

1 **Sigmoidal Water Retention Function with Improved Behavior in Dry**  
2 **and Wet Soils**

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14 **Abstract**

15

16 A popular parameterized soil water retention curve (SWRC) has a hydraulic conductivity  
17 curve associated with it that can have a physically unacceptable infinite slope at saturation.  
18 The problem was eliminated before by giving the SWRC a non-zero air-entry value. This  
19 improved version still has an asymptote at the dry end, which limits its usefulness for dry  
20 conditions and causes its integral to diverge for commonly occurring parameter values. We  
21 therefore joined the parameterizations' sigmoid mid-section to a logarithmic dry section  
22 ending at zero water content for a finite matric potential, as was done previously for a  
23 power-law type SWRC. We selected five SWRC parameterizations that had been proven to  
24 produce unproblematic near-saturation conductivities and fitted these and our new curve  
25 to data from 21 soils. The logarithmic dry branch gave more realistic extrapolations into the  
26 dry end of both the retention and the conductivity curves than an asymptotic dry branch.  
27 We tested the original curve, its first improvement, and our second improvement by feeding  
28 them into a numerical model that calculated evapotranspiration and deep drainage for nine  
29 combinations of soils and climates. The new curve was more robust than the other two. The  
30 new curve was better able to produce a conductivity curve with a substantial drop during  
31 the early stages of drying than the earlier improvement. It therefore generated smaller  
32 amounts of more evenly distributed deep drainage compared to the spiked response to  
33 rainfall produced by the earlier improvement.

34 **Introduction**

35 The soil water retention function introduced by van Genuchten (1980) has been the  
36 most popular parameterization (denoted VGN below; these and other abbreviations are  
37 listed in Appendix A) to describe the SWRC in numerical models for unsaturated flow for  
38 the past few decades (e.g., Kroes et al., 2017; Šimůnek and Bradford, 2008; Šimůnek et al.,  
39 2016):

40

41 
$$\theta(h) = \theta_r + (\theta_s - \theta_r)(1 + |\alpha h|^n)^{\frac{1}{n}-1}, \quad h \leq 0 \quad (1)$$

42

43 where  $h$  denotes the matric potential in equivalent water column [L]. The volumetric water  
44 content is denoted by  $\theta$ , with the subscript 's' denoting its value at saturation and the  
45 subscript 'r' its residual, or irreducible, value. Parameters  $\alpha$  [ $L^{-1}$ ] and  $n$  are shape  
46 parameters.

47 Van Genuchten (1980) combined Eq. (1) with Mualem's (1976) conductivity model  
48 and derived an analytical expression for the unsaturated hydraulic conductivity curve:

49

50 
$$K(h) = K_s \frac{\left[1 - |\alpha h|^{n-1} (1 + |\alpha h|^n)^{\frac{1}{n}-1}\right]^2}{(1 + |\alpha h|^n)^{\frac{1}{2} - \frac{1}{2n}}} \quad (2)$$

51

52 where  $K$  [ $LT^{-1}$ ] is the soil hydraulic conductivity and  $K_s$  [ $LT^{-1}$ ] its value at saturation.

53 Hysteretic (Kool and Parker, 1987) and multimodal versions (Durner, 1994) of Eq.  
54 (1) are available. Apart from the convenience of having analytical expressions for the  
55 retention as well as the conductivity curve, the function's popularity derives from its

56 continuous derivative and its inflection point, which gives it considerable flexibility in  
57 fitting observations.

58 Fuentes et al. (1991) warned that the asymptotic residual water content at the dry  
59 end could lead to a non-converging integral of the retention curve when the integration is  
60 carried out between the saturated water content and a water content that approximates the  
61 residual water content in the limit. In that case, the area below the retention curve becomes  
62 infinite. Fuentes et al. (1991) showed that this would lead to an unlimited amount of water  
63 stored in a column of a finite length at hydrostatic equilibrium if its height was such that the  
64 residual water content was approximated closely at the top of the column. This physically  
65 impossible case is only avoided if  $n > 2$  in Eq. (1), a condition which is often not satisfied.

66 Near saturation, the slope  $d\theta/dh$  is not zero at zero matric potential. This implies  
67 that the soil has pores that have at least one infinite principal radius according to the  
68 Laplace–Young Law (Hillel, 1998, p. 46), which is physically unacceptable (see also Iden et  
69 al., 2015). Durner (1994) noted this could lead to an infinite slope in the hydraulic  
70 conductivity function of Mualem (1976) when the matric potential approached zero, and  
71 Ippisch et al. (2006) showed that if  $n < 2$  this would indeed be the case. The more recent  
72 sigmoid curve of Fredlund and Xing (1994) and its modification by Wang et al. (2016), used  
73 by Wang et al. (2018) and Rudiyanto et al. (2020) have the same problem (see Appendix B  
74 for the proof). The curve of Assouline et al. (1998) is based on the Weibull distribution, and  
75 therefore has a non-zero slope at zero matric potential when its fitting parameter  $\eta$  is  
76 smaller than 2, which was the case for 75% of the soils for which it was fitted. None of these  
77 curves therefore offers a remedy to the problem associated with VGN.

