Interactive comment on “Latitude and bathymetry modify lake warming under ice” by Cintia L. Ramón et al.

Response to Referee #2

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By application of a circulation model in an idealized domain mimicking the heating of an ice-covered lake of irregular morphometry by solar radiation, the authors arrive at an insightful demonstration of the rotation effects on the radial density flows produced by differential heating between shallow and deep lake areas. Rotational gravity flows are widespread in geophysical fluids and an advance in their quantification makes a valuable contribution to earth and planetary fluid dynamics. The ice-covered lakes represent rare natural examples, where these flows can be observed and investigated in detail at their whole range of scales, undisturbed by more energetic flows, usually persisting in open water, oceans, or the atmosphere. In that sense, the authors discuss an intriguing problem, of interest for a wide research community. The modeling methods are relevant, and the results are presented in a well-structured way.

We would like to thank Referee #2 for his/her insightful review. It has greatly helped us identify points that needed better clarification.

I had the opportunity to read the previous comment and generally share the concerns of the Reviewer1: my major criticism refers to the weak connection of the model configuration to the real conditions met in lakes and, as a result, misleading, superfluous, and over-generalized conclusions made by the authors.
Instead of nondimensionalization of the problem with regard to the rotational forces prior applying a numerical model, the authors voluntary choose the domain dimensions of O(10^2) m and vary the Coriolis parameter within 2 (!) orders of magnitude. It is left for the reader’s inspiration to imagine where on Earth f = O(10^{-2}) s^{-1} can be observed (Line 139, Table 1). By using a posteriori re-scaling based on the Rossby number (Eq. 4, Line 148), a conclusion can be drawn that the ageostrophic regime (Ro = O(10^{-1}), Fig. 3, first column), similar to that described by Ulloa et al. (2019), can be found only in small ponds with an area of several ha. In lakes with characteristic length scales of O(1) km (Ro = O(10^{-2}), Fig. 3, second column) and longer (Ro = O(10^{-3}), Fig. 3, second column), the shallow near-shore areas are effectively decoupled from the lake interior by rotation. The modeling results do not however provide a final proof for the importance of differential heating even in small ponds: they are typically much shallower than the modeled domain and have the background mixing intensities higher than those adopted in the model (Lines 108-111).
Herewith, the following outcomes of the study must be made clear: 1. For the vast majority of ice-covered lakes, differential heating does not contribute to the vertical mixing in the lake interior. 2. The previous findings of Ulloa et al. (2019) must be reconsidered taking into account the new results. 3. All variations of the Rossby number should be clearly related to corresponding variations in lake horizontal dimensions. Any mentioning of latitudinal effects should be removed, since for all seasonally ice-covered lakes f = O(10^{-4}) s^{-1}.

AR (Authors’ response) 1: We appreciate the reviewer’s comment. However, some of the interpretations of the reviewer are not aligned with what we actually show throughout the manuscript. We believe that this reflects that our manuscript needs clarifications, which will be addressed.

- First: Starting already in the introduction we presented the Rossby number as the non-dimensional parameter to evaluate "*the importance of Earth rotation on horizontal flows*". So this non-dimensional number defines the rotational regime. The selection of the different simulations in Table 1 is based on this parameter. In our simulations we certainly modify *f* and not the size of the lake, but this decision is based on computational efficiency in terms of the needed computational resources. Modifying *f* instead of *L* is, for example, common practise in laboratory (e.g., Afanasyev and Zhang, 2018; Cenedese and Adduce, 2008 (doi: 10.1017/S0022112008001237);Fultz et al., 1959; Wells and Cossu, 2013 (doi: 10.1098/rsta.2012.0366) and modeling (e.g., Carpenter and Timmermans, 2014 (10.1175/JPO-D-13-098.1); Pal and Chalamalla, 2020 (10.1017/jfm.2020.94); Ulloa et al., 2015 (doi: 10.1017/jfm.2015.311)) works studying rotational effects . We never stated in the text that a value of $f = O(10^{-2}\text{-}10^{-3})\ s^{-1}$ is representative of Coriolis values on Earth, which obviously is not. We are sorry if our writing leads to that interpretation. What is representative of lakes on Earth is the order of magnitude of the Rossby number (as shown in Fig. 5d) and this was stated when we defined the scenarios "*We investigate three scenarios ranging from weak (Ro $O(10^{-1})$) to stronger (Ro $O(10^{-3})$) rotational influence (Table 1 and see Sect. 2.4 and Sect. S1.1 in the supplementary material for the Ro calculations). This range of Ro spans the expected range of values typical of the varying size and latitudinal distribution of ice-covered lakes on Earth (see Sect. 4).*" We believe that the source of confusion comes from the order in which we presented the information in the Methods section. We will combine the last paragraph in section 2.3 in Methods with the section 2.4 (*"Rossby number"*) to help clarify this point. We will also remove the column of *f* values in Table 1.

