Response to comments of Anonymous Referee 2

We would like to thank the referee for the valuable comments and suggestions, which improved the quality of the paper. Below you will find our response in regard to the comments and suggestions.

Comments to the Authors:

The manuscript provides a closed-form expression for the transfer function in the frequency domain of a confined aquifer with variable thickness submitted to a variable recharge rate. The mathematical development is well described and quite easy to follow.

I suggest intermediate revisions.

Here are some general comments:

1) I believe that a conceptual scheme of the system including the main variables of the problem would helpfully accompany the mathematical development.

Response

To make the focus of this study clear, we have changed the first paragraph in Section 2, "Problem Statement," to read as follows

"In certain areas, aquifer recharge can vary greatly over time, so determining the discharge of the aquifer at the outlet for regional groundwater problems, which involves transferring recharge at the aquifer outcrop over a relatively large space scale, can be quite difficult. However, it is very important for planning and management of regional groundwater resources that require knowledge of discharge at the aquifer outlet over a long period of time. This study is therefore devoted to quantifying the discharge response of the confined aquifer at the outlet to the temporal variation in aquifer recharge.

In this study, a confined aquifer with variable thickness is considered as a linear block-box system with a stochastic rainfall recharge input and therefore a stochastic runoff output. Both inputs and outputs are variable in time. In a linear system, the output of the system can be represented as a linear combination of the responses to each of the basic inputs through the convolution integral on a continuous time scale as (e.g., Rugh, 1981; Rinaldo and Marani, 1987)

$$Q(t)=\int_{0}^{t}\varphi(t,\tau)R(\tau)d\tau, \tag{1}$$

where Q and R denote the output flow (discharge) rate and the input flow (recharge) rate of the system, respectively, and φ is the impulse response function of the system. As shown in Fig. 1, once an appropriate impulse response function can be specified at the scale of the aquifer, it is possible to evaluate the system response from records of the input without the need to specify smaller scale heterogeneity. As will be shown below, the transfer function of the system can be used to characterize the uncertainty (variability) expected in applying the convolution integral Eq. (1) to the regional groundwater flow problems.

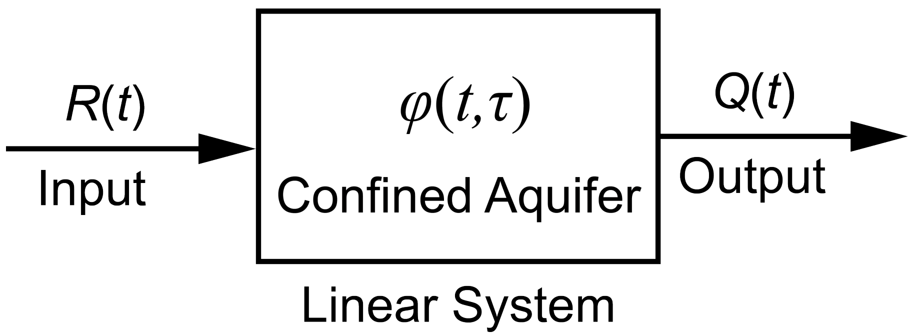

Figure 1. Schematic representation of a linear block-box system."

2) The assumptions should be discussed to better assess the capabilities and limitations of the solution proposed: - Homogeneity: what kind of systems (possibly known aquifers) would be fairly well modelled by the solution proposed? To what extent? - One-dimensional flow: I expect strong limitations of this assumption given the thickness variability and possible convergence or divergence of flow. One would obviously not expect a homogeneous and 1D solution to represent the complexity of natural systems. However, it can still be used as a practical rough approximation.

Response

a. In regional aquifers, the relatively small depth (compared to the horizontal dimensions) allows modelers to simplify the 3-D flow problem in a 2-D problem. Under such a condition, the 2-D flow equation (please see Eq. (11)) is governed by the transmissivity (defined as the hydraulic conductivity times the depth) and depth-averaged hydraulic head (please see Eq. (12)). Transmissivity and depth-averaged head account for variability in both thickness and conductivity in the x_3 direction. Furthermore, the assumption of unidirectional mean regional groundwater flow in the x_1 direction allows the 2-D flow equation to be simplified to a 1-D flow equation. The approximation of 1-D flow equation is simply due to the fact that the flow domain in the x_1 direction is much larger than that in the x_2 direction. The transmissivity and depth-averaged head appearing in 1-D flow equation still account for variability in the x_1 and x_3 directions.

b. "The use of the depth-averaged hydraulic head operator for modeling regional groundwater flow is valid when the variation in aquifer thickness is much smaller than the average thickness (Bear, 1979; Bear and Cheng, 2010). The error introduced by the use of this operator is very small in most cases of practical interest, greatly simplifying the analysis of flow in confined aquifers."

The above sentences are added on the page 9 (Line 144).

c. "It is worth noting that a one-dimensional flow equation with the transmissivity parameter has been widely used to predict the regional groundwater flow fields in the downstream region of the aquifer in field applications (e.g., Gelhar, 1974; Onder, 1998; Molénat et al., 1999; Russian et al., 2013)."

The above note is added on page 10 (Line160).

Gelhar, L.: Stochastic analysis of phreatic aquifers, Water Resour. Res., 10(3), 539-545, 1974.

Molénat, J., Davy, P., Gascuel-Odoux, C., and Durand, P.: Study of three subsurface hydrologic systems based on spectral and cross-spectral analysis of time series, J. Hydrol., 222(1-4), 152-164, 1999.

Onder, H.: One-dimensional transient flow in a finite fractured aquifer system, Hydrol. Sci. J., 43(2), 243-265, 1998.

