



1 **Sigmoidal Water Retention Function with Improved Behavior in Dry**
2 **and Wet Soils**

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14 **Abstract**

15

16 A popular parameterized soil water retention curve (SWRC) has a hydraulic conductivity
17 curve associated with it that can have an infinite slope at saturation. The problem was
18 eliminated before by giving the SWRC a non-zero air-entry value. This improved version
19 still has an asymptote at the dry end, which limits its usefulness for dry conditions and
20 causes its integral to diverge for commonly occurring parameter values. We therefore
21 joined the parameterizations' sigmoid mid-section to a logarithmic dry section ending at
22 zero water content for a finite matric potential, as was done previously for a power-law
23 type SWRC. We selected five SWRC parameterizations that had been proven to produce
24 unproblematic near-saturation conductivities and fitted these and our new curve to data
25 from 21 soils. The logarithmic dry branch gave more realistic extrapolations into the dry
26 end of both the retention and the conductivity curves than an asymptotic dry branch. We
27 tested the original curve, its first improvement, and our second improvement by feeding
28 them into a numerical model that calculated evapotranspiration and deep drainage for nine
29 combinations of soils and climates. The new curve was more robust than the other two. The
30 new curve was better able to produce a conductivity curve with a substantial drop during
31 the early stages of drying than the earlier improvement. It therefore generated smaller
32 amounts of more evenly distributed deep drainage compared to the spiked response to
33 rainfall produced by the earlier improvement.



34 Introduction

35 The soil water retention function introduced by van Genuchten (1980) has been the
36 most popular parameterization (denoted VGN below; these and other abbreviations are
37 listed in Appendix A) to describe the SWRC in numerical models for unsaturated flow for
38 the past few decades (e.g., Kroes et al., 2017; Šimůnek and Bradford, 2008; Šimůnek et al.,
39 2016):

$$41 \quad \theta(h) = \theta_r + (\theta_s - \theta_r)(1 + |\alpha h|^n)^{\frac{1}{n}-1}, \quad h \leq 0 \quad (1)$$

42
43 where h denotes the matric potential in equivalent water column [L]. The volumetric water
44 content is denoted by θ , with the subscript 's' denoting its value at saturation and the
45 subscript 'r' its residual, or irreducible, value. Parameters α [L^{-1}] and n are shape
46 parameters.

47 Van Genuchten (1980) combined Eq. (1) with Mualem's (1976) conductivity model
48 and derived an analytical expression for the unsaturated hydraulic conductivity curve:

$$50 \quad K(h) = K_s \frac{\left[1 - |\alpha h|^{n-1} (1 + |\alpha h|^n)^{\frac{1}{n}-1}\right]^2}{(1 + |\alpha h|^n)^{\frac{1}{2} - \frac{1}{2n}}} \quad (2)$$

51
52 where K [LT^{-1}] is the soil hydraulic conductivity and K_s [LT^{-1}] its value at saturation.

53 Hysteretic (Kool and Parker, 1987) and multimodal versions (Durner, 1994) of Eq.
54 (1) are available. Apart from the convenience of having analytical expressions for the
55 retention as well as the conductivity curve, the function's popularity derives from its



56 continuous derivative and its inflection point, which gives it considerable flexibility in
57 fitting observations.

58 Fuentes et al. (1991) warned that the asymptotic residual water content at the dry
59 end could lead to a non-converging integral of the retention curve, and showed how this
60 would mathematically lead to a physically impossible unlimited water uptake capacity of a
61 finite soil column. From their analysis follows that this can only be prevented if $n > 2$ in Eq.
62 (1), a condition which is often not satisfied.

63 Near saturation, the slope $d\theta/dh$ is not zero at zero matric potential. This implies
64 that the soil has pores that have at least one infinite principal radius according to the
65 Laplace-Young Law (Hillel, 1998, p. 46), which is physically unacceptable (see also Iden et
66 al., 2015). Durner (1994) noted this could lead to an infinite slope in the hydraulic
67 conductivity function of Mualem (1976) when the matric potential approached zero, and
68 Ippisch et al. (2006) showed that if $n < 2$ this would indeed be the case. The more recent
69 sigmoid curve of Fredlund and Xing (1994) and its modification by Wang et al. (2016), used
70 by Wang et al. (2018) and Rudiyanto et al. (2020) have the same problem (see Appendix B
71 for the proof). The curve of Assouline et al. (1998) is based on the Weibull distribution, and
72 therefore has a non-zero slope at zero matric potential when its fitting parameter η is
73 smaller than 2, which was the case for 75% of the soils for which it was fitted. None of these
74 curves therefore offers a remedy to the problem associated with VGN.

75 Corrections for the conductivity curve were proposed by Vogel et al. (2001), Schaap
76 and van Genuchten (2006), and Iden et al. (2015), but these leave the effect of the non-
77 physical, very large pores on the SWRC intact and create an inconsistency between the
78 retention model and the conductivity model. For instance, Iden et al. (2015) clipped the



79 integral in the conductivity function at a matric potential h_c somewhat below zero. In the
80 range between h_c and zero, their modified unsaturated hydraulic conductivity increased
81 linearly with the water content (Iden et al. (2015), Fig. 1), which is not physically realistic
82 because the pore sizes that are being filled are increasing in size according to Eq. (1) or its
83 multimodal version (Peters et al., 2011). Only Ippisch et al. (2006) addressed the
84 underlying problem in the SWRC by introducing a non-zero air-entry value, thereby
85 eliminating excessively large pores whilst maintaining the mathematical consistency
86 between the expressions for the retention and the conductivity curves. In doing so they
87 sacrificed the continuity of the derivative of the VGN curve. Iden et al. (2015) suspected this
88 would pose a challenge to numerical solves of Richards' equation, but Ippisch et al.'s (2006)
89 numerical simulations ran without difficulty. Their equation scaled the sigmoid curve by its
90 value at the air-entry value h_{ae} [L] and introduced a saturated section for $h > h_{ae}$.

91

$$92 \quad \theta(h) = \begin{cases} \theta_r + (\theta_s - \theta_r) \left(\frac{1+|\alpha h|^n}{1+|\alpha h_{ae}|^n} \right)^{\frac{1}{n}-1}, & h \leq h_{ae} \\ \theta_s, & h > h_{ae} \end{cases} \quad (3)$$

93

94 This function is denoted VGA below.

95 The smooth, sigmoidal shape of VGN resembles many observed curves for which the
96 data points in the wet range were obtained by equilibrating cylindrical soil samples at well-
97 defined matric potentials and determining the corresponding water content by weighing
98 the sample (Klute, 1986, p. 644–647). Liu and Dane (1995) took into account the vertical
99 variation of the water content in such samples and demonstrated that a power-law SWRC
100 with a well-defined air-entry matric value but without inflection point can produce a



101 sigmoid-type apparent SWRC if the non-uniform distribution of water in the sample is
102 ignored. A series of data points suggesting a smooth SWRC therefore does not intrinsically
103 contradict the existence of a discrete non-zero air-entry value, corroborating the
104 correction to VGN by Ippisch et al. (2006).

105 In a separate development, several researchers argued that in the dry range, water is
106 bound to the soil by adsorptive rather than capillary forces. Usually, a logarithmic term that
107 allowed the adsorbed water content to go to zero at a prescribed matric potential was
108 added to a capillary term. The former would dominate in the dry range and become
109 negligible as the soil became wetter (e.g., Campbell and Shiozawa, 1992; Fayer and
110 Simmons, 1995; Khlosi et al., 2006; Peters, 2013). The logarithmic relationship was based
111 on the sorption theory of Bradley (1936). It removed the asymptote and the associated
112 problem of the non-converging integral of the SWRC that Fuentes et al. (1991) warned
113 about. Rossi and Nimmo (1994) presented a junction model in which a critical matric
114 potential separated purely capillary bound water (described by a variation of the Brooks-
115 Corey (1964) model) from solely adsorbed water. By doing so they avoided the problem of
116 many of the other models that would have some capillary bound water still present in the
117 soil below the matric potential at which the adsorbed water content had gone to zero.

118 Madi et al. (2018) generalized the analysis of Ippisch et al. (2006) and applied it to
119 18 parameterizations of the SWRC to verify that the slope of the hydraulic conductivity near
120 saturation would remain finite. Apart from Eq. (3), only the expressions developed by
121 Brooks and Corey (1964) (denoted BCO), Fayer and Simmons (1995) (denoted FSB), and
122 the junction model of Rossi and Nimmo (1994) (denoted RNA) satisfied this requirement.
123 In the latter case, the equation had to be modified by removing a modification that



124 smoothed the curve near saturation. All these equations have a power law relationship
125 between the water content and the matric potential, and therefore do not have the sigmoid
126 shape of VGN and VGA.

127 As noted above, the introduction of a non-zero air-entry value by Ippisch et al.
128 (2006) eliminated the unphysically large slopes of the hydraulic conductivity according to
129 Mualem (1976). The approach of Rossi and Nimmo (1994) resolved the issue of the
130 asymptotic behavior in the dry range. The objective of this paper therefore is to combine
131 Rossi and Nimmo's (1994) model for the dry range with the VGA model of Ippisch et al.
132 (2006) to arrive at a SWRC (denoted RIA) that has a non-zero air-entry value, a sigmoid
133 shape in the intermediate range, a dry branch that can reach zero water content at a finite
134 matric potential, and therefore a finite integral. We will also develop an explicit expression
135 for the unsaturated hydraulic conductivity based on this SWRC. For completeness, a
136 generalized expression for multimodal SWRCs will also be derived.

137 Together with the other functions that lead to physically acceptable behavior of the
138 hydraulic conductivity near saturation (BCO, FSB, RNA, and VGA), RIA will be fitted to 21
139 soils selected from the UNSODA database (National Agricultural Library, 2017; Nemes et al.,
140 2001) that cover a wide range of textures (Madi et al., 2018). For comparison VGN is also
141 included, in view of its de facto status as the standard parameterization for the SWRC. All
142 three versions with the sigmoid shape (VGN, VGA, and RIA) will be tested in a simulation
143 study for different combinations of soil types and climates.

144

145 **Theory**

146 **The Soil Water Retention Curve**



147 The junction model of Rossi and Nimmo (1994) has a SWRC with a logarithmic dry
 148 branch without residual water content. The parameterization proposed by Ippisch et al.
 149 (2006) combines the sigmoid shape of van Genuchten (1980) with a non-zero air-entry
 150 value. By setting the θ_r in Ippisch et al. (2006) to zero, we can combine the two models to
 151 give the following parameterization:

152

$$153 \quad \theta(h) = \begin{cases} 0, & h \leq h_d \\ \theta_s \beta \ln\left(\frac{h_d}{h}\right), & h_d < h \leq h_j \\ \theta_s \left(\frac{1+|\alpha h|^n}{1+|\alpha h_{ae}|^n}\right)^{\frac{1}{n}-1}, & h_j < h \leq h_{ae} \\ \theta_s, & h > h_{ae} \end{cases} \quad (4)$$

154

155 where subscripts 'd' and 'ae' denote the value at which the water content reaches zero and
 156 the air-entry value, respectively, and subscript 'j' indicates the value at which the
 157 logarithmic and sigmoid branch are joined. Joining instead of adding the the logarithmic
 158 and the sigmoid functions avoids potentially non-monotonic behavior (Peters et al., 2011).