- Second: As shown in Fig. 5d (calculated with the data of lakes in the HydroLAKES database) and discussed in the text, we expect the ageostrophic regime to be more common than the geostrophic regime. This suggests that for the vast majority of ice-covered lakes, the circulation studied by Ulloa et al. (2019) applies and differential heating could potentially affect the warming rates in the lake interior. Considering radial velocities in the range of $O(10^{-3}\text{-}10^{-2})$ m/s as reported for ice-covered lakes, to obtain values of *Ro* in the range $0.1 < Ro < 1$, the length *L* from the littoral region to the lake interior in the *Ro* calculations could be up to several kilometers even at high latitudes (see figure below). We will clarify this point in the text. We propose to include this clarification in the subsection "Rossby number" in the Methods section.

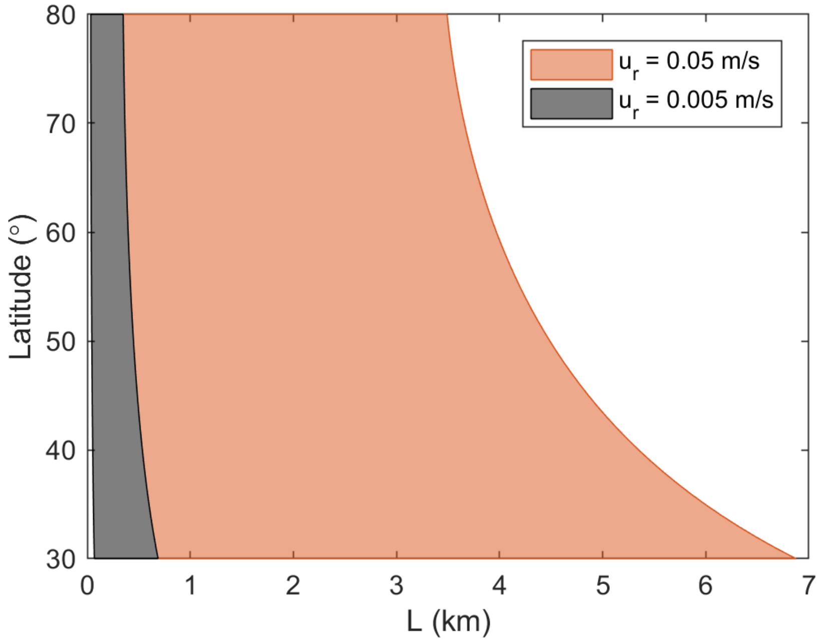

Figure. Example of range of lengths (here lake radius) and latitudes for the ageostrophic regime ($0.1 \lesssim Ro < 1$) for radial velocities of 0.005 m/s (black) and 0.05 m/s (red)

- The objective of this work is to provide a general framework of the importance that the lake bathymetry and rotation may have on the warming of lakes under the ice. Although we did not provide a specific "field site" validation, we validated the RANS model with a LES model in the supporting information and we provided examples in the discussion section that support the circulation pattern described here and which provide qualitative validation of this work.

Thus, based on these points, we have to disagree with the main two outcomes as expressed by the reviewer. Differential heating could indeed contribute to the warming of the lake interior. However, we agree with the reviewer on the fact that *Ro* depends predominantly on the horizontal dimensions of the lake. While *L* could vary by several orders of magnitude, *f* remains O(10^{-4}) s^{-1}. Still, as shown in the figure above, the variability in *f* alone will allow that bigger lakes lie within the ageostrophic regime as we move towards lower latitudes. See for example how a lake at 30°N could double the size of a lake at latitudes > 70°N and still be in the ageostrophic regime. The relative importance of *L* and *f* will be clarified in the text. As suggested by reviewer 1, we will also reorder the title of the manuscript.