78 Corrections for the conductivity curve were proposed by Vogel et al. (2001), Schaap  
 79 and van Genuchten (2006), and Iden et al. (2015), but these leave the effect of the non-  
 80 physical, very large pores on the SWRC intact and create an inconsistency between the  
 81 retention model and the conductivity model. For instance, Iden et al. (2015) clipped the  
 82 integral in the conductivity function at a matric potential  $h_c$  somewhat below zero. In the  
 83 range between  $h_c$  and zero, their modified unsaturated hydraulic conductivity increased  
 84 linearly with the water content (Iden et al. (2015), Fig. 1), which is not physically realistic  
 85 because the pore sizes that are being filled are increasing in size according to Eq. (1) or its  
 86 multimodal version (Peters et al., 2011). Only Ippisch et al. (2006) addressed the  
 87 underlying problem in the SWRC by introducing a non-zero air-entry value, thereby  
 88 eliminating excessively large pores whilst maintaining the mathematical consistency  
 89 between the expressions for the retention and the conductivity curves. In doing so they  
 90 sacrificed the continuity of the derivative of the VGN curve. Iden et al. (2015) suspected this  
 91 would pose a challenge to numerical solves of Richards' equation, but Ippisch et al.'s (2006)  
 92 numerical simulations ran without difficulty. Their equation scaled the sigmoid curve by its  
 93 value at the air-entry value  $h_{ae}$  [L] and introduced a saturated section for  $h > h_{ae}$ .

94

$$95 \quad \theta(h) = \begin{cases} \theta_r + (\theta_s - \theta_r) \left( \frac{1+|\alpha h|^n}{1+|\alpha h_{ae}|^n} \right)^{\frac{1}{n}-1}, & h \leq h_{ae} \\ \theta_s, & h > h_{ae} \end{cases} \quad (3)$$

96

97 This function is denoted VGA below.

98 The smooth, sigmoidal shape of VGN resembles many observed curves for which the  
 99 data points in the wet range were obtained by equilibrating vertically placed cylindrical soil

100 samples at well-defined matric potentials and determining the corresponding water  
101 content by weighing the sample (Klute, 1986, p. 644–647). Liu and Dane (1995) took into  
102 account the vertical variation of the water content in such samples and demonstrated that a  
103 power-law SWRC with a well-defined air-entry matric value but without inflection point  
104 can produce a sigmoid-type apparent SWRC if the non-uniform distribution of water in the  
105 sample is ignored. A series of data points suggesting a smooth SWRC therefore does not  
106 intrinsically contradict the existence of a discrete non-zero air-entry value, corroborating  
107 the correction to VGN by Ippisch et al. (2006).

108 Madi et al. (2018) generalized the analysis of Ippisch et al. (2006) and applied it to  
109 18 parameterizations of the SWRC to verify that the slope of the hydraulic conductivity near  
110 saturation would remain finite. Apart from Eq. (3), only the expressions developed by  
111 Brooks and Corey (1964) (denoted BCO), Fayer and Simmons (1995) (denoted FSB), and  
112 the junction model of Rossi and Nimmo (1994) (denoted RNA) satisfied this requirement.  
113 In the latter case, **a modification that smoothed the curve near saturation needed to be**  
114 **removed**. All these equations have a power law relationship between the water content and  
115 the matric potential, and therefore do not have the sigmoid shape of VGN and VGA.

116 In a separate development, several researchers argued that in the dry range, water is  
117 bound to the soil by adsorptive rather than capillary forces. Usually, a logarithmic term that  
118 allowed the adsorbed water content to go to zero at a prescribed matric potential was  
119 added to a capillary term. The former would dominate in the dry range and become  
120 negligible as the soil became wetter (e.g., Campbell and Shiozawa, 1992; Fayer and  
121 Simmons, 1995; Khlosoi et al., 2006; Peters, 2013). The logarithmic relationship was based  
122 on the sorption theory of Bradley (1936). It removed the asymptote and the associated

123 problem of the non-converging integral of the SWRC that Fuentes et al. (1991) warned  
124 about. Rossi and Nimmo (1994) presented a junction model in which a critical matric  
125 potential separated purely capillary bound water from solely adsorbed water. The  
126 modified junction model of Rossi and Nimmo (1994) is