159 The derivative of Eq. (4) is

160

$$161 \quad \frac{d\theta}{dh}(h) = \begin{cases} 0, & h \leq h_d \\ \frac{\theta_s \beta}{h}, & h_d < h \leq h_j \\ \theta_s \alpha (n-1) |\alpha h|^{n-1} (1+|\alpha h_{ae}|^n)^{1-\frac{1}{n}} (1+|\alpha h|^n)^{\frac{1}{n}-2}, & h_j < h \leq h_{ae} \\ 0, & h > h_{ae} \end{cases} \quad (5)$$

162

163 Mass continuity dictates that the SWRC is continuous. At the air-entry value, this condition
 164 is met irrespective of the parameter values. At h_j , continuity requires the following equality
 165 to hold:



166

$$167 \quad \beta \ln \left(\frac{h_d}{h_j} \right) = \left(\frac{1 + |\alpha h_j|^n}{1 + |\alpha h_{ae}|^n} \right)^{\frac{1}{n} - 1} \quad (6)$$

168

169 In accordance with Rossi and Nimmo (1994) we also require the derivatives at h_j to match,
 170 leading to

171

$$172 \quad \beta = (n - 1) |\alpha h_j|^n (1 + |\alpha h_{ae}|^n)^{1 - \frac{1}{n}} (1 + |\alpha h_j|^n)^{\frac{1}{n} - 2} \quad (7)$$

173

174 Combining Eqs. (6) and (7) and solving for h_d gives:

175

$$176 \quad h_d = h_j \exp \left(\frac{1 + |\alpha h_j|^{-n}}{n - 1} \right) \quad (8)$$

177

178 This leaves h_{ae} , h_j , θ_s , α , and n as fitting parameters.

179 The derivation of a multimodal curve analogous to that of Durner (1994) and
 180 Zurmühl and Durner (1998) is straightforward if the values of h_{ae} and h_j are kept the same
 181 for all contributing terms:

$$182 \quad \theta(h) = \begin{cases} 0, & h \leq h_d \\ \theta_s \beta \ln \left(\frac{h_d}{h} \right), & h_d < h \leq h_j \\ \theta_s \sum_{i=1}^k w_i \left(\frac{1 + |\alpha_i h|^{n_i}}{1 + |\alpha_i h_{ae}|^{n_i}} \right)^{\frac{1}{n_i} - 1}, & h_j < h \leq h_{ae} \\ \theta_s, & h > h_{ae} \end{cases} \quad (9)$$

183



184 where the modality is indicated by k . The weighting factors w_i are bounded on the interval
 185 $[0,1]$ and their sum must equal 1 (Durner, 1994; Zurmühl and Durner, 1998). Requiring the
 186 logarithmic and the multimodal branch as well as their derivatives to match at h_j leads to
 187

$$188 \quad \beta = \sum_{i=1}^k w_i (n_i - 1) |\alpha_i h_j|^{n_i} (1 + |\alpha_i h_{ae}|^{n_i})^{1 - \frac{1}{n_i}} (1 + |\alpha_i h_j|^{n_i})^{\frac{1}{n_i} - 2} \quad (10)$$

189

190 and

191

$$192 \quad h_d = h_j \exp \left[\frac{\sum_{i=1}^k w_i (1 + |\alpha_i h_{ae}|^{n_i})^{1 - \frac{1}{n_i}} (1 + |\alpha_i h_j|^{n_i})^{\frac{1}{n_i} - 1}}{\sum_{i=1}^k w_i (n_i - 1) |\alpha_i h_j|^{n_i} (1 + |\alpha_i h_{ae}|^{n_i})^{1 - \frac{1}{n_i}} (1 + |\alpha_i h_j|^{n_i})^{\frac{1}{n_i} - 2}} \right] \quad (11)$$

193

194 The fitting parameters are h_{ae} , h_j , θ_s , α_i , n_i , and w_i .

195

196 The Unsaturated Hydraulic Conductivity Curve

197 Kosugi (1999) proposed the following conductivity model (see also Ippisch et al.,
 198 2006):

199

$$200 \quad K(h) = K(h(S_e)) = \begin{cases} K_s S_e^\tau \left(\frac{\int_{-\infty}^{h(S_e)} |h|^{-\kappa} \frac{dS}{dh} dh}{\int_{-\infty}^{h_{ae}} |h|^{-\kappa} \frac{dS}{dh} dh} \right)^\gamma, & h < h_{ae} \\ K_s, & h \geq h_{ae} \end{cases} \quad (12)$$

201

202 with γ , κ , and τ denoting shape parameters, S_e denoting the degree of saturation, and S

203 representing a variable running over all values of the degree of saturation from zero to its



204 actual value S_e . Mualem's (1976) conductivity model is a special case of Eq. (11), with $\gamma = 2$,
 205 $\kappa = 1$, and $\tau = 0.5$.

206 The integrals in Eq. (12) that arise when Eq. (5) is used to find dS/dh can be
 207 evaluated analytically if $\kappa = 1$. The resulting conductivity function is

208

$$K(h) = \begin{cases} 0, & h \leq h_d \\ K_s \left(\beta \ln \left| \frac{h_d}{h} \right| \right)^\tau \left[\frac{\frac{\beta}{|h_d|} - \frac{\beta}{|h|}}{\frac{\beta}{|h_d|} - \frac{\beta}{|h_j|} + \frac{|\alpha h_{ae}|^n}{|h_{ae}|} - (1 + |\alpha h_{ae}|^n)^{1-\frac{1}{n}} F(h_j)} \right]^\gamma, & h_d < h \leq h_j \\ K_s \left(\frac{1 + |\alpha h|^n}{1 + |\alpha h_{ae}|^n} \right)^{\frac{\tau}{n} - \tau} \left\{ \frac{\frac{\beta}{|h_d|} - \frac{\beta}{|h_j|} + (1 + |\alpha h_{ae}|^n)^{1-\frac{1}{n}} [F(h) - F(h_j)]}{\frac{\beta}{|h_d|} - \frac{\beta}{|h_j|} + \frac{|\alpha h_{ae}|^n}{|h_{ae}|} - (1 + |\alpha h_{ae}|^n)^{1-\frac{1}{n}} F(h_j)} \right\}^\gamma, & h_j < h \leq h_{ae} \\ K_s, & h > h_{ae} \end{cases} \quad (13a)$$

210

211

212 where

213

$$214 \quad F(h) = \frac{|\alpha h|^n (1 + |\alpha h|^n)^{\frac{1}{n} - 1}}{|h|} \quad (13b)$$

215

216 Other conductivity models are available and can be selected if desired, for instance using
 217 the framework presented by Weber et al. (2019).

218



219 The conductivity function associated with the multimodal soil water retention
220 function cannot be expressed in analytical form. For that case, the degree of saturation for
221 any h can be found with Eq. (9) and the corresponding hydraulic conductivity determined
222 with Eq. (12) or another conductivity model.

223

224 **Materials and Methods**

225 We selected 21 soils from the UNSODA database that had sufficient retention data
226 and together covered the textures represented in UNSODA. We then fitted Eq. (4) to these
227 soils using a Shuffled Complex Evolution algorithm (see Madi et al. (2018) and Appendix C
228 for details of the algorithm and the fitting procedure). We slightly modified the fitting code
229 used by Madi et al. (2018) to generate output that can more readily be converted to the
230 MATER.IN input file for Hydrus-1D, the numerical model used in this study. We therefore
231 refitted BCO, FSB, RNA, VGN, and VGA as well.

232 One-dimensional simulations for all combinations of three soils and three climates
233 were carried out to examine how the choice of parameterization affected fluxes at the soil
234 surface and in the subsoil. The model Hydrus-1D version 4.16.0110 (Šimůnek et al., 2013,
235 PC-progress website) was used for this purpose. The selected soils were a loamy sand
236 (UNSODA identifier: 2104), a silty loam (3261), and a clay (1181). Weather records were
237 generated from climate parameters based on weather data from Colombo (Sri Lanka,
238 monsoon climate), Tamale (Ghana, semi-arid climate), and Ukkel (Belgium, temperate
239 climate). Table 1 gives the most relevant statistics of the weather records. In order to
240 highlight the effects of the air-entry value and the logarithmic dry end of the SWRC, we
241 used the sigmoidal VGN, VGA, and RIA parameterizations in the simulations. The



242 Supplement details the generation of the weather records, the set-up of the simulations,
 243 and the simulation results.

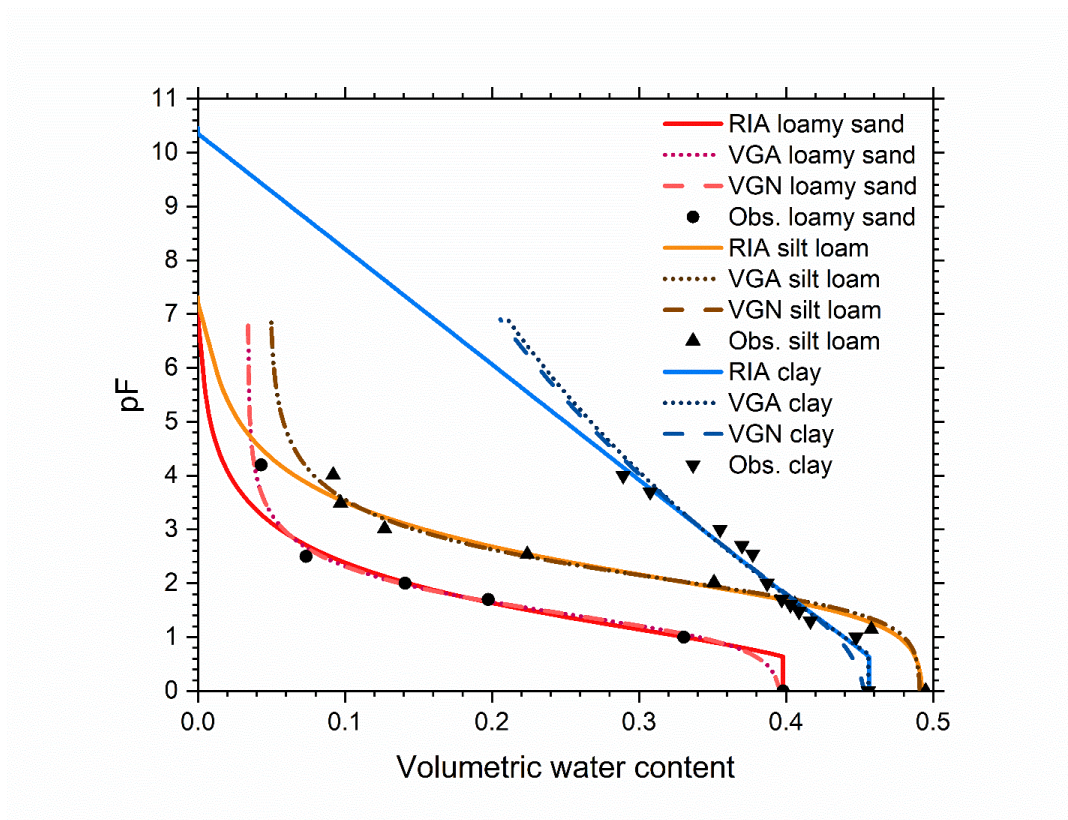
244

245 Table 1. Average cumulative monthly and annual rainfall and potential evapotranspiration
 246 of the three test climates calculated from 1000-year generated weather records.

