

Thanks for the editor and reviewer's comments. We have responded to each point in turn in the following sections. The comments are provided in blue text and our responses are organized point-by-point in black text. The manuscript text after the proposed changes is shown in "*black italics*". The number of the line, equation and section refers to the revised version of the manuscript without track changes, shown in yellow highlight.

Editors' comments:

I agree with the reviewer comments regarding the specifically mentioned sentences that are confusing and should be deleted.

We have deleted the mentioned sentences.

Regarding the other comment of referring to the BEAR method as pseudo-Bayesian, with the additional clarifications that the authors included in past review rounds, as far as I understand, it would depend on the technical details of how the BEAR algorithm is combined with a certain inference approach whether the final outcome is a formal or an informal Bayesian approach. Therefore, I think the need to strictly refer to the BEAR algorithm as a pseudo-Bayesian approach can be relativized. I therefore suggest the authors keep the current name of the algorithm.

Thanks for your suggestion. We agree with your opinion that whether it is a formal or an informal Bayesian approach depends on the inference approach the reordering strategy is built in. Considering the comments by the editor and the reviewers, we have kept 'BEAR' as the method name.

Additional comments:

I suggest the authors rearrange the structure of the manuscript such that the results are presented in a separate Section 4. In Section 3, the description of BwMod, as well as the setup of the case studies can be kept, but all the results should be clearly separated in a following section.

Thanks for your suggestion. We have rearranged the structure of the Case Study section as recommended. The titles of the rearranged sections are as follows:

3 Case studies

3.1 Water quality model: the build-up/wash-off model (BwMod)

3.2 Setup of synthetic case study

3.3 Setup of real case study

4 Results

4.1 Case study 1: Synthetic data suffering from input errors

4.2 Case study 2: Synthetic data suffering from input errors and output observation errors

4.3 Case study 3: Real data

We have also changed the related descriptions. The main changes are as follows:

“Section 3 describes the setup of these case studies and Sect. 4 demonstrates their results. Section 5 evaluates the BEAR method and its implementation. Finally, Section 6 outlines the main conclusions and recommendations for this work.” (line 65-67 in Introduction)

“To sum up, two synthetic case studies have been analysed: Case study 1 generates synthetic data only suffering from input errors to evaluate the effectiveness of the BEAR method in isolating the input error and the model parameter error; Case study 2 additionally considers output observation errors via synthetic data generation to evaluate the impacts of other sources of error on the BEAR method.” (line 260-263 in Case studies)

“Nash-Sutcliffe efficiency(NSE) is selected to measure the difference between the modified input in Case study 2 and the true input.” (line 318-319 in Results)

“Figure 4 compares the SDs of estimated input errors, the variances of model residual errors, and reliability and sharpness of model simulations among the four calibration scenarios and three calibration methods in the real case study.” (line 326-327 in Results)

The first paragraph of the conclusion section can be extended, and the second one shortened. The second paragraph is interesting and necessary, but some of the sentences are better placed in the discussion section (and maybe slightly extended).

Thanks for your suggestion. the conclusion section has been modified as follows:

“Observation uncertainty in input data is inherently independent of the model process and the input error model can be estimated prior to the model calibration and simulation by analysis of the data itself. Taking advantage of the prior information of an input error model, a new method, Bayesian error analysis with reordering (BEAR), is proposed to approach the time-varying input errors in WQM inference. It contains two main processes: sampling the errors from the assumed input error distribution and reordering them with the inferred ranks via the secant method. This approach is demonstrated in the case of TSS simulation via a conceptual water quality model, BwMod. Through the investigation of synthetic data and real data, the main findings are as follows:

(1) The estimation of the BEAR method focuses on the error rank rather than the error value in the existing methods, which can take advantage of the constraints of the known overall error distribution and then improve the precision of the input error estimation by optimising the error allocation in a time series.

(2) The introduction of the secant method addresses the nonlinearity in the WQM transformation and can effectively update the error rank of each input data according to minimizing its corresponding model residual.

(3) The ability of the BEAR method in decomposing the input error from model residual error is limited by the accuracy and selection of the input error model and is impacted by model structural uncertainty and output observation uncertainty.