127

$$128 \quad \theta(h) = \begin{cases} \theta_s \beta \ln\left(\frac{h_d}{h}\right), & h_d \leq h \leq h_j \\ \theta_s \left(\frac{h_0}{h}\right)^\lambda, & h_j \leq h \leq h_i \\ \theta_s \left[1 - c \left(\frac{h}{h_0}\right)^2\right], & h_i \leq h \leq 0 \end{cases} \quad (4)$$

129

130 The first (dry) branch is logarithmic, with  $h_d$  [L] the matric potential at which the water  
131 content reaches zero, and  $\beta$  a shape parameter. The middle branch is the power law  
132 adopted from Brooks and Corey (1964) (without smoothing near saturation), where  $\lambda$  is a  
133 shape parameter and  $h_0$  [L] is a fitting parameter. The final (wet) branch is a parabolic  
134 correction to avoid a discontinuity in the derivative at the air-entry value, with parameter  $c$   
135 a function of  $\lambda$  and  $h_0$  (Hutson and Cass, 1987). Madi et al. (2018) pointed out that this  
136 correction puts such strict constraints on the parameters of the conductivity curve that the  
137 usual models are ruled out. The first and middle branch are joined at  $h_j$  [L], the middle and  
138 final branch at  $h_i$  [L]. Rossi and Nimmo (1994) reduced the number of fitting parameters by  
139 requiring that the water contents and the first derivatives of the branches match at  $h_j$  and  
140  $h_i$ . This junction model avoids the problem of many of the other models that would have  
141 some capillary bound water still present in the soil below the matric potential at which the  
142 adsorbed water content had gone to zero.

143 As noted above, the introduction of a non-zero air-entry value by Ippisch et al.  
144 (2006) eliminated the unphysically large slopes of the hydraulic conductivity according to  
145 Mualem (1976). The approach of Rossi and Nimmo (1994) resolved the issue of the  
146 asymptotic behavior in the dry range. The objective of this paper therefore is to combine  
147 Rossi and Nimmo's (1994) model for the dry range with the VGA model of Ippisch et al.  
148 (2006) to arrive at a SWRC (denoted RIA) that has a non-zero air-entry value, a sigmoid  
149 shape in the intermediate range, a dry branch that can reach zero water content at a finite  
150 matric potential, and therefore a finite integral. We will also develop a closed-form  
151 expression for the unsaturated hydraulic conductivity based on this SWRC. For  
152 completeness, a generalized expression for multimodal SWRCs will also be derived.

153 Together with the other functions that lead to physically acceptable behavior of the  
154 hydraulic conductivity near saturation (BCO, FSB, RNA, and VGA), RIA will be fitted to 21  
155 soils selected from the UNSODA database (National Agricultural Library, 2017; Nemes et al.,  
156 2001) that cover a wide range of textures (Madi et al., 2018). For comparison VGN is also  
157 included, in view of its de facto status as the standard parameterization for the SWRC. All  
158 three versions with the sigmoid shape (VGN, VGA, and RIA) will be tested in a simulation  
159 study for different combinations of soil types and climates.

160

## 161 **Theory**

### 162 **The Soil Water Retention Curve**

163 The junction model of Rossi and Nimmo (1994) has a SWRC with a logarithmic dry  
164 branch without residual water content. The parameterization proposed by Ippisch et al.  
165 (2006) combines the sigmoid shape of van Genuchten (1980) with a non-zero air-entry

166 value. By setting the  $\theta$  in Ippisch et al. (2006) to zero, we can combine the two models to  
 167 give the following parameterization:

168

$$169 \quad \theta(h) = \begin{cases} 0, & h \leq h_d \\ \theta_s \beta \ln \left( \frac{h_d}{h} \right), & h_d < h \leq h_j \\ \theta_s \left( \frac{1+|\alpha h|^n}{1+|\alpha h_{ae}|^n} \right)^{\frac{1}{n}-1}, & h_j < h \leq h_{ae} \\ \theta_s, & h > h_{ae} \end{cases} \quad (5)$$

170

171 where subscripts 'd' and 'ae' denote the value at which the water content reaches zero and  
 172 the air-entry value, respectively, and subscript 'j' indicates the value at which the  
 173 logarithmic and sigmoid branch are joined. The first branch ensures that no water can be  
 174 present at matric potentials below  $h_d$ . The second, logarithmic, branch is identical to that of  
 175 Eq. (4). The third, sigmoidal, branch between  $h_j$  and  $h_{ae}$ , and the fourth branch, for matric  
 176 potentials larger than the air-entry value, are from Eq. (3). Joining instead of adding the  
 177 logarithmic and the sigmoid functions avoids potentially non-monotonic behavior (Peters  
 178 et al., 2011).