	Monsoon		Semi-arid		Temperate	
Month	Rain (mm)	ET_{pot} (mm)	Rain (mm)	ET_{pot} (mm)	Rain (mm)	ET_{pot} (mm)
Jan	94.0	160.5	14.3	159.4	69.0	12.7
Feb	83.3	156.0	12.9	158.1	55.6	18.8
Mar	217.6	166.8	13.7	185.0	59.4	34.4
Apr	233.8	160.0	34.1	177.7	57.1	51.1
May	237.4	164.0	117.6	166.6	24.5	73.9
Jun	138.5	164.7	140.9	151.4	24.4	83.3
Jul	139.4	172.4	141.8	153.2	23.2	86.3
Aug	137.1	174.4	213.5	145.0	20.9	73.7
Sep	133.0	164.7	214.7	136.9	23.8	50.6
Oct	325.8	142.3	76.6	154.1	38.0	30.8
Nov	328.8	128.1	12.9	149.3	49.1	16.1
Dec	112.0	150.9	12.9	151.9	51.2	11.2
Annual sum	2180.5	1904.8	1005.8	1888.7	496.1	542.9

247

248



249

250 Figure 1. Soil water retention data of the soils used in the numerical simulations, and the
251 retention curves fitted to these data for three parameterizations with a sigmoid midsection
252 of the curve: the original model by van Genuchten (1980) (VGN), the modification thereof
253 by Ippisch et al. (2006) (VGA), and the further modification introduced in this paper (RIA).

254

255 The different parameterizations of the SWRCs (Table 2, Fig. 1) were used to generate
256 tables of the soil water retention and conductivity curves that were provided as input to
257 Hydrus-1D. Mualem's (1976) model for the unsaturated soil hydraulic conductivity was
258 used to generate relative hydraulic conductivities (scaled by K_s). These were converted to



259 unscaled conductivities (Fig. 2) by multiplying with the K_s -value according to the UNSODA
 260 database, and then included in the tables.

261

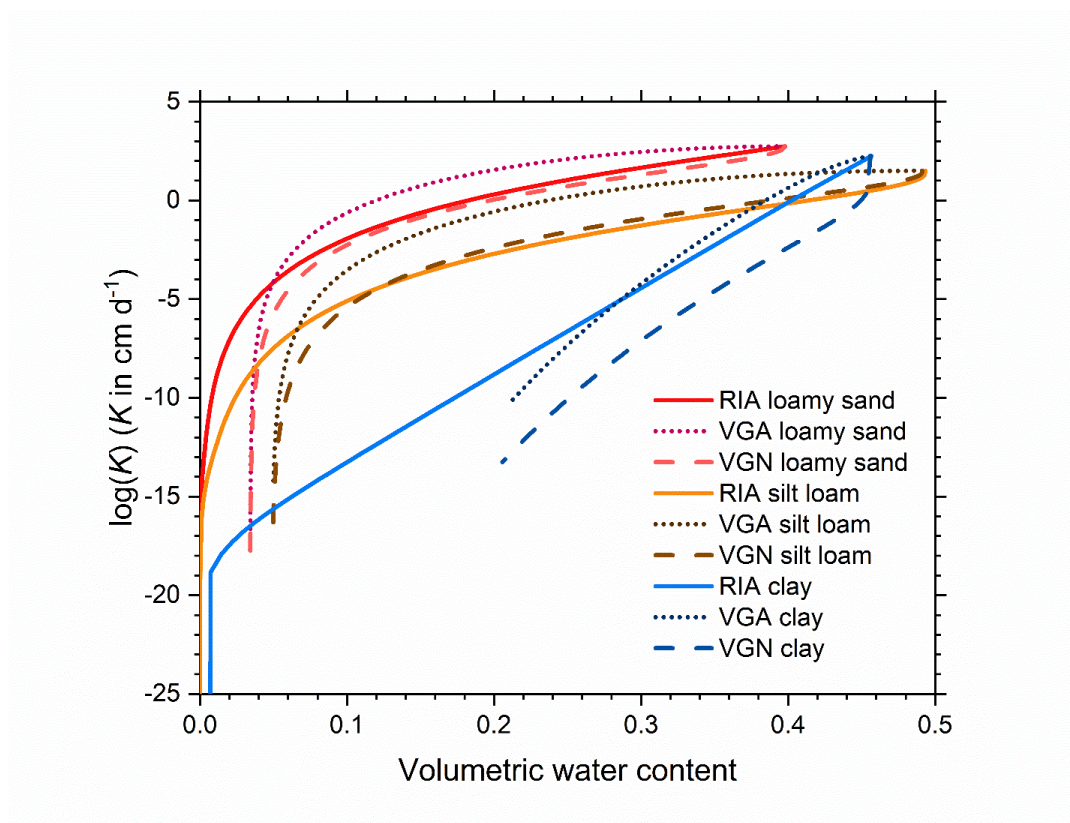
262 Table 2. Parameters of the fit to data of the three parameterizations of the soil water
 263 retention curve used for the numerical simulations.

Soil	Parameter	θ_r	θ_s	α (cm ⁻¹)	n	h_{ae} (cm)	h_j (cm)
Clay	RIA	–	0.45666	0.70200	1.0543	–4.2523	–205.65
	VGA	2.12×10^{-6}	0.45603	1.7047	1.0543	–4.8324	–
	VGN	3.34×10^{-6}	0.45616	0.14265	1.0571	–	–
Silt loam	RIA	–	0.49346	0.023340	1.3691	-2.361×10^{-3}	-1.0739×10^6
	VGA	0.048871	0.49122	0.018365	1.5158	-2.081×10^{-3}	–
	VGN	0.048816	0.49134	0.018425	1.5149	–	–
Loamy sand	RIA	–	0.39801	0.17096	1.4106	–4.3581	-7.7464×10^5
	VGA	0.034209	0.39771	0.069661	1.6395	–0.016234	–
	VGN	0.034133	0.39772	0.069707	1.6389	–	–

264

265

266 For the clay soil, the recorded value (178 cm d⁻¹) was such that it can be assumed
 267 that macropore flow contributed to its value. We therefore also ran simulations with a K_s –
 268 value of 1.25 cm d⁻¹. This value was adopted from soil 1182 from the UNSODA database,
 269 from the same location.



270

271 Figure 2. The unsaturated soil hydraulic conductivity curves derived from the fitted
272 retention curves depicted in Fig.1 and scaled by soil-specific values of the hydraulic
273 conductivity at saturation.

274

275 Results and Discussion

276 Fitted Curves for 21 Soils

277 The fitted parameter values (Tables C1–C4) and the associated curves (Figs. C2–C5)
278 are presented in Appendix C. The extra parameter of RIA compared to RNA and FSB gives it
279 a clear advantage in fitting the full water content range. The sigmoid shape of RIA provides
280 a better fit near the air entry value while still providing good fits of the drier data points



281 (e.g. 1121 in Fig. C3, all soils in Figs. C4–C5). In the wet range, the sigmoid curves (VGN,
282 VGA, RIA) outperform the power-law curves (BCO, FSB, RNA). In almost all cases, VGA and
283 RIA look very similar. Figure C3 shows multimodality in the data for five out of six soils that
284 cannot be reproduced by any of the parameterizations.

285 Many of the UNSODA soils have one retention data point at saturation and the next
286 at $h = -10$ cm. The air-entry value of many sandy soils is within that range. The fitting
287 routine struggles to fit h_{ae} to these data for VGA and RIA because the sigmoid shape of the
288 van Genuchten parameterization has such flexibility that it can fit the intermediate range
289 well for a range of h_{ae} -values. We therefore recommend making $\theta(h)$ -measurements at one
290 or two matric potentials between zero and -10 cm for coarse-textured soils.

291 Eleven of the 21 soils have residual water contents for VGN and/or VGA that exceed
292 0.05 (up to 0.263), mostly in loamy sands, loams, and clays (Tables C1–C4). All of these and
293 several others have dry-end data points with water contents that appear too high (Figs. C2–
294 C5). These water contents may have been overestimated due to the lack of equilibrium
295 reported by Bitelli and Flury (2009). The parameterizations without asymptote (FSB, RNA,
296 RIA) generally have more plausible fits based on visual inspection of the graphs, but
297 because they do not follow the upward tail of the data in the dry end, their RMSE values for
298 the cases with suspicious dry-end data points are larger than those of VGN and VGA (Tables
299 C5–C8).

300 Especially for fine-textured soils, the lack of data points for air dryness or oven
301 dryness (Figs. C3–C5) causes the fitting procedure to treat θ_r for the asymptotic dry
302 branches and h_d for the logarithmic dry branches as pure fitting parameters for the drier
303 range of the data. This range exceeds pF4.2 in only one case and in several cases likely



304 suffers from lack of equilibrium (Bitelli and Flury, 2009). This leads to unrealistically high
305 values for both in many cases. If applications are envisioned for which low water contents
306 are expected, it would be better to have some data in the dry range and ensure equilibrium
307 has been achieved before fitting RIA. One could even add a virtual data point with zero
308 water content at $pF_{6.8}$, since this is the value that is approximated by many laboratory
309 ovens (Schneider and Goss, 2012). We did not do so here to be able to observe how the
310 various parameterizations perform on frequently reported data ranges.

311 Figure C5 showcases a peculiarity of FSB. The original version by Fayer and
312 Simmons (1995) allowed capillary bound water to be present even if the adsorbed water
313 content was down to zero. We adopted the correction by Madi et al. (2018, Eq. (S12a)) that
314 forced the amount of water bound by capillary forces to zero if the adsorbed water content
315 goes reaches zero at $pF_{6.8}$. In the case of clay soils, the parameter values are such that a
316 substantial amount of capillary bound water is still present at $pF_{6.8}$, leading to a sudden
317 cut-off at $pF_{6.8}$.

318 The hydraulic conductivity curves based on water retention curve parameters and
319 K_s very poorly match the data in most cases (Fig. C6–C9). We note here that all but one (soil
320 4450, Fig. C8) sets of unsaturated conductivity data were obtained in the field, while all
321 retention data were laboratory data on drying samples. The reported K_s -values were
322 probably measured in a separate experiment (possibly in the laboratory), in which case a
323 mismatch between K_s and the unsaturated conductivity data is to be expected. The poor
324 match with field data notwithstanding, the graphs can be used to study the effect of the
325 parameterization on the shape of the $K(\theta)$ curve. The comparison between measured and
326 modeled shapes of the conductivity curves is inconclusive.



327 In many soils, regardless of texture, RIA's $K(\theta)$ curve drops of much slower in the
328 dry range than those of VGN and VGA (Fig. C6–C9), a consequence of the ability of the
329 underlying SWRC to reach zero water content at a finite matric potential. Near saturation,
330 RIA's $K(\theta)$ curve often drops off sharply before leveling off, in stark contrast to that of VGA,
331 which remains high in the wet range. Given the similarity in the $\theta(h)$ curves of VGA and RIA,
332 the difference in their $K(\theta)$ curves is remarkable. RIA's $K(\theta)$ curve is the only one that can
333 drop off sharply near saturation, level out somewhat in the mid-range and drop ever more
334 sharply in the dry range. It is below many of the other curves in the wet and mid-range, and
335 above most of them in the dry range.

336 Peters et al. (2011) developed parameter constraints to ensure physically plausible
337 shapes of the SRWC and the conductivity curve. For FSB, the criterion for non-monotonicity
338 of the conductivity curve is not met for soil 4450 (Fig. C8), resulting in K increasing with
339 decreasing water content near saturation.

340

341 **Model Simulations**

342 In total, 21 out of 36 combinations of soil–climate–parameterization ran to
343 completion. Runoff did not occur in any of the successful model runs. Convergence was not
344 achieved for any of the runs for the clay soil with the reduced K_s -value. For the clay soil
345 with the high K_s -value, only RIA ran to completion. The discontinuity of the first derivative
346 at the air-entry value did not cause numerical problems. In fact, the replacement of any
347 parametric expression by a look-up table creates a discontinuity of the first derivative at
348 every point of the look-up table.