We propose that Section 2.4 will be now called "Rossby number and test cases" and that the text in this section now reads: "*Since we are interested in evaluating the advection of heat from the littoral to the lake interior, the surface radius, R, was selected as the characteristic length scale in the calculations of Ro. Ro was calculated using the maximum radial velocity in the littoral region,* $U_{rs\text{-}max}$*, as the characteristic velocity scale (see details in Sect. S1.1 in supplementary material).*

$$Ro = \frac{U_{rs-max}}{fR} . \qquad (4)$$

$R/U_{rs\text{-}max}$ is the nominal time required for a gravity current at speed $U_{rs\text{-}max}$ to reach the center of the lake. When $R/U_{rs\text{-}max} > f^{-1}$, gravity currents are affected by Earth rotation (e.g., Davarpanah Jazi et al., 2020). While R could vary several orders of magnitude among the ice-covered lakes on Earth, f remains O(10^{-4}) s^{-1}. Thus, Ro depends predominantly on the horizontal dimensions of the lake. Still, the variability in f alone allows that bigger lakes lie within the same rotation regime as one moves towards lower latitudes. Note that for a given value of $U_{rs\text{-}max}$, to obtain the same value of Ro, Eq. (4) shows that

a lake at 30° N ($f = 0.7\times10^{-4}\ s^{-1}$) would still double the size of a lake at latitudes ≥ 70° N ($f \geq 1.4\times10^{-4}\ s^{-1}$)

The model was first used to simulate rotational effects as in Ulloa et al. (2019), with a characteristic Ro $O(10^{-1})$. This corresponds to run 1 in Table1. For a range of measured radial velocities of $O(10^{-3}–10^{-2})$ m s^{-1} under ice (Forrest et al., 2013; Kirillin et al., 2015; Rizk et al., 2014), a value of Ro $O(10^{-1})$ could be representative of lakes ranging from several tens of meters to several kms in length. To analyze the effect of rotation in the lake circulation and in the warming of the CML, two additional simulations were conducted where we increased rotational effects by decreasing Ro up to two orders of magnitude (runs 2 and 3 in Table 1). To analyze bathymetric effects (differential heating), an additional simulation was conducted (reference simulation) where forcing was kept as in run 1, but the bathymetry was modified to obtain a cylinder of depth D = H. Each run spans 12 radiative cycles (12 days). This number of cycles was long enough to expose and analyze the effect of rotation and bathymetry on lake warming under ice."

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***Other remarks**:*

L68 The geometrical factor G (Eq. 1 and Eq. 8) is of little predictive power as long as the hypsometry (the shape of the basin) is not included. When derived in a strict way, G incorporates a "shape factor" S = 0..1, which is found as an integral $S = \int_0^1 D(x)dx$, where D(x) = 0..1 is dimensionless depth, x = 0..1 is the relative distance from the shore to the lake center. For vertical walls S = 0, for linear slope S = 0.5, for the typical "bowl"-shaped lake S ≈ 0.3, and for the authors' tanh-approximation S ≈ 0:6. Hence, application of uncorrected G to different basin shapes can lead to ≥ 2 times differences in the result. Removal of G and related discussion is strongly recommended unless the basin shape is incorporated in the scaling.

AR2: We would like to highlight that the *G* parameter is accounting for the advective transport of heat from the littoral region within the CML. Therefore, it is not the "basin-scale" hypsometry that is affecting *G*, but the "fraction" shallower than the depth of the CML. We agree with the reviewer on the importance of the lake hypsometry, and, lake hypsometry is directly included in the formulation of the *G* parameter. The geometrical factor *G* in Eq. 1 is expressed as

$$G = \left|\frac{A_{shallow}}{A_{total}}\left(\frac{\bar{h}}{h_{cml}} - 1\right)\right|$$

For a given lake bathymetry, we calculate $A_{shallow}$ as the surface area of the water columns of the lake that are shallower than the depth of the convective mixed layer, and as shown in Fig. S1, the average depth of the littoral region $\bar{h}$ is calculated as

$$\bar{h} = \frac{V_{shallow}}{A_{shallow}}$$

where $V_{shallow}$ is the volume of the littoral region with an arbitrary morphology. So $\bar{h}$ would be an average value for the whole littoral region, not for a specific cross-section. Then, $A_{shallow}$, $V_{shallow}$ and $\bar{h}$ depend on the lake hypsometry. For the extreme case of vertical walls, *S* = 1 and, in Eq. 1, $A_{shallow}$ would be equal to zero, so *G* = 0. In Eq. 8, this G parameter was expressed for the specific case of a circular surface area, but again $L_{shallow}$ and $\bar{h}$ are average values for the whole basin.