179 The derivative of Eq. (5) is

180

$$181 \quad \frac{d\theta}{dh}(h) = \begin{cases} 0, & h \leq h_d \\ \frac{\theta_s \beta}{h}, & h_d < h \leq h_j \\ \theta_s \alpha(n-1)|\alpha h|^{n-1}(1+|\alpha h_{ae}|^n)^{1-\frac{1}{n}}(1+|\alpha h|^n)^{\frac{1}{n}-2}, & h_j < h \leq h_{ae} \\ 0, & h > h_{ae} \end{cases} \quad (6)$$

182

183 Mass continuity dictates that the SWRC is continuous. At the air–entry value, this condition  
184 is met irrespective of the parameter values. At  $h_j$ , continuity requires the following equality  
185 to hold:

186

$$187 \beta \ln \left( \frac{h_d}{h_j} \right) = \left( \frac{1+|\alpha h_j|^n}{1+|\alpha h_{ae}|^n} \right)^{\frac{1}{n}-1} \quad (7)$$

188

189 In accordance with Rossi and Nimmo (1994) we also require the derivatives at  $h_j$  to match,  
190 leading to

191

$$192 \beta = (n-1)|\alpha h_j|^n (1+|\alpha h_{ae}|^n)^{1-\frac{1}{n}} (1+|\alpha h_j|^n)^{\frac{1}{n}-2} \quad (8)$$

193

194 Combining Eqs. (7) and (8) and solving for  $h_d$  gives:

195

$$196 h_d = h_j \exp \left( \frac{1+|\alpha h_j|^{-n}}{n-1} \right) \quad (9)$$

197

198 This leaves  $h_{ae}$ ,  $h_j$ ,  $\theta_s$ ,  $\alpha$ , and  $n$  as fitting parameters.

199 The derivation of a multimodal curve analogous to that of Durner (1994) and  
200 Zurmühl and Durner (1998) is straightforward if the values of  $h_{ae}$  and  $h_j$  are kept the same  
201 for all contributing terms:

202 
$$\theta(h) = \begin{cases} 0, & h \leq h_d \\ \theta_s \beta \ln \left( \frac{h_d}{h} \right), & h_d < h \leq h_j \\ \theta_s \sum_{i=1}^k w_i \left( \frac{1+|\alpha_i h|^{n_i}}{1+|\alpha_i h_{ae}|^{n_i}} \right)^{\frac{1}{n_i}-1}, & h_j < h \leq h_{ae} \\ \theta_s, & h > h_{ae} \end{cases} \quad (10)$$

203  
 204 where the modality is indicated by  $k$ . The weighting factors  $w_i$  are bounded on the interval  
 205  $[0,1]$  and their sum must equal 1 (Durner, 1994; Zurmühl and Durner, 1998). Requiring the  
 206 logarithmic and the multimodal branch as well as their derivatives to match at  $h_j$  leads to  
 207

208 
$$\beta = \sum_{i=1}^k w_i (n_i - 1) |\alpha_i h_j|^{n_i} (1 + |\alpha_i h_{ae}|^{n_i})^{1-\frac{1}{n_i}} (1 + |\alpha_i h_j|^{n_i})^{\frac{1}{n_i}-2} \quad (11)$$

209  
 210 and  
 211

212 
$$h_d = h_j \exp \left[ \frac{\sum_{i=1}^k w_i (1 + |\alpha_i h_{ae}|^{n_i})^{1-\frac{1}{n_i}} (1 + |\alpha_i h_j|^{n_i})^{\frac{1}{n_i}-1}}{\sum_{i=1}^k w_i (n_i - 1) |\alpha_i h_j|^{n_i} (1 + |\alpha_i h_{ae}|^{n_i})^{1-\frac{1}{n_i}} (1 + |\alpha_i h_j|^{n_i})^{\frac{1}{n_i}-2}} \right] \quad (12)$$

213  
 214 The fitting parameters are  $h_{ae}$ ,  $h_j$ ,  $\theta_s$ ,  $\alpha_i$ ,  $n_i$  and  $w_i$   
 215

## 216 The Unsaturated Hydraulic Conductivity Curve

217 The primary focus of this paper is on the SWRC. Nevertheless, it is worthwhile to  
 218 find a closed-form expression for the unsaturated hydraulic conductivity that can be used in  
 219 association with Eq. (5). Kosugi (1999) proposed the following conductivity model (see  
 220 also Ippisch et al., 2006):

221

$$222 \quad K(h) = K(h(S_e)) = \begin{cases} K_s S_e^\tau \left( \frac{\int_{-\infty}^{h(S_e)} |h|^{-\kappa \frac{dS}{dh}} dh}{\int_{-\infty}^{h_{ae}} |h|^{-\kappa \frac{dS}{dh}} dh} \right)^\gamma, & h < h_{ae} \\ K_s, & h \geq h_{ae} \end{cases} \quad (13)$$

223

224 with  $\gamma$ ,  $\kappa$  and  $\tau$  denoting shape parameters,  $S_e$  denoting the degree of saturation  $(\theta - \theta_r)/(\theta_s - \theta_r)$ , and  $S$  representing a variable running over all values of the degree of saturation from zero to its actual value  $S_e$ . Mualem's (1976) conductivity model is a special case of Eq. (13), with  $\gamma = 2$ ,  $\kappa = 1$ , and  $\tau = 0.5$ . Madi et al. (2018) give parameter combinations for additional models.