349 Table 3 lists the main mean annual fluxes calculated from the simulation study. The
350 median flux was produced by RIA for all combinations of soil and climate. The mean annual
351 actual transpiration between the three parameterizations differed by more than 10% from
352 the median only for the loamy sand under a temperate climate. Actual evaporation never
353 deviated more than 10%. The amount of water leaving the soil profile differed substantially
354 between parameterizations for the semi-arid and temperate, especially for VGA (20–46%
355 deviation from the median).

356 The daily data revealed significant differences on smaller time scales that will be
357 relevant if reactive solute transport is of interest (see the Supplement). The fluxes
358 generated by the VGA parameterizations responded more quickly and strongly to the
359 rainfall signal, with VGN and RIA giving a more delayed and smooth response. The SWRCs
360 (Fig. 1) offer no explanation for this, but Fig. 2 shows that VGA's hydraulic conductivity in
361 the wet and intermediate water content range for all three soils is considerably higher than
362 that of VGN and RIA, except for clay, where it is only moderately higher than RIA's and
363 drops below RIA at a water content of 0.28. Thus, small differences between SWRCs can
364 have a significant influence on soil water flow simulations through their effect on the soil
365 hydraulic conductivity curve, an effect that the reviews by Leij et al. (1997) and Assouline
366 and Or (2013) took into consideration to some degree, but was not considered in several
367 other studies that compared different parameterizations (e.g., Rossi and Nimmo, 1994;
368 Assouline et al., 1998; Cornelis et al., 2005; Khlosi et al., 2008).



369 Table 3. Average of the annual sums of the actual transpiration and evaporation, and of the
 370 outflow across the lower boundary of the simulated soil profiles. The averages were
 371 calculated for the final six years of the simulation periods. The values in parentheses are
 372 scaled with respect to the corresponding value for RIA.

Climate	Soil	Actual transpiration (mm)			Actual evaporation (mm)			Downward flux at 2 m depth (mm)		
		RIA	VGA	VGN	RIA	VGA	VGN	RIA	VGA	VGN
Monsoon	clay	962	–	–	543	–	–	876	–	–
	silt	1114	1044	1112	541	554	534	711	763	718
	loam		(0.94)	(1.06)		(1.02)	(0.96)		(1.07)	(0.94)
	loamy	1037	976	1024	480	440	461	863	959	896
	sand		(0.94)	(1.05)		(0.92)	(1.05)		(1.11)	(0.93)
Semi-arid	clay	585	–	–	319	–	–	294	–	–
	silt	657	588	653	297	303	287	164	226	177
	loam		(0.90)	(1.11)		(1.02)	(0.95)		(1.37)	(0.78)
	loamy	585	540	572	248	222	234	290	363	319
	sand		(0.92)	(1.06)		(0.89)	(1.06)		(1.25)	(0.88)
Temperate	clay	202	–	–	163	–	–	167	–	–
	silt	281	236	271	150	151	143	99	144	114
	loam		(0.84)	(1.15)		(1.01)	(0.95)		(1.46)	(0.79)
	loamy	224	205	214	130	118	124	174	208	190
	sand		(0.92)	(1.04)		(0.91)	(1.05)		(1.20)	(0.91)



374 **Summary and Conclusions**

375 The improvements incorporated in the RIA parameterization for the first time
376 remove problems of the popular VGN model in both the wet and the dry range while
377 retaining the desirable sigmoid shape in the mid-range. This shape allows its multimodal
378 version to represent SWRCs with multiple humps. RIA offers a wider range of shapes for the
379 conductivity curve than any other parameterization that does not lead to the unphysical
380 behavior near saturation that was revealed by Durner (1994) and Ippisch et al. (2006) for
381 VGN and by Madi et al. (2018) for 14 other parameterizations. RIA also proved to be more
382 robust during numerical simulations than VGN itself as well as its modification VGA, which
383 still has a non-physical asymptote at a non-zero residual water content. The deep drainage
384 generated by RIA was more spread out and smaller than the spiked response to rainfall
385 produced by VGA, probably because RIA was better able to produce a conductivity curve
386 with a substantial drop during the early stages of drying. We therefore hope that RIA or its
387 multimodal version will be adopted for use in numerical simulations of soil water flow. The
388 catalogue of parameters for 21 soils in Appendix C may be of help for such simulations.



389 **Appendix A. List of Abbreviations**

390

391 BC: Brooks and Corey (1964)

392 BCO: parameterization of the SWRC according to the original Brooks and Corey (1964)

393 equation

394 FSB: parameterization of the SWRC according to the BC-based version of Fayers and

395 Simmons (1995)

396 RIA: parameterization of the SWRC that combines RNA and VGA

397 RNA: parameterization of the SWRC according to the junction model of Rossi and Nimmo

398 (1994)

399 SWRC: soil water retention curve

400 RMSE: root mean square error

401 UNSODA: unsaturated soil hydraulic properties database

402 VGA: parameterization of the SWRC according to Ippisch et al. (2016)

403 VGN: parameterization of the SWRC according to the original van Genuchten (1980)

404 equation

405



406 **Appendix B. Assessing the near-saturation behavior of recently**
407 **developed soil water retention and hydraulic conductivity curves**

408

409 Madi et al. (2018) developed a criterion that needs to be met to avoid unphysical
410 behavior of the unsaturated soil hydraulic conductivity curve near saturation:

411

$$412 \lim_{h \uparrow 0} \left(|h|^{-\kappa} \frac{d\theta}{dh} \right) = 0 \quad (B1)$$

413

414 where κ is a fitting parameter (> 0) that appears in a several parameterizations of the
415 unsaturated soil hydraulic conductivity curve based on the capillary bundle
416 conceptualization.

417 Fredlund and Xing (1994) introduced the following parameterization for the soil
418 water retention curve:

419

$$420 \theta(h) = \left[\frac{-\ln\left(1 + \frac{|h|}{|h_r|}\right)}{\ln\left(1 + \frac{b}{|h_r|}\right)} + 1 \right] \frac{\theta_s}{\left\{ \ln\left[e + \left(\frac{|h|}{a}\right)^n \right] \right\}^m} \quad (B2)$$

421

422 where the subscript 'r' denotes the value when the residual water content is reached, and a
423 [L], n , and m are fitting parameters. The first term on the right-hand side is a correction
424 term that forces the water content to zero for $h = -b$, with b equal to 10^7 cm (Fredlund and
425 Xing, 1994) or 6.3×10^6 cm (Wang et al., 2016). Wang et al. (2016) also modified the
426 correction factor to give



427

$$428 \quad \theta(h) = \left[\frac{-\ln\left(1 + \frac{c|h|}{|h_r|}\right)}{\ln\left(1 + \frac{bc}{|h_r|}\right)} + 1 \right] \frac{\theta_s}{\left\{ \ln\left[e + \left(\frac{|h|}{a}\right)^n \right] \right\}^m} \quad (B3)$$

429

430 where c has to be small and positive. The derivative of Eq. (B3) is

431

$$432 \quad \frac{d\theta}{dh} = \frac{-\theta_s}{\left\{ \ln\left[e + \left(\frac{|h|}{a}\right)^n \right] \right\}^m} \left\{ \frac{mn|h|^{n-1} \left[\ln\left(1 + \frac{bc}{|h_r|}\right) \ln\left(1 + \frac{c|h|}{|h_r|}\right) - 1 \right]}{a^n \left[e + \left(\frac{|h|}{a}\right)^n \right] \ln\left[e + \left(\frac{|h|}{a}\right)^n \right]} - \frac{\ln\left(1 + \frac{bc}{|h_r|}\right)}{\frac{|h_r|}{c} + |h|} \right\} \quad (B4)$$

433

434 The derivative of Eq. (B2) follows by setting c equal to 1 in Eq. (B4). Combining Eq.(B4)

435 with Eq. (B1) results in the following requirement:

436

$$437 \quad \lim_{h \uparrow 0} \left[\frac{\theta_s mn}{ea^n} |h|^{-\kappa+n-1} + \frac{c\theta_s \ln\left(1 + \frac{bc}{|h_r|}\right)}{|h_r|} |h|^{-\kappa} \right] = 0 \quad (B5)$$

438

439 The limit goes to zero if and only if the exponents of $|h|$ in both terms are positive. Hence, $\kappa <$

440 $n-1$ and $\kappa < 0$. The first requirement may be met for some soils, but the second violates

441 the physical constraint that κ cannot be negative (Madi et al., 2018). Therefore, neither

442 Fredlund and Xing's (1994) nor Wang et al.'s (2016) parameterization lead to unsaturated

443 hydraulic conductivity curves that exhibit physically realistic behavior near saturation.

444 Wang et al. (2018) added a modification in the dry end of Wang et al. (2016), and

445 Rudiyanto et al. (2020) in turn used Wang et al.'s (2018) curves. Because the problem near

446 saturation was not resolved, these two hydraulic conductivity models suffer from the same

447 problem near saturation.



448 **Appendix C. Fitted Parameters and Root Mean Square Error for Six**
449 **Parameterizations of the Soil Water Retention Curve applied to Data**
450 **of 21 Soils**

451

452 The UNSODA soils selected for parameter fitting are grouped in Tables C1–C4
453 according to their texture classification according to Twarakavi et al. (2010). Sand is a
454 major constituent of the mineral soil in Tables C1 and C2, silt in Table C3, and clay in Table
455 C4. Figure C1 shows the textural composition of the soils. For each soil, the parameter
456 values for six parameterizations are given, resulting in a total of 126 SWRC
457 parameterizations. The Root Mean Square Errors (RMSE) for all fits are listed in Tables C5–
458 C8. The saturated hydraulic conductivities of the soils as given by the UNSODA database are
459 given in Table C9.

460 Madi et al. (2018) provide an analysis of the underlying functions of all
461 parameterizations tested here, except RIA. The meaning of all variables except λ is
462 explained in the main text. Variable λ is the power to which the factor (h / h_{ae}) is raised in
463 the power-law segments of the SWRCs of BCO, FSB, and RNA. In RNA, λ is expressed as a
464 function of the fitting parameters, and therefore does not appear in the tables.