Therefore, the study identifies several areas which need further analysis. Firstly, the availability of

prior knowledge of the input error model is important. When this information is not reliable or even cannot be estimated, a significant issue is the selection of a suitable error distribution. Thus, a general measure should be found to judge whether an error model is appropriate, especially in real cases where the “true” information is limited. Secondly, this study focuses on identifying the input errors in model calibration and the derivation of the BEAR method is based on the assumption that the input error is dominant in the residual error. If the reordering strategy is developed within a more comprehensive framework to quantify multiple sources of error, this assumption will be relaxed, the interactions amongst these error sources might be well-identified and the quantification of individual errors might be improved. This study provides a starting point for developing the rank estimation via the secant method to identify input error. Further study is necessary to modify the algorithm and improve confidence in extended case studies or model scenarios.” (line 418-441)

Specific comments:

Line 35: Replace “For the surrogate error, its probability distribution” by “The probability distribution of the surrogate error”

This has been replaced. (line 39)

Line 37: Replace “more” by “larger”

This has been replaced. (line 41)

Line 52: Replace “input data” by “input data point”, if this is what is meant here

Yes, this has been replaced. (line 56)

Line 60: Replace “two synthetic cases and a real case” by “two synthetic and one real case study”. Check also other instances where this change is appropriate.

Yes, this has been replaced. (line 63). In addition, the contents have been replaced by “one synthetic and one real case study” (line 87).

Line 85: Replace “appealing” by “common”

This has been replaced. (line 89)

Line 147: Replace “can be” by “is often”

This has been replaced. (line 151)

Reviewer's comments:

Thanks to the authors for their effort in improving the manuscript. Overall, the changes made have increased clarity and do help to better understand the working principles behind BEAR and its purpose. Yet, some of the revised sections point into a direction that I struggle to comply with: "Therefore, a modification should be made in the IBUNE approach to improve the accuracy of input error identification." (l. 54-55) I disagree with the revised version of this sentence and therefore suggest to delete it. It indicates that there was a flaw in the IBUNE method that has to be fixed. However, as Bayesian approach, IBUNE simply samples errors without processing them unlike BEAR, which is also stated in l. 344ff.: "In the IBUNE framework (Ajami et al., 2007), the errors are also sampled from the error distribution, but not reordered. Thus, the error precision at each time step cannot be guaranteed. In the BEAR method, adjusting the sampled errors according to the inferred error rank reduces the randomness of the error allocation in the IBUNE framework..." This randomness in sampling is part of the fully Bayesian approach and not something that per se has to be fixed. That said, I find the second sentence about a "guaranteed error precision" unclear and obsolete, and suggest deleting it.

Thanks for your comments. We have deleted "Therefore, a modification should be made in the IBUNE approach to improve the accuracy of input error identification." in line 54-55 and "Thus, the error precision at each time step cannot be guaranteed." in line 344.

Overall, this pertains to the general question of whether BEAR is a full Bayesian approach that I referred to as "arbitrary error treatment" in earlier rounds of reviewing and that Reviewer 2 highlighted in great detail pointing also at the theoretical problems underlying the approach. This is clarified in l. 154 ff. to some degree, i.e. "the BEAR method does not provide formal Bayesian inference". However, this statement is still somewhat hidden in the article and therefore I would like to second the suggestion of reviewer 2 in generally renaming the method a "Pseudo-Bayesian error analysis with reordering". Renaming BEAR to pBEAR (for example) might not appear too appealing to the authors but it would be a more honest name of the method and therefore increase scientific soundness. Hence, I suggest publishing the manuscript with this minor name modification all over the manuscript. I stick to my former evaluation that this manuscript is a valuable contribution to the general discussion of input error treatment. The methodology of BEAR is an idea worth publishing and with the label "pseudo-Bayesian" this will also be a clear contribution to a broad audience.

General: rename the method from "Bayesian" to "pseudo-Bayesian"

We appreciate the reviewer's concerns about the BEAR name. Balancing the authors request with the editor's comment, we have retained BEAR as the method name, as we believe the formal/informal Bayesian nature of the algorithm will depend on the inference approach the reordering strategy is built in.