229 The integrals in Eq. (13) that arise when Eq. (6) is used to find  $dS/dh$  can be evaluated analytically if  $\kappa = 1$ . The resulting conductivity function is

231

$$232 \quad K(h) = \begin{cases} 0, & h \leq h_d \\ K_s \left( \beta \ln \left| \frac{h_d}{h} \right| \right)^\tau \left[ \frac{\frac{\beta}{|h_d|} - \frac{\beta}{|h|}}{\frac{\beta}{|h_d|} - \frac{\beta}{|h_j|} + \frac{|\alpha h_{ae}|^n}{|h_{ae}|} - (1 + |\alpha h_{ae}|^n)^{1-\frac{1}{n}} F(h_j)} \right]^\gamma, & h_d < h \leq h_j \\ K_s \left( \frac{1 + |\alpha h|^n}{1 + |\alpha h_{ae}|^n} \right)^{\frac{\tau}{n} - \tau} \left\{ \frac{\frac{\beta}{|h_d|} - \frac{\beta}{|h_j|} + (1 + |\alpha h_{ae}|^n)^{1-\frac{1}{n}} [F(h) - F(h_j)]}{\frac{\beta}{|h_d|} - \frac{\beta}{|h_j|} + \frac{|\alpha h_{ae}|^n}{|h_{ae}|} - (1 + |\alpha h_{ae}|^n)^{1-\frac{1}{n}} F(h_j)} \right\}^\gamma, & h_j < h \leq h_{ae} \\ K_s, & h > h_{ae} \end{cases} \quad (14a)$$

233

234

235 where

236

237 
$$F(h) = \frac{|\alpha h|^n (1+|\alpha h|^n)^{\frac{1}{n}-1}}{|h|} \quad (14b)$$

238

239 If only the hydraulic conductivity at saturation is available, one can use the values  
240 for  $\alpha$  and  $n$  of the water retention curve and fix parameters  $\gamma$  and  $\tau$  according to one of  
241 several conductivity models (Madi et al., 2018) to define an unsaturated hydraulic  
242 conductivity curve, but this is not recommended. It is better to have additional data points  
243 of the hydraulic conductivity curve so that parameters  $\gamma$  and  $\tau$ , and perhaps even  
244 conductivity-specific values of  $\alpha$  and  $n$ , can be fitted directly to the conductivity data.

245 Equations (14a) and (14b) have the advantage that they can be expressed in closed  
246 form but they do not account for non-capillary flow in dry soils, vapor flow, or sequences of  
247 evaporation and condensation in soils with pockets of water and soil air. Hence, they are of  
248 limited value for water flows in dry soils. For such conditions more sophisticated  
249 conductivity could be selected, for instance using the framework presented by Weber et al.  
250 (2019).

251 The conductivity function associated with the multimodal soil water retention  
252 function cannot be expressed in closed form. For that case, the degree of saturation for any  
253  $h$  can be found with Eq. (10) and the corresponding hydraulic conductivity determined with  
254 Eq. (13) or another conductivity model.

255

256 **Materials and Methods**

257        We selected 21 soils from the UNSODA database that had sufficient retention data  
258        and together covered the textures represented in UNSODA. We then fitted Eq. (5) to these  
259        soils using a Shuffled Complex Evolution algorithm (see Madi et al. (2018) and Appendix C  
260        for details of the algorithm and the fitting procedure). We slightly modified the fitting code  
261        used by Madi et al. (2018) to generate output that can more readily be converted to the  
262        MATER.IN input file for Hydrus-1D, the numerical model used in this study. We therefore  
263        refitted BCO, FSB, RNA, VGN, and VGA as well.

264        One-dimensional simulations for all combinations of three soils and three climates  
265        were carried out to examine how the choice of parameterization affected fluxes at the soil  
266        surface and in the subsoil. The model Hydrus-1D version 4.16.0110 (Šimůnek et al., 2013,  
267        PC-progress website) was used for this purpose. The selected soils were a loamy sand  
268        (UNSOADA identifier: 2104), a silty loam (3261), and a clay (1181). Weather records were  
269        generated from climate parameters based on weather data from Colombo (Sri Lanka,  
270        monsoon climate), Tamale (Ghana, semi-arid climate), and Ukkel (Belgium, temperate  
271        climate). Table 1 gives the most relevant statistics of the weather records. In order to  
272        highlight the effects of the air-entry value and the logarithmic dry end of the SWRC, we  
273        used the sigmoidal VGN, VGA, and RIA parameterizations in the simulations. The  
274        Supplement details the generation of the weather records, the set-up of the simulations,  
275        and the simulation results.