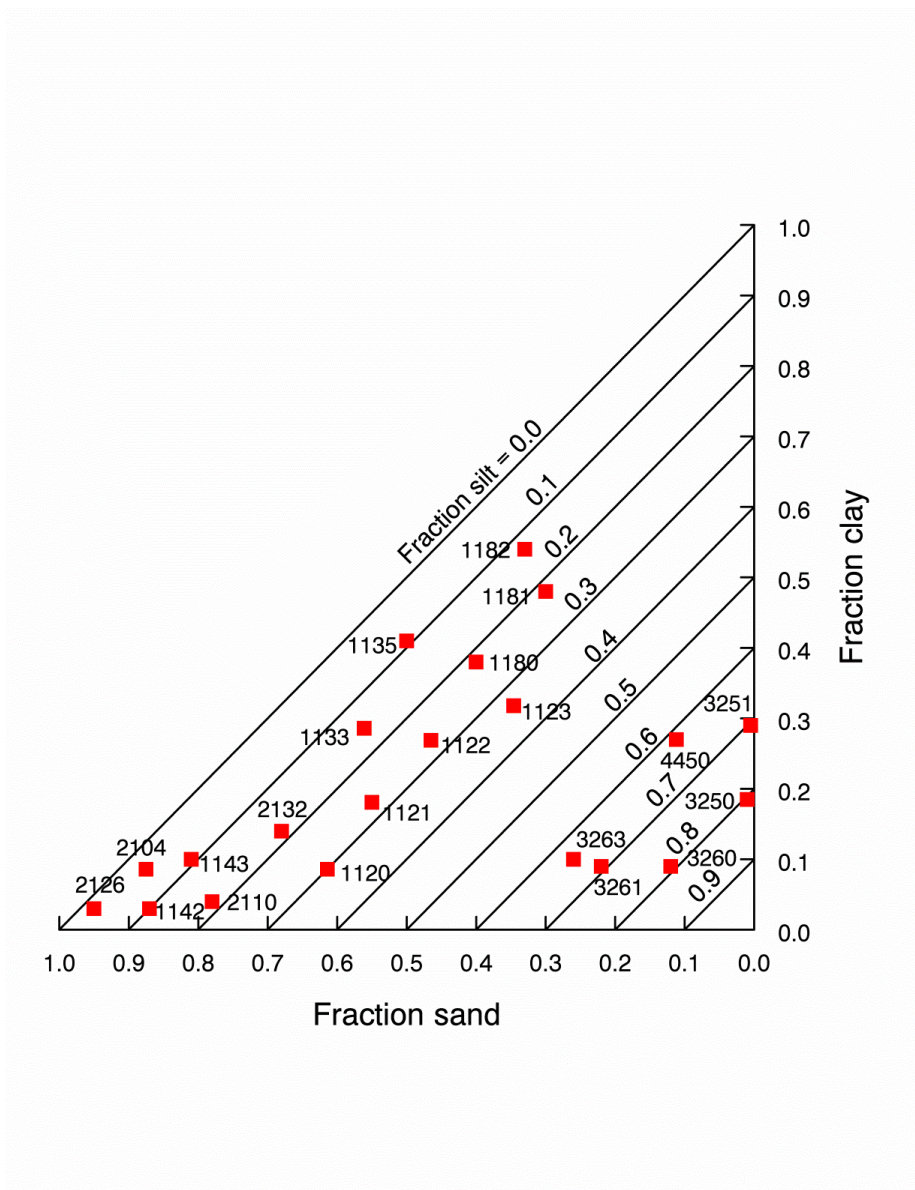
465 The SWRCs defined by these parameterizations together with the data points on
466 which they are based are given in Figs. C2–C5. The hydraulic conductivity curves according
467 to Mualem (1976) that can be derived from the parameterizations (Eqs. (13a) and (13b)
468 for RIA, equations for the other parameterizations in Madi et al. (2018)) are plotted in Figs.
469 C6–C9. We plotted K as a function of θ because this relationship is less hysteretic than $K(h)$



470 (Koorevaar et al., 1983, p. 141) and some of the UNSODA data only provided $K(\theta)$ data
471 points. The conductivity data available in UNSODA are also plotted, but these were not used
472 to fit the curves to.

473 The comparison of theoretical conductivity curves based on water retention data
474 with the measured values showed a sometimes substantial deviation between hydraulic
475 conductivities measured at saturation and the K_s value used to create the theoretical curves.
476 The latter value was obtained by a query that made the database return a K_s value for the
477 soils that met all other criteria also included in the query. The conductivity data displayed
478 in the plots were separately obtained by requesting the database to return a report of
479 tabular data for each of the 21 soils. The reasons for the discrepancies between the queried
480 value and the tabulated may reflect separate experiments for measuring unsaturated and
481 saturated values of K .

482 There also are obvious differences between the water content at saturation between
483 the retention and the conductivity data. In all but one case (soil 4450, Fig. C8) the hydraulic
484 conductivity observations were made in the field whereas the retention data used were all
485 obtained from drying experiments in the laboratory. Scale differences, effects of air
486 enclosure and hysteresis in the field, and differences between different measurement
487 techniques explain these differences. We can therefore only compare the shape of the
488 theoretical curves with those of the data clouds.



489

490 Figure C1. The positions in the texture triangle of the selected soils, indicated by their

491 UNSODA identifier. (Taken from Madi et al., 2018)

492



493 The RMSE was calculated from the weighted sum of squares of the differences
494 between calculated and observed water contents and pressure heads. The weights equaled
495 the estimated scaled standard deviations of the individual water retention observations
496 (pairs of matric potential and water content values). The standard deviations of the water
497 content observations were scaled to have an average value of 0.2. The scale factor needed to
498 arrive at this value was then applied to the standard deviations of the matric potential
499 values as well. This ensured that the weighting of water contents and matric potential
500 values was consistent with the original standard deviations of both. The scaling greatly
501 improved the efficiency of the parameter fitting procedure. The squared difference between
502 a single observed water content and its fitted value during a given iteration of the
503 parameter fitting algorithm, taking into account observation errors in both the water
504 content and the matric potential, is

505

$$506 \left(\frac{\theta_{\text{fit}} - \theta_{\text{obs}}}{\sigma_{\theta, \text{scaled}} + \sigma_{h, \text{scaled}} \left| \frac{d\theta}{dh} \right|_{h_{\text{obs}}}} \right)^2 \quad (C1)$$

507

508 where $\sigma_{a, \text{scaled}}$ is the scaled standard deviation of the variable a , subscript 'fit' signifies a
509 fitted value of the subscripted variable, and subscript 'obs' a measured value (Madi et al.,
510 2018). The slope of the SWRC in the denominator is estimated from the fitted
511 parameterization using the parameter values that have been fitted in the iteration that is
512 currently being tested.

513 The scale factors applied to the observation error standard deviations varied
514 between soils but not between parameterizations fitted to the same soil. RMSE values of



515 different parameterizations valid for a particular soil can therefore be readily compared.
516 Comparisons between soils only give a rough indication. When the water contents at which
517 measurements were taken differ strongly from one soil to another, the comparison of their
518 RMSE values is less reliable.

519 In case the measured water contents were obtained at hydrostatic equilibrium, the
520 fitted water content calculated directly from the matric potential could not be compared to
521 the observed water content. To approximate the soil sample on which the observation was
522 made, a hypothetical soil slab of the same height as the sample was divided into 20
523 horizontal layers. If UNSODA did not specify the sample height, it was assumed to be 5.0 cm.
524 The matric potential in the center of each layer was determined from the given matric
525 potential, which was assumed to apply to the center of the sample. The water content of the
526 soil slab was then calculated as the average water content of its 20 layers. This water
527 content was used to calculate the difference between the observed and the fitted water
528 content. Figure C10 shows a comparison of retention points calculated for three soils based
529 directly on the RIA parameterization and based on the same parameterization applied to a
530 hypothetical sample of 5.0 cm height at hydrostatic equilibrium with the nominal matric
531 potential valid at the sample center. Deviations are small, even for the loamy sand.



532 Table C1. The fitting parameters and their values for six parameterizations for sandy soils in the A1
 533 and A2 classifications of Twarakavi et al. (2010) from the UNSODA database (National Agricultural
 534 Library, 2017; Nemes et al., 2001). The three-character parameterization label is explained in the
 535 main text.

		Soil (UNSODA identifier and classification according to Twarakavi et al. (2010))			
			2126 A1	1142 A2	2104 A2
Paramete- rization	Parame- ter	Unit			
BCO	θ_r	-	1.63E-2	4.92E-5	2.27E-2
	θ_s	-	0.377	0.250	0.398
	h_{ae}	cm	-6.78	-7.00	-6.79
	λ	-	0.846	0.210	0.434
FSB	θ_s	-	0.377	0.250	0.398
	θ_a	-	2.58E-2	6.26E-5	5.46E-2
	h_{ae}	cm	-6.76	-7.00	-6.73
	λ	-	0.861	0.211	0.468
RNA	θ_s	-	0.378	0.250	0.398
	h_{ae}	cm	-6.37	-7.00	-6.17
	h_j	cm	-8.87E4	-9.24E4	-6.45E4
	h_d	cm	-3.60E5	-1.07E7	-9.73E5
VGN	θ_r	-	3.39E-2	9.42E-2	3.41E-2
	θ_s	-	0.376	0.242	0.398
	α	cm ⁻¹	6.85E-2	1.99E-2	6.97E-2
	n	-	2.73	2.93	1.64
VGA	θ_r	-	3.39E-2	9.64E-2	3.42E-2
	θ_s	-	0.376	0.242	0.398
	α	cm ⁻¹	6.84E-2	1.98E-2	6.97E-2
	n	-	2.73	3.05	1.64
	h_{ae}	cm	-0.101	-0.240	-1.62E-2
RIA	θ_s	-	0.378	0.245	0.398
	α	cm ⁻¹	0.239	2.68E-2	0.171
	n	-	1.77	1.47	1.41
	h_{ae}	cm	-5.82	-7.00	-4.36
	h_j	cm	-1.76E6	-4.35E5	-7.75E5



537 Table C2. The fitting parameters and their values for six parameterizations for sandy soils in the A3
 538 and A4 classifications of Twarakavi et al. (2010) from the UNSODA database (National Agricultural
 539 Library, 2017; Nemes et al., 2001). The three-character parameterization label is explained in the
 540 main text.

			Soil (UNSODA identifier and classification according to Twarakavi et al. (2010))					
			1120 A3	1143 A3	2110 A3	2132 A3	1121 A4	1133 A4
Para- meteri- zation	Para- me- ter	Unit						
BCO	θ_r	-	6.15E-6	2.71E-5	0.103	4.10E-5	2.64E-5	5.37E-5
	θ_s	-	0.311	0.279	0.348	0.303	0.350	0.330
	h_{ae}	cm	-10.0	-7.00	-25.5	-8.00	-10.0	-206
	λ	-	0.204	0.169	0.537	0.107	0.118	0.103
FSB	θ_s	-	0.311	0.279	0.348	0.308	0.346	0.330
	θ_a	-	4.33E-5	4.26E-4	0.213	0.298	0.324	0.310
	h_{ae}	cm	-10.0	-7.00	-25.7	-3.24	-10.0	-206
	λ	-	0.204	0.169	0.763	0.422	0.377	0.213
RNA	θ_s	-	0.311	0.279	0.351	0.303	0.352	0.330
	h_{ae}	cm	-10.0	-7.00	-20.3	-8.00	-10.0	-220
	h_j	cm	-7.32E4	-8.46E4	-6.44E4	-7.18E4	-8.73E4	-6.77E4
	h_d	cm	-9.95E6	-3.18E7	-2.35E6	-8.06E8	-3.88E8	-8.68E8
VGN	θ_r	-	7.22E-2	9.24E-2	0.126	1.25E-4	1.70E-5	0.202
	θ_s	-	0.305	0.278	0.360	0.305	0.339	0.324
	α	cm ⁻¹	1.72E-2	4.66E-2	2.63E-2	5.73E-2	7.21E-3	7.34E-4
	n	-	1.69	1.49	1.84	1.14	1.26	3.02
VGA	θ_r	-	7.37E-2	0.112	0.106	6.61E-4	1.47E-5	0.202
	θ_s	-	0.303	0.276	0.348	0.306	0.339	0.324
	α	cm ⁻¹	1.72E-2	4.41E-2	0.230	6.06E-2	7.16E-3	7.35E-4
	n	-	1.71	1.66	1.56	1.14	1.27	3.00
	h_{ae}	cm	-9.37	-6.81	-25.3	-8.47E-4	-2.73E-2	-12.5
RIA	θ_s	-	0.308	0.280	0.360	0.306	0.339	0.328
	α	cm ⁻¹	3.01E-2	6.39E-2	4.18E-2	6.08E-2	7.13E-2	1.30E-3
	n	-	1.29	1.23	1.33	1.14	1.27	1.20
	h_{ae}	cm	-1.24E-3	-4.96E-4	-7.85	-7.44E-4	-9.11E-4	-220
	h_j	cm	-1.71E6	-7.01E6	-9.37E6	-7.81E6	-2.04E6	-5.07E6



541 Table C3. The fitting parameters and their values for six parameterizations for silty soils from the
 542 UNSODA database (National Agricultural Library, 2017; Nemes et al., 2001). The three-character
 543 parameterization label is explained in the main text.

