agricultural and Forest Meteorology, 275, 59-68, 10.1016/j.agrformet.2019.05.001, 2019.

Ning, T., Li, Z., Feng, Q., Chen, W., and Li, Z.: Effects of forest cover change on catchment evapotranspiration variation in China, Hydrological Processes, 34, 2219-2228, 10.1002/hyp.13719, 2020a. Ning, T., Liu, W., Li, Z., and Feng, Q.: Modelling and attributing evapotranspiration changes on China's Loess Plateau with Budyko framework considering vegetation dynamics and climate seasonality, Stochastic Environmental Research and Risk Assessment, 10.1007/s00477-020-01813-0, 2020b.

Ol'Dekop, E.: On evaporation from the surface of river basins, Transactions on meteorological observations, 4, 200, 1911.

Reaver, N. G. F., Kaplan, D. A., Klammler, H., and Jawitz, J. W.: Reinterpreting the Budyko Framework, Hydrol. Earth Syst. Sci. Discuss., 2020, 1-31, 10.5194/hess-2020-584, 2020.

Roderick, M. L., and Farquhar, G. D.: A simple framework for relating variations in runoff to variations in climatic conditions and catchment properties, Water Resources Research, 47, n/a-n/a, 10.1029/2010wr009826, 2011.

Schaake, J., Cong, S., and Duan, Q.: The US MOPEX data set, IAHS publication, 307, 9, 2006.

Schreiber, P.: Über die Beziehungen zwischen dem Niederschlag und der Wasserführung der Flüsse in Mitteleuropa, Z. Meteorol, 21, 441-452, 1904.

Shao, Q., Traylen, A., and Zhang, L.: Nonparametric method for estimating the effects of climatic and catchment characteristics on mean annual evapotranspiration, Water Resources Research, 48, 10.1029/2010wr009610, 2012.

Shen, Q., Cong, Z., and Lei, H.: Evaluating the impact of climate and underlying surface change on runoff within the Budyko framework: A study across 224 catchments in China, Journal of Hydrology, 554, 251-262, 10.1016/j.jhydrol.2017.09.023, 2017.

Wang, C., Wang, S., Fu, B., and Zhang, L.: Advances in hydrological modelling with the Budyko framework: A review, Progress in Physical Geography, 40, 409-430, 10.1177/0309133315620997, 2016a.

Wang, D., and Hejazi, M.: Quantifying the relative contribution of the climate and direct human impacts on mean annual streamflow in the contiguous United States, Water Resources Research, 47, n/a-n/a, 10.1029/2010wr010283, 2011.

Wang, W., Zou, S., Shao, Q., Xing, W., Chen, X., Jiao, X., Luo, Y., Yong, B., and Yu, Z.: The analytical derivation of multiple elasticities of runoff to climate change and catchment characteristics alteration, Journal of Hydrology, 541, 1042-1056, 10.1016/j.jhydrol.2016.08.014, 2016b.

Xing, W., Wang, W., Shao, Q., and Yong, B.: Identification of dominant interactions between climatic seasonality, catchment characteristics and agricultural activities on Budyko-type equation parameter estimation, Journal of Hydrology, 556, 585-599, 10.1016/j.jhydrol.2017.11.048, 2018.

Xu, X., Liu, W., Scanlon, B. R., Zhang, L., and Pan, M.: Local and global factors controlling water-energy balances within the Budyko framework, Geophysical Research Letters, 40, 6123-6129, 10.1002/2013gl058324, 2013.

Yang, D., Sun, F., Liu, Z., Cong, Z., Ni, G., and Lei, Z.: Analyzing spatial and temporal variability of annual water-energy balance in nonhumid regions of China using the Budyko hypothesis, Water Resources Research, 43, n/a-n/a, 10.1029/2006wr005224, 2007.

Yang, D., Shao, W., Yeh, P. J. F., Yang, H., Kanae, S., and Oki, T.: Impact of vegetation coverage on regional water balance in the nonhumid regions of China, Water Resources Research, 45, 10.1029/2008wr006948, 2009.

Yang, H., Yang, D., Lei, Z., and Sun, F.: New analytical derivation of the mean annual water-energy balance equation, Water Resources Research, 44, n/a-n/a, 10.1029/2007wr006135, 2008.

Yang, H., and Yang, D.: Derivation of climate elasticity of runoff to assess the effects of climate change on annual runoff, Water Resources Research, 47, 10.1029/2010wr009287, 2011.

Yang, Y., Donohue, R. J., and McVicar, T. R.: Global estimation of effective plant rooting depth: Implications for hydrological modeling, Water Resources Research, 52, 8260-8276, 10.1002/2016wr019392, 2016.

Zhang, L., Hickel, K., Dawes, W. R., Chiew, F. H. S., Western, A. W., and Briggs, P. R.: A rational function approach for estimating mean annual evapotranspiration, Water Resources Research, 40, n/a-n/a, 10.1029/2003wr002710, 2004.

Zhang, S., Yang, H., Yang, D., and Jayawardena, A.: Quantifying the effect of vegetation change on the regional water balance within the Budyko framework, Geophysical Research Letters, 43, 1140-1148, 2016. Zhang, S., Yang, Y., McVicar, T. R., and Yang, D.: An Analytical Solution for the Impact of Vegetation Changes on Hydrological Partitioning Within the Budyko Framework, Water Resources Research, n/a-n/a, 10.1002/2017WR022028, 2018.

Zhang, X., Dong, Q., Costa, V., and Wang, X.: A hierarchical Bayesian model for decomposing the impacts of human activities and climate change on water resources in China, Sci Total Environ, 665, 836-847, 10.1016/j.scitotenv.2019.02.189, 2019.

Zhao, J., Huang, S., Huang, Q., Leng, G., Wang, H., and Li, P.: Watershed water-energy balance dynamics and their association with diverse influencing factors at multiple time scales, Sci Total Environ, 711, 135189, 10.1016/j.scitotenv.2019.135189, 2020.

Zhou, S., Yu, B., Zhang, L., Huang, Y., Pan, M., and Wang, G.: A new method to partition climate and catchment effect on the mean annual runoff based on the Budyko complementary relationship, Water Resources Research, 10.1002/2016wr019046, 2016.