276

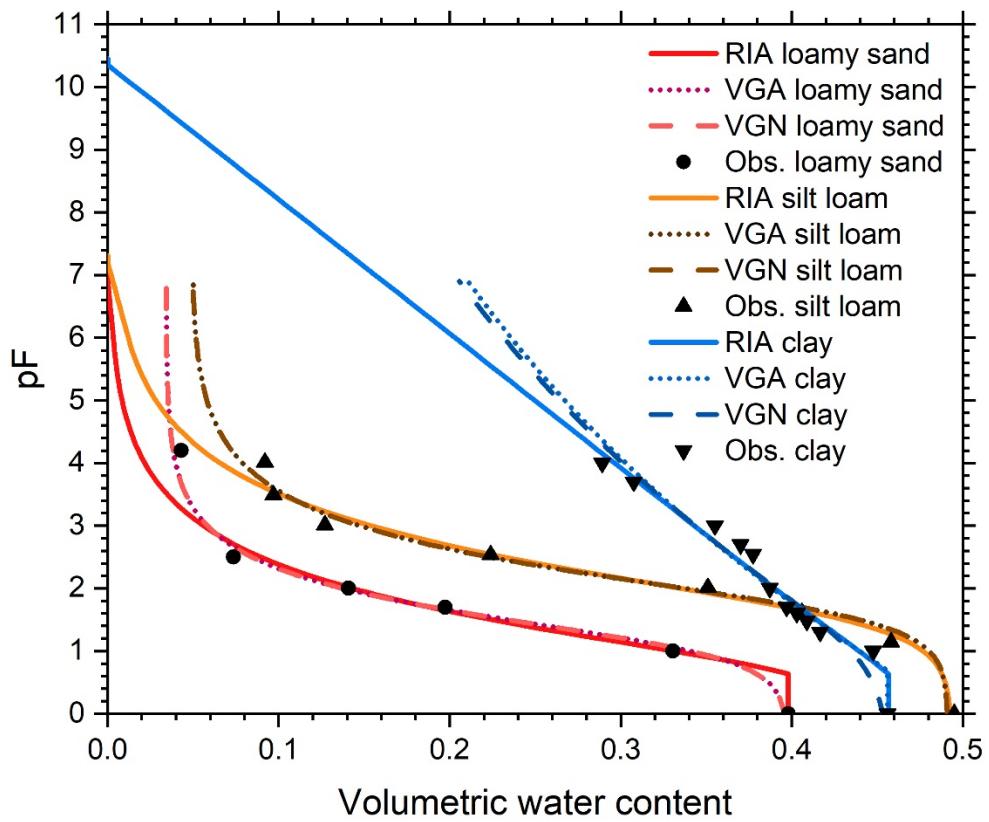
277 Table 1. Average cumulative monthly and annual rainfall and potential evapotranspiration  
 278 of the three test climates calculated from 1000-year generated weather records.

	Monsoon		Semi-arid		Temperate	
Month	Rain (mm)	$ET_{pot}$ (mm)	Rain (mm)	$ET_{pot}$ (mm)	Rain (mm)	$ET_{pot}$ (mm)
Jan	94.0	160.5	14.3	159.4	69.0	12.7
Feb	83.3	156.0	12.9	158.1	55.6	18.8
Mar	217.6	166.8	13.7	185.0	59.4	34.4
Apr	233.8	160.0	34.1	177.7	57.1	51.1
May	237.4	164.0	117.6	166.6	24.5	73.9
Jun	138.5	164.7	140.9	151.4	24.4	83.3
Jul	139.4	172.4	141.8	153.2	23.2	86.3
Aug	137.1	174.4	213.5	145.0	20.9	73.7
Sep	133.0	164.7	214.7	136.9	23.8	50.6
Oct	325.8	142.3	76.6	154.1	38.0	30.8
Nov	328.8	128.1	12.9	149.3	49.1	16.1
Dec	112.0	150.9	12.9	151.9	51.2	11.2
Annual	2180.5	1904.8	1005.8	1888.7	496.1	542.9
sum						

279

280

281



282

283 Figure 1. Soil water retention data of the soils used in the numerical simulations, and the  
 284 retention curves fitted to these data for three parameterizations with a sigmoid midsection  
 285 of the curve: the original model by van Genuchten (1980) (VGN), the modification thereof  
 286 by Ippisch et al. (2006) (VGA), and the further modification introduced in this paper (RIA).