			Soil (UNSODA identifier and classification according to Twarakavi et al. (2010))					
			3260 B2	3261 B2	3263 B2	3250 B4	3251 B4	4450 B4
Para- meteri- zation	Para- me- ter	Unit						
BCO	θ_t	-	2.85E-6	3.42E-6	3.85E-7	3.85E-6	1.94E-6	2.77E-5
	θ_s	-	0.470	0.499	0.460	0.540	0.500	0.380
	h_{ae}	cm	-28.6	-13.5	-28.8	-30.5	-18.2	-4.80
	λ	-	0.281	0.256	0.255	0.183	9.56E-2	9.51E-2
FSB	θ_s	-	0.470	0.499	0.460	0.540	0.500	0.380
	θ_a	-	1.33E-5	6.49E-5	1.33E-5	0.173	0.431	0.320
	h_{ae}	cm	-28.6	-13.5	-28.8	-30.0	-10.9	-0.882
	λ	-	0.281	0.256	0.255	0.241	0.197	0.196
RNA	θ_s	-	0.470	0.499	0.460	0.540	0.500	0.380
	h_{ae}	cm	-28.6	-13.5	-28.8	-30.5	-18.2	-4.81
	h_j	cm	-9.23E4	-9.43E4	-7.48E4	-7.87E4	-1.97E4	-2.63E4
	h_d	cm	-3.23E6	-4.96E6	-3.75E6	-6.86E6	-7.66E8	-9.69E8
VGN	θ_t	-	5.25E-2	4.88E-2	4.48E-2	3.10E-2	1.60E-5	1.16E-6
	θ_s	-	0.472	0.491	0.461	0.540	0.501	0.379
	α	cm ⁻¹	1.62E-2	1.84E-2	1.53E-2	1.21E-2	2.62E-2	0.164
	n	-	1.47	1.51	1.41	1.28	1.11	1.10
VGA	θ_t	-	5.27E-2	4.89E-2	6.12E-2	3.80E-4	7.21E-5	2.80E-5
	θ_s	-	0.472	0.491	0.457	0.540	0.500	0.379
	α	cm ⁻¹	1.62E-2	1.84E-2	1.49E-2	1.33E-2	3.66E-2	1.26
	n	-	1.47	1.52	1.46	1.25	1.11	1.10
	h_{ae}	cm	-3.26E-3	-2.08E-3	-15.1	-4.87	-7.31	-4.54
RIA	θ_s	-	0.474	0.493	0.463	0.540	0.500	0.379
	α	cm ⁻¹	2.04E-2	2.33E-2	1.86E-2	1.33E-2	3.57E-2	0.164
	n	-	1.33	1.37	1.31	1.25	1.11	1.10
	h_{ae}	cm	-5.95E-3	-2.36E-3	-2.43E-3	-4.80	-7.12	-1.21E-3
	h_j	cm	-1.49E6	-1.07E6	-8.62E6	-8.02E6	-8.33E6	-9.89E6



545 Table C4. The fitting parameters and their values for six parameterizations for clayey soils from the
 546 UNSODA database (National Agricultural Library, 2017; Nemes et al., 2001). The three-character
 547 parameterization label is explained in the main text.

			Soil (UNSODA identifier and classification according to Twarakavi et al. (2010))					
			1135 C2	1182 C2	1122 C4	1123 C4	1180 C4	1181 C4
Para- meteri- zation	Para- meter	Unit						
BCO	θ_t	-	3.94E-4	1.79E-4	2.64E-4	2.13E-4	5.22E-4	1.24E-5
	θ_s	-	0.420	0.549	0.362	0.358	0.497	0.456
	h_{ae}	cm	-106	-0.977	-10.0	-10.0	-11.1	-5.17
	λ	-	7.85E-2	4.41E-2	3.37E-2	2.69E-2	5.65E-2	5.40E-2
FSB	θ_s	-	0.420	0.548	0.360	0.356	0.495	0.456
	θ_a	-	0.400	0.307	0.350	0.340	0.491	0.345
	h_{ae}	cm	-106	-0.230	-5.75	-10.0	-8.58	-13.2
	λ	-	0.172	5.64E-2	6.60E-2	5.69E-2	100	8.08E-2
RNA	θ_s	-	0.420	0.549	0.370	0.370	0.497	0.456
	h_{ae}	cm	-106	-3.61	-9.99	-9.99	-0.150	-7.66
	h_j	cm	-107	-12.2	-10.5	-10.7	-24.1	-22.1
	h_d	cm	-1.64E8	-1.00E9	-1.00E9	-1.00E9	-1.00E9	-1.00E9
VGN	θ_t	-	0.263	8.94E-6	1.16E-4	0.210	0.255	3.34E-6
	θ_s	-	0.413	0.548	0.359	0.354	0.496	0.456
	α	cm ⁻¹	1.02E-3	0.753	1.37E-2	2.92E-3	0.805	0.143
	n	-	2.37	1.05	1.05	1.21	1.26	1.06
VGA	θ_t	-	0.263	1.19E-5	5.78E-2	0.188	2.63E-2	2.12E-6
	θ_s	-	0.413	0.548	0.359	0.354	0.498	0.456
	α	cm ⁻¹	1.02E-3	1.21	1.37E-2	3.22E-3	10.1	1.70
	n	-	2.37	1.05	1.07	1.17	1.06	1.05
	h_{ae}	cm	-1.03	-0.467	-4.18E-2	-10.0	-1.11E-2	-4.83
RIA	θ_s	-	0.416	0.548	0.359	0.354	0.497	0.457
	α	cm ⁻¹	1.86E-3	1.31	1.39E-2	4.00E-3	14.2	0.702
	n	-	1.16	1.05	1.05	1.06	1.06	1.05
	h_{ae}	cm	-106	-0.525	-3.80E-3	-9.99	-4.41E-2	-4.25
	h_j	cm	-7.57E6	-9.97E6	-9.56E4	-4.43E6	-415	-206



549 Table C5. Root mean square errors of the parameter fits for the sandy or loamy soils (A1 and A2
550 soils according to Twarakavi et al., 2010)

Parameterization	Soil (UNSODA identifier and classification according to Twarakavi et al. (2010))		
	2126 A1	1142 A2	2104 A2
BCO	0.0620	0.0990	0.0481
FSB	0.0626	0.0990	0.0517
RNA	0.0659	0.0989	0.0553
VGN	0.0330	0.0252	0.0278
VGA	0.0330	0.0250	0.0278
RIA	0.0652	0.0504	0.0542

551

552



553 Table C6. Root mean square errors of the parameter fits for the sandy soils (A3 and A4 soils
 554 according to Twarakavi et al., 2010)

Parameterization	Soil (UNSODA identifier and classification according to Twarakavi et al. (2010))					
	1120	1143	2110	2132	1121	1133
	A3	A3	A3	A3	A4	A4
BCO	0.0926	0.0501	0.0445	0.0356	0.1288	0.0803
FSB	0.0926	0.0500	0.0445	0.0292	0.1054	0.0700
RNA	0.0926	0.0500	0.0457	0.0356	0.1286	0.0775
VGN	0.0446	0.0333	0.0378	0.0204	0.0720	0.0175
VGA	0.0489	0.0396	0.0445	0.0203	0.0720	0.0175
RIA	0.0643	0.0346	0.0491	0.0203	0.0720	0.0530

555

556



557 Table C7. Root mean square errors of the parameter fits for the silty soils.

Parameterization	Soil (UNSODA identifier and classification according to Twarakavi et al. (2010))					
	3260	3261	3263	3250	3251	4450
	B2	B2	B2	B4	B4	B4
BCO	0.0793	0.1316	0.0973	0.0822	0.0551	0.0499
FSB	0.0794	0.1316	0.0973	0.0815	0.0395	0.0445
RNA	0.0793	0.1316	0.0973	0.0822	0.0551	0.0499
VGN	0.0456	0.0607	0.0638	0.0413	0.0474	0.0485
VGA	0.0455	0.0607	0.0769	0.0412	0.0466	0.0497
RIA	0.0543	0.0698	0.0668	0.0412	0.0466	0.0485

558

559



560 Table C8. Root mean square errors of the parameter fits for the clayey soils.

Parameterization	Soil (UNSODA identifier and classification according to Twarakavi et al. (2010))					
	1135	1182	1122	1123	1180	1181
	C2	C2	C4	C4	C4	C4
BCO	0.0913	0.0494	0.0349	0.0489	0.0187	0.0428
FSB	0.0721	0.0441	0.0212	0.0321	0.1196	0.0360
RNA	0.0812	0.0913	0.1235	0.1501	0.0347	0.0570
VGN	0.0208	0.0488	0.0197	0.0244	0.0411	0.0433
VGA	0.0208	0.0485	0.0198	0.0243	0.0197	0.0429
RIA	0.0519	0.0485	0.0197	0.0244	0.0180	0.0391

561

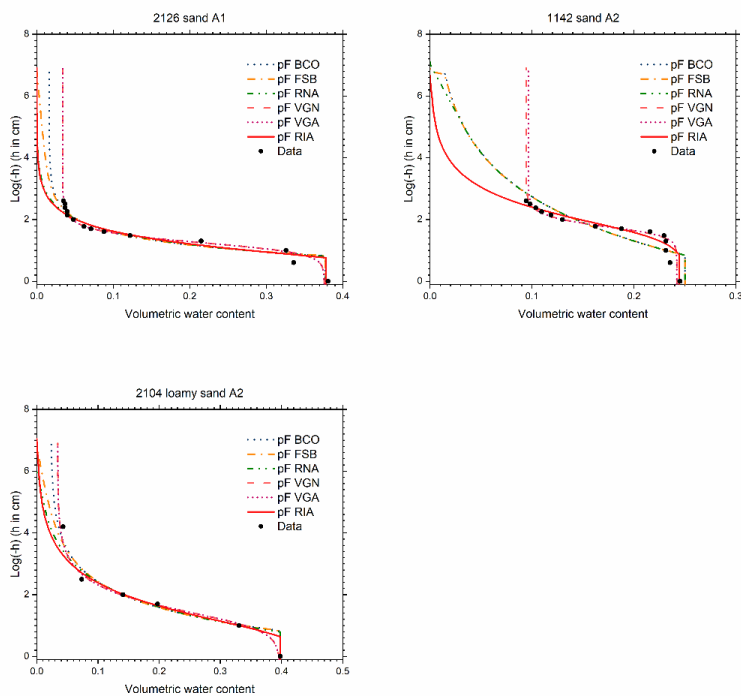
562



563 Table C9. Values for the hydraulic conductivity at saturation (K_s) for the selected soils from
564 the UNSODA database. The soils are identified by their UNSODA identifier. Their classification
565 according to Twarakavi et al. (2010) is also given.