L131: $1/\lambda = 2.5\ m^{-1}$ (< 1 m Secchi depth) is rather turbid than moderately clear and is not typical for the majority of ice-covered lakes. Would the differential heating increase in more transparent waters? How the transparency affects the rotation effects? Make it clear in the text.

AR3: Yes, the reviewer is right. A lake with a light attenuation of 2.5 m^{-1} is better classified as turbid. We will modify this in the text.

There are different scenarios with respect to the attenuation of solar radiation: (1) The penetration depth of solar radiation is shallower than the depth of the littoral region and h_{cml}; (2) the penetration depth is deeper than the depth of the littoral region but shallower than h_{cml}; and (3) the penetration depth is deeper than h_{cml}. In the absence of horizontal advection of heat, the vertically-integrated rate of change of temperature in a water column of depth d in the littoral region would be

$$\frac{\partial \overline{T_L}}{\partial t} \approx \frac{I_0\left(1-e^{-d/\lambda}\right)}{d}$$

And in the lake interior (considering that the background stratification suppresses the vertical transport of heat at h_{cml})

$$\frac{\partial \overline{T_I}}{\partial t} \approx \frac{I_0\left(1-e^{-h_{cml}/\lambda}\right)}{h_{cml}}$$

Thus, the subtraction of the two gives the rate of change of the temperature between the two regions, that is:

$$\frac{\partial (\overline{T_L}-\overline{T_I})}{\partial t} \approx \frac{I_0\left(1-e^{-d/\lambda}\right)}{d} - \frac{I_0\left(1-e^{-h_{cml}/\lambda}\right)}{h_{cml}}$$

In the first scenario, $\lambda << d$; thus, $e^{-d/\lambda} \approx e^{-hcml/\lambda} \approx 0$ for all possible values of λ and the lateral temperature gradient does not depend on the light attenuation: $\frac{\partial (\overline{T_L}-\overline{T_I})}{\partial t} \approx I_0(\frac{1}{d} - \frac{1}{h_{cml}})$. In this scenario, more vigorous convection is expected as attenuation increases, but differential heating should not be affected. In the second scenario, if by decreasing the attenuation an important fraction of the solar radiation reaches the sediment of the littoral region and is absorbed there without being emitted back into the lake (no sediment heat flux), then $e^{-d/\lambda}$ may not be negligible and $\frac{\partial (\overline{T_L}-\overline{T_I})}{\partial t} \approx I_0(\frac{1-e^{-d/\lambda}}{d} - \frac{1}{h_{cml}})$. For a given radiative flux, differential heating would be weakened as the water becomes clearer. If otherwise we consider that all the incoming heat is retained in the water column, differential heating would not be modified. Finally, in the third scenario, $e^{-hcml/\lambda}$ may not be negligible, so if $(1-e^{-d/\lambda}) \sim 1$ (perfect insulator), differential heating would be enhanced as the water becomes clearer: $\frac{\partial (\overline{T_L}-\overline{T_I})}{\partial t} \approx I_0(\frac{1}{d} - \frac{1-e^{-h_{cml}/\lambda}}{h_{cml}})$.

Attenuation could then influence the magnitude of the radial velocity (and thus *Ro*), but we expect this effect to be secondary to the effect of the magnitude of I_0 and the geometry of the littoral region. We will write some lines about this in the Method section. We propose that the text in section 2.4 now reads *"Here the time t is expressed in days, I_0 (= 1×10^{-5} ºC m s^{-1}) is the water surface radiative forcing, F(t) = sin(2πt) during the day (t < 0.5) and zero otherwise, and 1/λ (= 2.5 m^{-1}) is the depth scale for light attenuation. The order of magnitude of I_0 and the value for λ selected, are representative of late-winter conditions in turbid waters (Leppäranta et al., 2003; Bouffard et al., 2019; Ulloa et al., 2019). **The intensity of convection is expected to decrease as λ increases (e.g., Winters et al., 2019), but the effect of light attenuation on differential heating and on the magnitude of the radial velocities remains secondary compared to the effect of the magnitude of I_0 and the geometry of the littoral region.** For visualization purposes only, our results are shifted 0.25 days so the peak in the radiative heat flux matches midday (Fig. 1)."*

Leppäranta, M., Reinart, A., Erm, A., Arst, H., Hussainov, M. and Sipelgas, L.: Investigation of ice and water properties and under-ice light fields in fresh and Brackish water bodies, Nord. Hydrol., 34(3), 245–266, doi:10.2166/nh.2003.0006, 2003.