287

288 The different parameterizations of the SWRCs (Table 2, Fig. 1) were used to generate  
 289 tables of the soil water retention and conductivity curves that were provided as input to  
 290 Hydrus-1D. Equations (14a) and (14b) with  $\gamma = 2$  and  $\tau = 0.5$  (Mualem, 1976) were used to  
 291 generate relative hydraulic conductivities (scaled by  $K_s$ ). These were converted to unscaled

292 conductivities (Fig. 2) by multiplying with the  $K_s$ -value according to the UNSODA database,  
293 and then included in the tables.

294

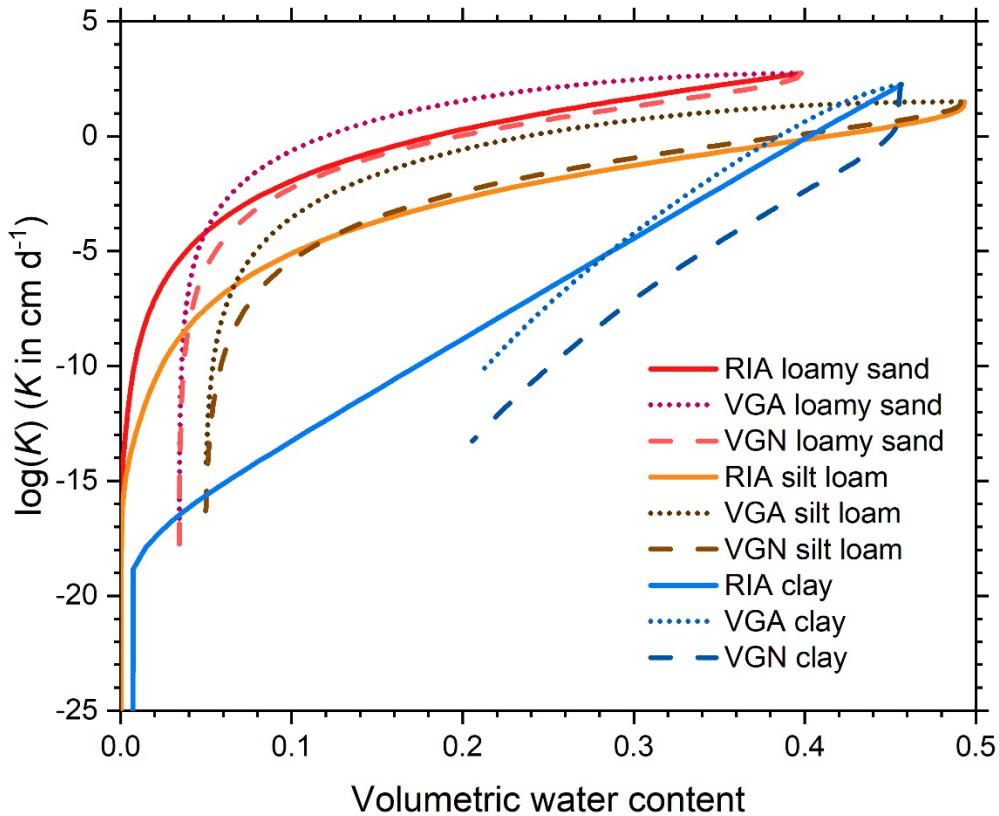
295 Table 2. Parameters of the fit to data of the three parameterizations of the soil water  
296 retention curve used for the numerical simulations.

Soil	Parameter	$\theta_r$	$\theta_s$	$\alpha$ (cm $^{-1}$ )	$n$	$h_{ae}$ (cm)	$h_j$ (cm)
Clay	RIA	–	0.45666	0.70200	1.0543	–4.2523	–205.65
	VGA	$2.12 \times 10^{-6}$	0.45603	1.7047	1.0543	–4.8324	–
	VGN	$3.34 \times 10^{-6}$	0.45616	0.14265	1.0571	–	–
Silt	RIA	–	0.49346	0.023340	1.3691	$-2.361 \times 10^{-3}$	$-1.0739 \times 10^6$
	VGA	0.048871	0.49122	0.018365	1.5158	$-2.081 \times 10^{-3}$	–
	VGN	0.048816	0.49134	0.018425	1.5149	–	–
Loamy sand	RIA	–	0.39801	0.17096	1.4106	–4.3581	$-7.7464 \times 10^5$
	VGA	0.034209	0.39771	0.069661	1.6395	–0.016234	–
	VGN	0.034133	0.39772	0.069707	1.6389	–	–

297

298

299 For the clay soil, the recorded value (178 cm d $^{-1}$ ) was such that it can be assumed  
300 that macropore flow contributed to its value. We therefore also ran simulations with a  $K_s$  –  
301 value of 1.25 cm d $^{-1}$ . This value was adopted from soil 1182 from the UNSODA database,  
302 from the same location.