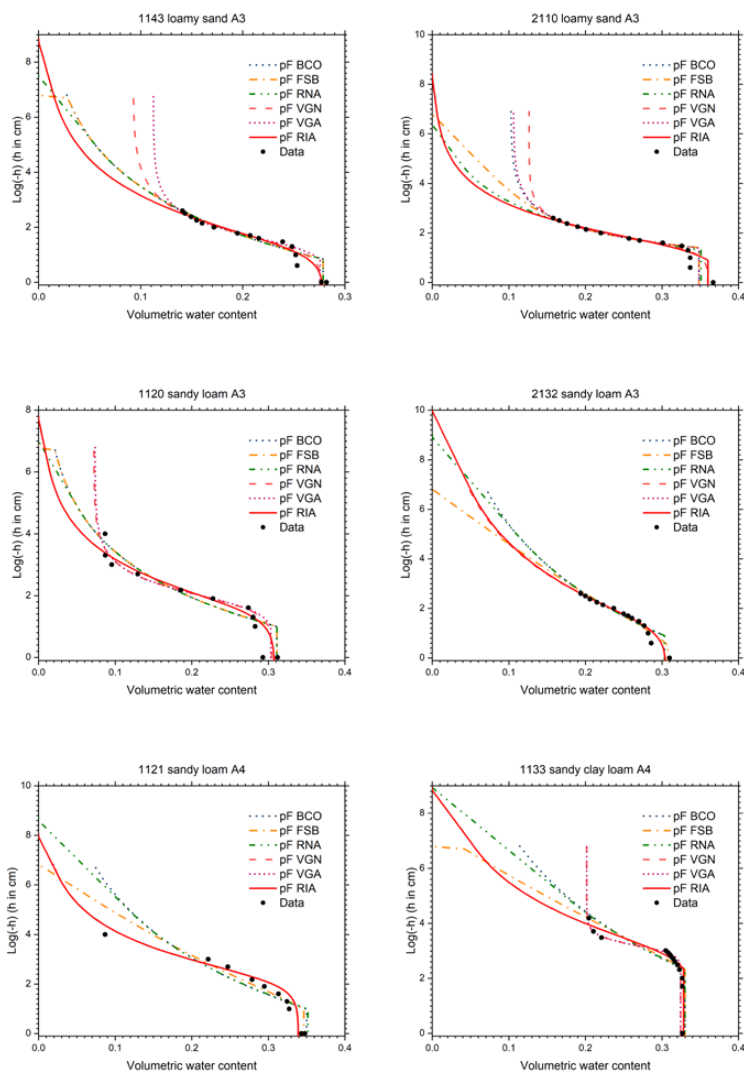
UNSODA identifier	Texture classification	K_s (cm d ⁻¹)
2126	A1	1.10E3
1142	A2	13.4
2104	A2	553
1120	A3	37.9
1143	A3	23.5
2110	A3	16.3
2132	A3	5.52
1121	A4	7.13
1133	A4	7.13
3260	B2	10.8
3261	B2	32.0
3263	B2	54.0
3250	B4	1.51
3251	B4	2.74
4450	B4	1.20
1135	C2	0.142
1182	C2	1.25
1122	C4	2.92
1123	C4	0.740
1180	C4	215
1181	C4	178

566



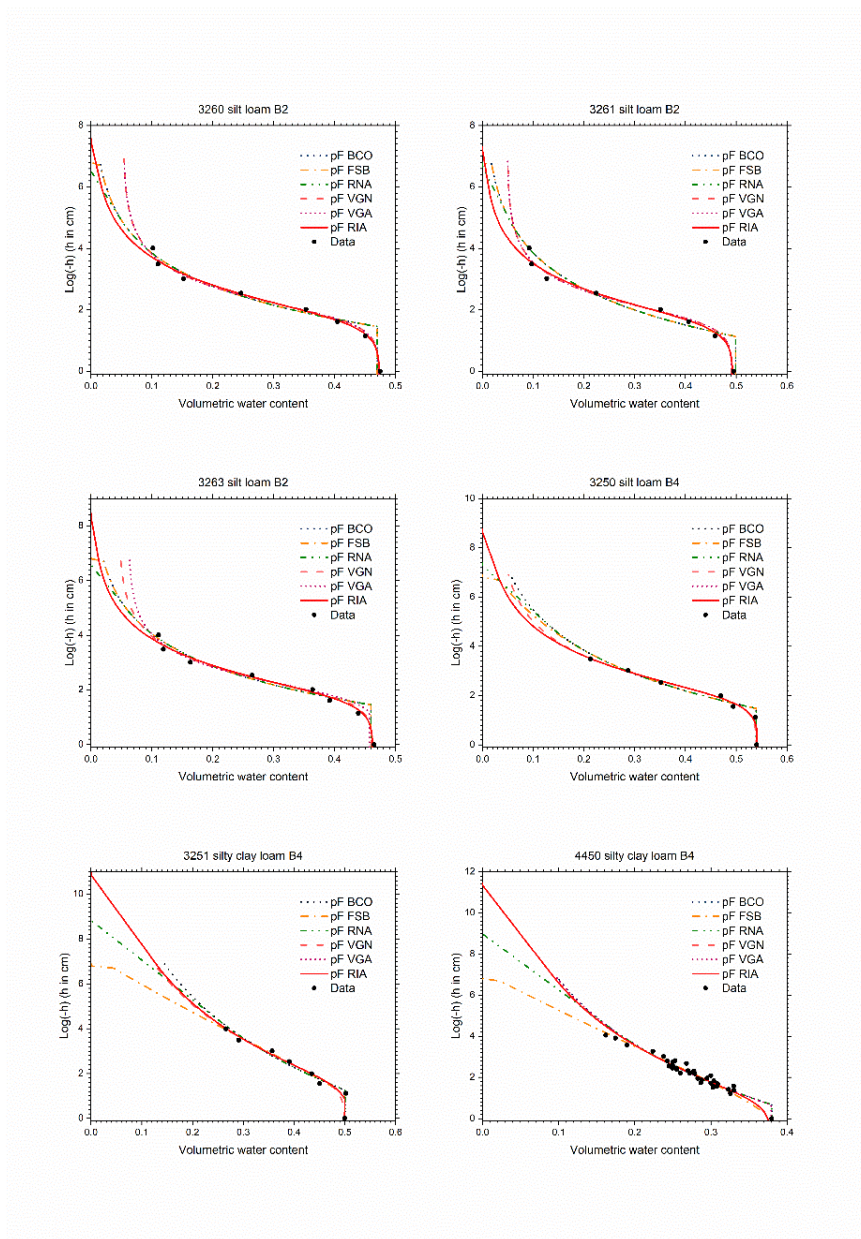
567

568 Figure C2. Retention data and fitted soil water retention curves according to six
569 parameterizations for selected UNSODA soils with Twarakavi et al.'s (2010) A1 or A2
570 classification. The parameterizations are explained in the text.



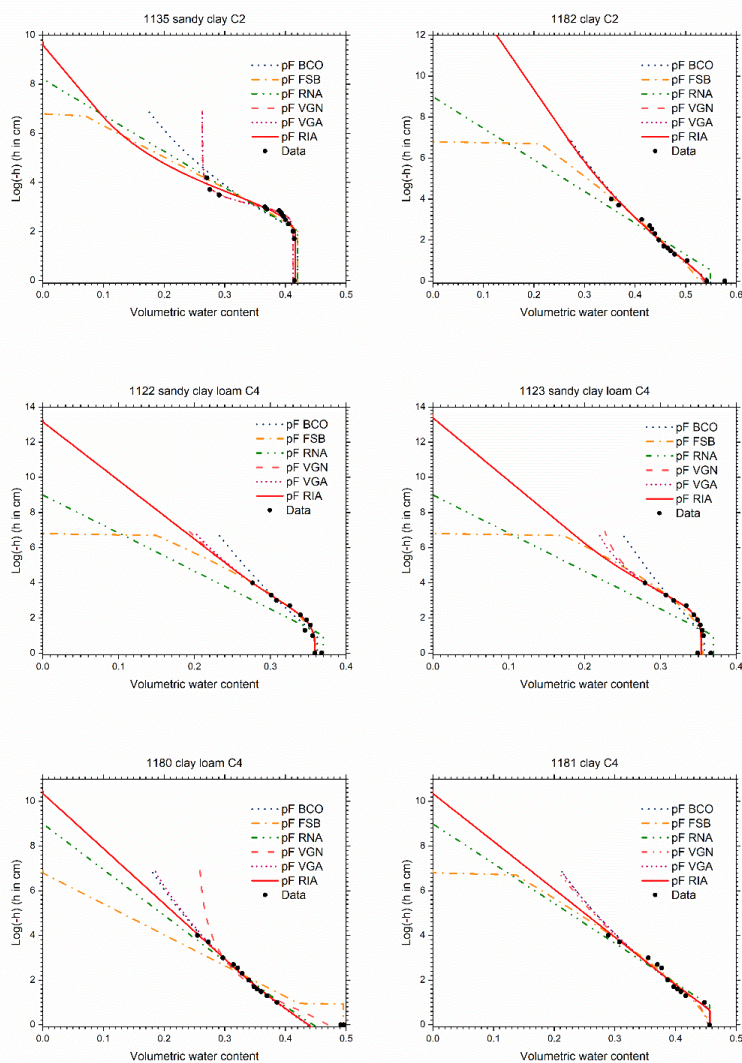
571

572 Figure C3. Retention data and fitted soil water retention curves according to six
573 parameterizations for selected UNSODA soils with Twarakavi et al.'s (2010) A3 or A4
574 classification. The parameterizations are explained in the text.



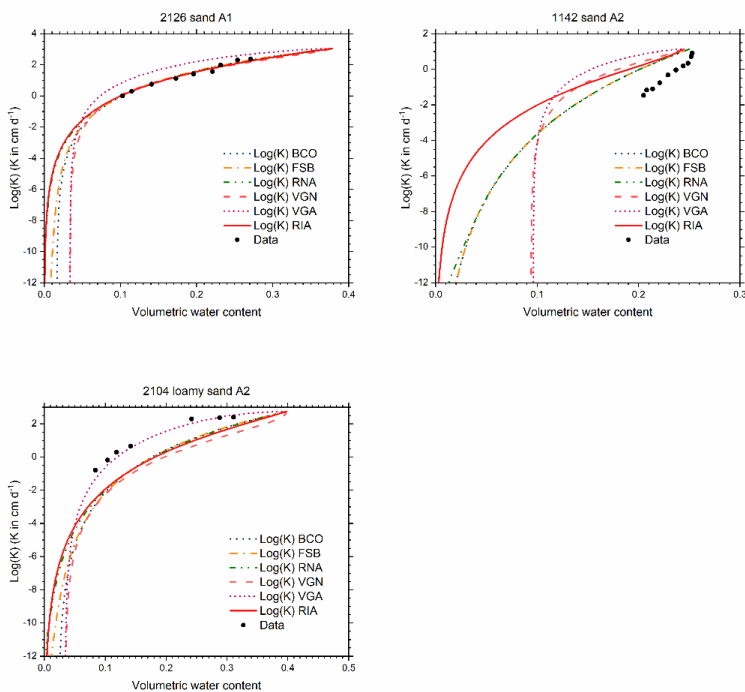
575

576 Figure C4. Retention data and fitted soil water retention curves according to six
577 parameterizations for selected UNSODA soils with Twarakavi et al.'s (2010) B2 or B4
578 classification. The parameterizations are explained in the text.