Winters, K. B., Ulloa, H. N., Wüest, A. and Bouffard, D.: Energetics of radiatively-heated ice-covered lakes, Geophys. Res. Lett., 2019GL084182, doi:10.1029/2019GL084182, 2019.

Although this is indeed an interesting topic, we consider that including the full explanation (the three scenarios described above) would deviate from the main point of the manuscript. In conclusion, we prefer not developing extensively this point in the main manuscript unless the referee and the handling editor think otherwise.

L202-206: The geostrophic balance does not hold true in the bottom boundary layer (BBL), and the Coriolis effect on bottom-slope currents is strongly reduced by bottom friction. How good is BBL is reproduced by the model?

AR4: We set no-slip conditions on the bottom boundaries. Also, in our simulations h_{cml} reaches values of ~7 m after 12 days. The vertical resolution of our grid in this region is of 0.05 (up to a depth of 2 m) and 0.1 m (up to 10 m). For an expected BBL height of O(m), this means that our model resolves the BBL with at least 10 grid points.

L314 Avoid the term "fjord-type" lakes, because the effect of the non-unity horizontal aspect ratio was not investigated in the study.

AR5: We agree with the reviewer on the fact that using the term "fjord" will lead the reader to think not only on high slopes, but most probably, on an elongated lake basin. The text will be modified and will refer now only to lakes with "more vertical walls".

L320 ". . .Peruvian Andes . . . ": Any example of a seasonally ice-covered lake at latitudes below 15°? A lake at 20° lat or lower can develop ice cover only at altitudes where liquid water is extremely rare. The whole discussion on the latitudinal variability is vague and should be rethought in terms of lake size (see above).

AR6: Yes, we will restrict our discussion in the text and Fig. 5d to lakes starting at ~30°N (Lakes in the Himalayas and Tibet Plateau). Still, there are examples of lakes that seasonally freeze in the North of Chile, like Licancabur Lake (22°S) (Hock et al., 2002; bibcode 2002AGUFM.P71A0435H). With respect to the discussion about the importance of *L* vs. *f*, please see answer AR1.

L321-323 The sentence is unsupported and—to be straight—wrong and misleading. Has to be removed.

AR7: We believe that this sentence is not wrong or misleading and that the confusion is coming from the reviewer's interpretation that the ageostrophic regime only occurs in small shallow ponds (please, see details in answer AR1).

L330-332 It is quite an interesting point deserving more discussion in view of the presented results. If the littoral water temperatures reach the maximum density point (TMD), but the lake interior stays colder than TMD, the ice cover will quickly melt over the shallows, forming the well-known "moats". As long as the rest of the lake stays ice covered, water temperatures in these open areas will be close to TMD without long-lasting stable stratification. "Moat" formation has been traditionally referred to terrestrial heat fluxes; the role of heat capture by rotation was never considered in this context but deserves a closer discussion.

AR8: Yes, as the reviewer points out, the excess heat retained in the littoral region could act as an internal accelerator for the ice melting process, and thus, potentially, for moat formation. Although this is indeed an interesting topic, we believe that an extended discussion linking the retention of heat in the shallows and moat formation would overstate the significance of the results of this numerical study. Instead, we propose that we add a new line in this section establishing a possible connection between the two processes. We propose that the text reads: "*Due to the retention of heat in the littoral region, water temperature there could potentially reach values* $\geq T_{md}$. *This would lead to the development of a stable stratification in the littoral region and to the suppression of convection that, in contrast, would continue in the lake interior.* ***This could*** *have implications for early formation of thermal bars and/or* ***contribute to the formation of moats (e.g., Nolan et al., 2002)***".

Presentation in form of a HESS publication of the otherwise well-performed and insightful study to a wider community can be recommended only after resolving the above mentioned issues.