Russian, A., Dentz, M., Le Borgne, T., Carrera, J., and Jimenez-Martinez, J.: Temporal scaling of groundwater discharge in dual and multicontinuum catchment models, Water Resour. Res., 49(12), 8552-8564, 2013.

3) The sensitivity analysis going along with figures 1, 2 and 3 would greatly benefit from a mechanistic interpretation and/or (a short) comparison with existing works. It would reinforce the validity and usefulness of the solution proposed. This needs more effort.

Response

a. To our knowledge, modeling of the natural recharge-discharge process in confined aquifers of nonuniform thickness as a nonstationary process has not been previously presented in the literature. No data are available to compare the results of this work. An application of the proposed model to predict the outflow discharge is added on page 22 (Line 361) as

"**4.2 Application in the prediction of outflow discharge**

The usefulness of the stochastic theory presented here lies in its essentially predictive nature. The variance can be used as a quantification of the uncertainty associated with the prediction in field situations using the linear system model. In this sense, the solution of Eq. (1) ± two times the square root of the variance provides a rational framework for predicting discharge over a relatively large spatial scale where direct observations of such a dependent variable are not possible.

For large times, the first term in Eq. (37) dominates the sum of the other terms, and therefore the transfer function can be approximated by

$$\left|\Lambda_q(t;\omega)\right|^2 = \frac{256}{\pi^2} L^2 \vartheta^2 [\pi \cos(\pi \Upsilon) - \mu \sin(\pi \Upsilon)]^2 \frac{1}{\Theta_1^2 + \Gamma^2} \frac{1}{\forall^2 + \Gamma^2}$$

$$\left[1 + 2\forall \Xi T_s + T_s^2 - 2(\forall \Xi + T_s)\cos(\Omega_t) - 2\Xi\,\Omega_t \sin(\Omega_t) + \Xi^2\right], \qquad (42)$$

where $\Theta_l = 1 + \pi^2\eta^2$, $\forall = \overline{K}\,\tau_0(\pi^2+\mu^2)/(S_s L^2)$, $\Xi = [\exp(-\Theta_l t/\tau_0) - T_s]/(\forall - \Theta_l)$, and $T_s = \exp(-\forall t/\tau_0)$. If the variation of the rainfall event is generated by a random white noise forcing, the variance of the outflow discharge at large times can then be calculated using Eq. (42) as

$$\sigma_q^2(t) = \int_{-\infty}^{\infty} S_{qq}(t;\omega)\,d\omega = \int_{-\infty}^{\infty} \left|\Lambda_q(t;\omega)\right|^2 S_{\xi\xi}(\omega)\,d\omega$$

$$= \frac{256}{\pi^2} L^2 \vartheta^2 G_0 [\pi \cos(\pi \Upsilon) - \mu \sin(\pi \Upsilon)]^2 \left\{ \frac{\Xi^2}{\Theta_1} + \frac{1 + 2\forall \Xi T_s + T_s^2}{\forall \Theta_1 (\forall + \Theta_1)} \right.$$

$$\left. -2(\forall \Xi + T_s) \frac{\Theta_1 \cosh(\forall t/\tau_0) - \forall \cosh(\Theta_1 t/\tau_0)}{\forall \Theta_1^3 + \Theta_1 \forall^3} - 2\Xi \frac{\sinh(\Theta_1 t/\tau_0) - \sinh(\forall t/\tau_0)}{\Theta_1^2 - \forall^2} \right\}, \qquad (43)$$

where G_0 represents a constant spectral density of a white noise process. Note that white noise is a signal that contains all frequencies in equal proportions, that is, a signal whose spectrum is flat.

After observing the recharge rate $R(t)$ over time at the outcrop of the aquifer and identifying input parameters such as the specific storage coefficient, mean hydraulic conductivity and geometrical parameters of the aquifer and the characteristic time and length scales of the rainfall event for a given area or region, the discharge can be determined under uncertainty in the far downstream aquifer area, Eq. (1) together with Eq. (26) ± two times the square root of Eq. (43). It provides an important basis for the rational management of regional groundwater resources in complex geologic settings under uncertainty.”

b. A note on the validity of the proposed model is added on page 17 (Line 290) as

"Note that the linearity in modeling the recharge-discharge response of a catchment in Eq. (1), which was originally developed for large catchments, increases with catchment area (e.g., Chow et al., 1988). This implies that the impulse responses and transfer functions derived here are valid in large confined aquifers."

Chow, V. T., Maidment, D. R., and Mays, L. W.: Applied hydrology, McGraw-Hill, 1988.

Some specific comments:

- There are no boundary conditions nor initial conditions for equation 10.

Response

In the development of Eq. (11) from Eq. (10), only the boundary conditions are used, namely the slip-free condition, at a fixed boundary the fluid has zero velocity. To clarify that, we add a note on page 9 (Line 141) as

"All terms involved in the fluxes in the directions of x_1 and x_2 at the boundaries are removed due to the no-slip condition at the boundaries."

- Line 91: may serve as a basis? not "service"

Response

The typo has been corrected.

- Line 119: I am not sure that b_1 and b_2 are defined

Response

The definitions of b_1 and b_2 were added on page 9 (Line L132) as

"$b_1(x_1,x_2)$ and $b_2(x_1,x_2)$ are the elevations of the fixed bottom and ceiling of the confined aquifer, respectively,"

- Line 149: "increases", not increase

Response

The typo has been corrected.

- Line 149: be more specific: "increase exponentially" (as shown in Eq.(18)).

Response

As suggested, it has been changed to "increases exponentially".