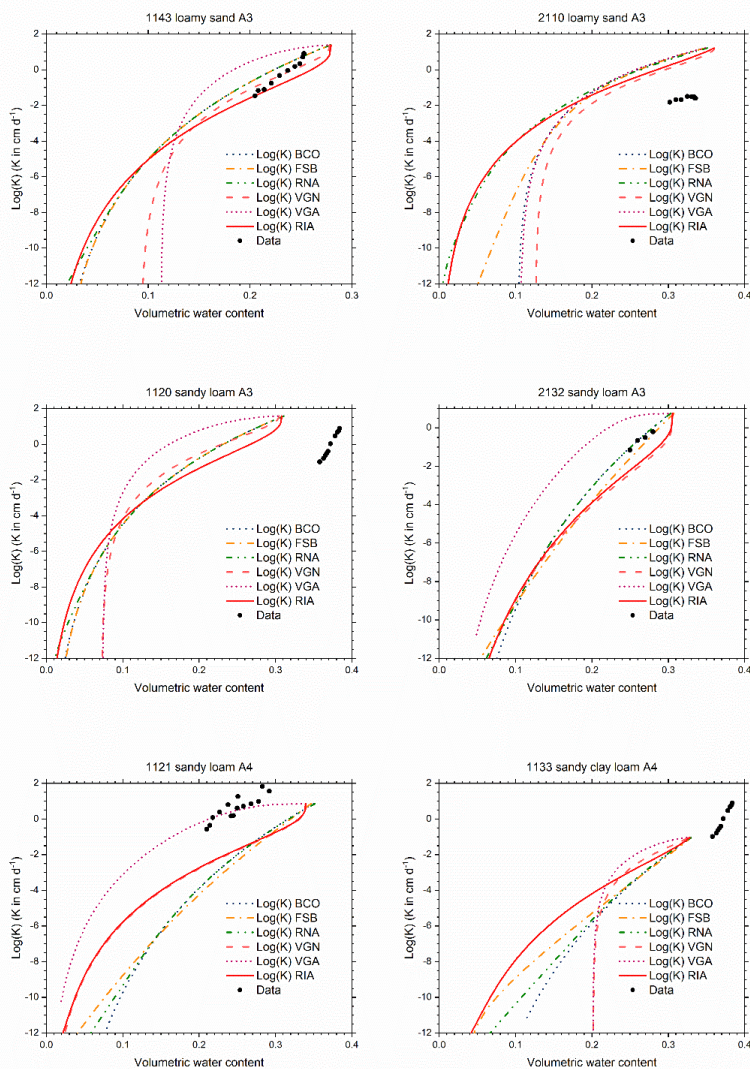
579

580 Figure C5. Retention data and fitted soil water retention curves according to six
581 parameterizations for selected UNSODA soils with Twarakavi et al.'s (2010) C2 or C4
582 classification. The parameterizations are explained in the text.



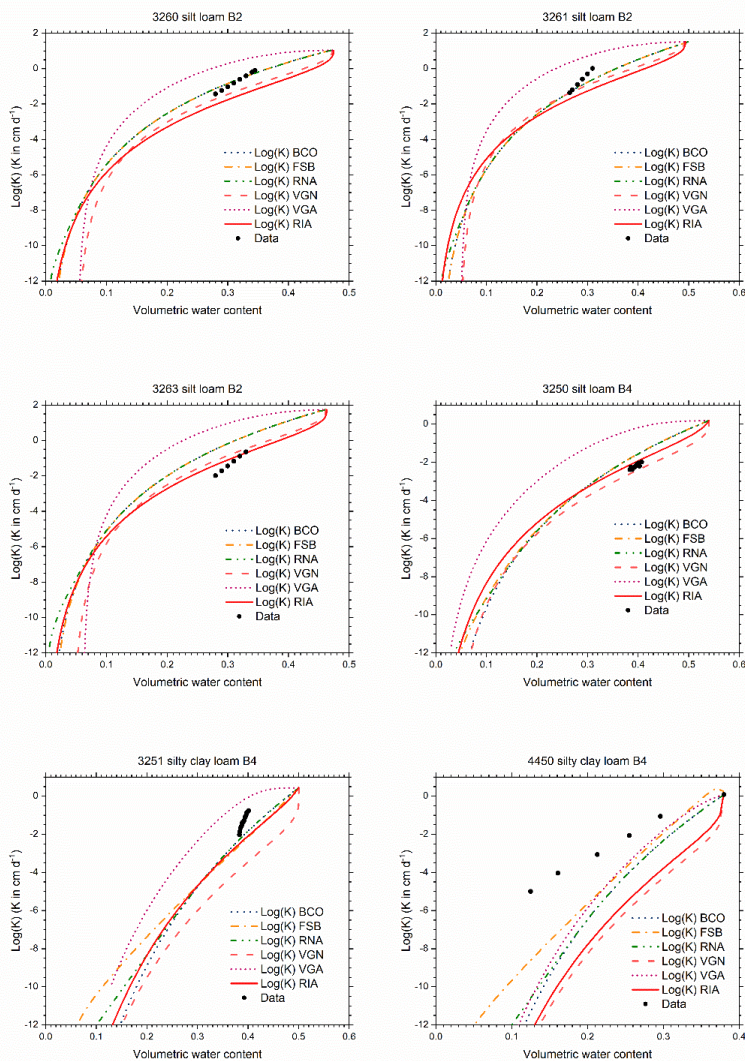
583

584 Figure C6. Conductivity data and conductivity curves derived from the retention curves of
585 Fig. C2 according to six parameterizations for selected UNSODA soils with Twarakavi et al.'s
586 (2010) A1 or A2 classification.



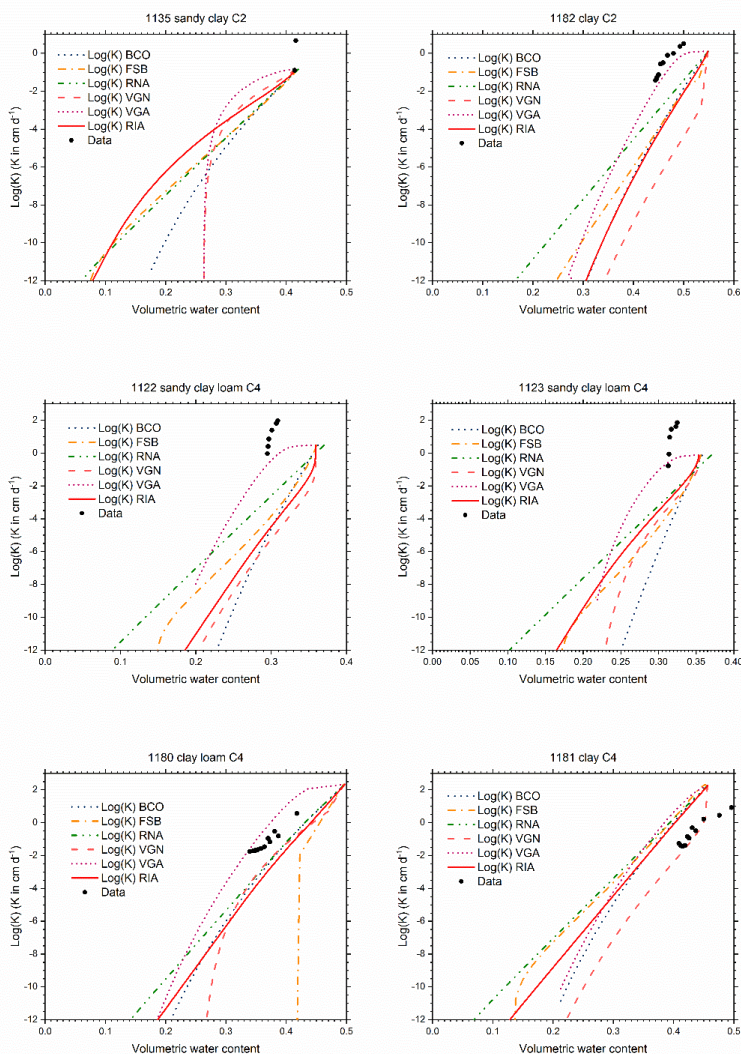
587

588 Figure C7. Conductivity data and conductivity curves derived from the retention curves of
589 Fig. C3 according to six parameterizations for selected UNSODA soils with Twarakavi et al.'s
590 (2010) A3 or A4 classification.



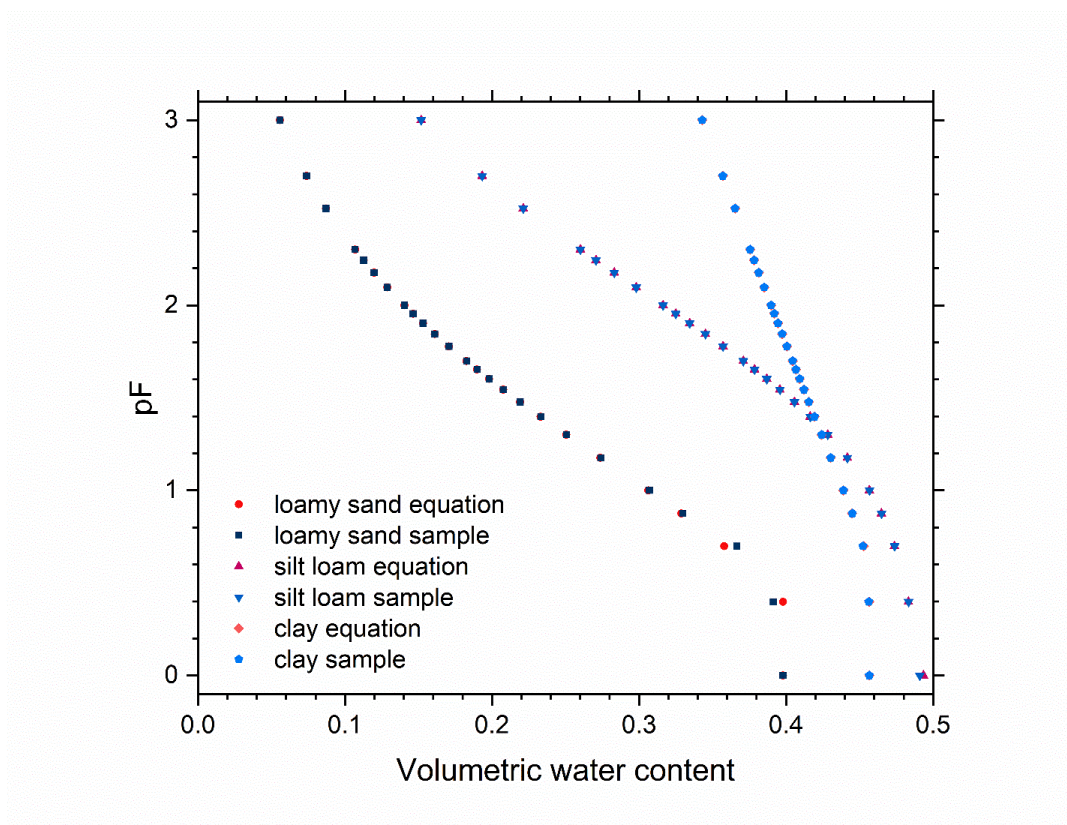
591

592 Figure C8. Conductivity data and conductivity curves derived from the retention curves of
593 Fig. C4 according to six parameterizations for selected UNSODA soils with Twarakavi et al.'s
594 (2010) B2 or B4 classification.



595

596 Figure C9. Conductivity data and conductivity curves derived from the retention curves of
597 Fig. C5 according to six parameterizations for selected UNSODA soils with Twarakavi et al.'s
598 (2010) C2 or AC4 classification.



599

600 Figure C10. Soil water retention points calculated from RIA parameterizations for loamy
601 sand (UNSODA identifier 2104, classification according to Twarakavi et al. (2010) A4), silt
602 loam (3261, B2), and clay (1181, C4). The points were either calculated directly for the
603 given pF value ('equation'), or by calculating the average water content in a sample of 5.0
604 cm height at hydrostatic equilibrium, with the matric potential at the center of the sample
605 corresponding to the indicated pF value ('sample'). N.B. The data points for zero matric
606 potential were plotted at $pF = 0$ instead of $pF = -\infty$.

607



608 **Author contributions**

609

610 GdR developed the new parameterization, assisted by RM. GdR identified the 21 soils from
611 the UNSODA database and designed the model test. JM developed the SCE code for
612 parameter identification. GdR coded the shell around the SCE core to make it suitable for
613 fitting various SWRC parameterizations. GdR collected the weather data and generated the
614 weather records. GdR and RM carried out the parameter fitting and the Hydrus-1D
615 simulations. GdR wrote the manuscript with contributions from JM and RM.

616

617 **Competing interests**

618

619 GdR is a member of the HESS Editorial Board.



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