305 Figure 2. The unsaturated soil hydraulic conductivity curves derived from the fitted  
 306 retention curves depicted in Fig.1 and scaled by soil-specific values of the hydraulic  
 307 conductivity at saturation.

## 309 Results and Discussion

### 310 Fitted Curves for 21 Soils

311 The fitted parameter values (Tables C1–C4) and the associated curves (Figs. C2–C5)  
 312 are presented in Appendix C. The extra parameter of RIA compared to RNA and FSB gives it  
 313 a clear advantage in fitting the full water content range. The sigmoid shape of RIA provides

314 a better fit near the air entry value while still providing good fits of the drier data points  
315 (e.g. 1121 in Fig. C3, all soils in Figs. C4–C5). In the wet range, the sigmoid curves (VGN,  
316 VGA, RIA) outperform the power–law curves (BCO, FSB, RNA). In almost all cases, VGA and  
317 RIA look very similar. Figure C3 shows multimodality in the data for five out of six soils that  
318 cannot be reproduced by any of the parameterizations.

319 Many of the UNSODA soils have one retention data point at saturation and the next  
320 at  $h = -10$  cm. The air–entry value of many sandy soils is within that range. The fitting  
321 routine struggles to fit  $h_{ae}$  to these data for VGA and RIA because the sigmoid shape of the  
322 van Genuchten parameterization has such flexibility that it can fit the intermediate range  
323 well for a range of  $h_{ae}$ –values. We therefore recommend making  $\theta(h)$ –measurements at one  
324 or two matric potentials between zero and  $-10$  cm for coarse–textured soils.

325 Eleven of the 21 soils have residual water contents for VGN and/or VGA that exceed  
326 0.05 (up to 0.263), mostly in loamy sands, loams, and clays (Tables C1–C4). All of these and  
327 several others have dry–end data points with water contents that appear too high (Figs. C2–  
328 C5). These water contents may have been overestimated due to the lack of equilibrium  
329 reported by Bittelli and Flury (2009). The parameterizations without asymptote (FSB, RNA,  
330 RIA) generally have more plausible fits based on visual inspection of the graphs, but  
331 because they do not follow the upward tail of the data in the dry end, their RMSE values for  
332 the cases with suspicious dry–end data points are larger than those of VGN and VGA (Tables  
333 C5–C8).

334 Especially for fine–textured soils, the lack of data points for air dryness or oven  
335 dryness (Figs. C3–C5) causes the fitting procedure to treat  $\theta_f$  for the asymptotic dry  
336 branches and  $h_d$  for the logarithmic dry branches as pure fitting parameters for the drier

337 range of the data. This range exceeds pF4.2 in only one case and in several cases likely  
338 suffers from lack of equilibrium (Bittelli and Flury, 2009). This leads to unrealistically high  
339 values for both in many cases. If applications are envisioned for which low water contents  
340 are expected, it would be better to have some data in the dry range and ensure equilibrium  
341 has been achieved before fitting RIA.

342 We tested this on the data of Bittelli and Flury (2009) for undisturbed samples taken  
343 at 0.15 m depth. We fitted RIA, VGA, and VGN with  $\theta_s$  fixed at 0.45 to their pressure plate  
344 data (which were unreliable in the dry range) and a combination of pressure plate data for  
345 matric potentials larger than  $-1000 \text{ cm H}_2\text{O}$  (pF3.0) and dew point data for higher pF  
346 values (Table 3). Figure 3 shows that all three parameterizations gave good fits for both  
347 data sets, and that the dry-end data points affect the entire SWRC. It is important to note  
348 that the pF value of  $h_d$  for the combination of pressure plate and dew point data equals  
349 6.428, close to the oven-dry value of pF6.8 approximated by most sample-drying ovens  
350 according to Schneider and Goss (2012).

351 If reliable data in the dry range are not available but a realistic fit in the dry range is  
352 necessary, one could add a virtual data point with zero water content at pF6.8 according to  
353 Schneider and Goss (2012) and give it a high weight to force the fitting algorithm to  
354 approximate it closely. We did not do so here to be able to observe how the various  
355 parameterizations perform on frequently reported data ranges.

356 Figure C5 showcases a peculiarity of FSB. The original version by Fayer and  
357 Simmons (1995) allowed capillary bound water to be present even if the adsorbed water  
358 content was down to zero. We adopted the correction by Madi et al. (2018, Eq. (S12a)) that  
359 forced the amount of water bound by capillary forces to zero if the adsorbed water content

360 reaches zero at pF6.8. In the case of clay soils, the parameter values are such that a  
361 substantial amount of capillary bound water is still present at pF6.8, leading to a sudden  
362 cut-off at pF6.8.

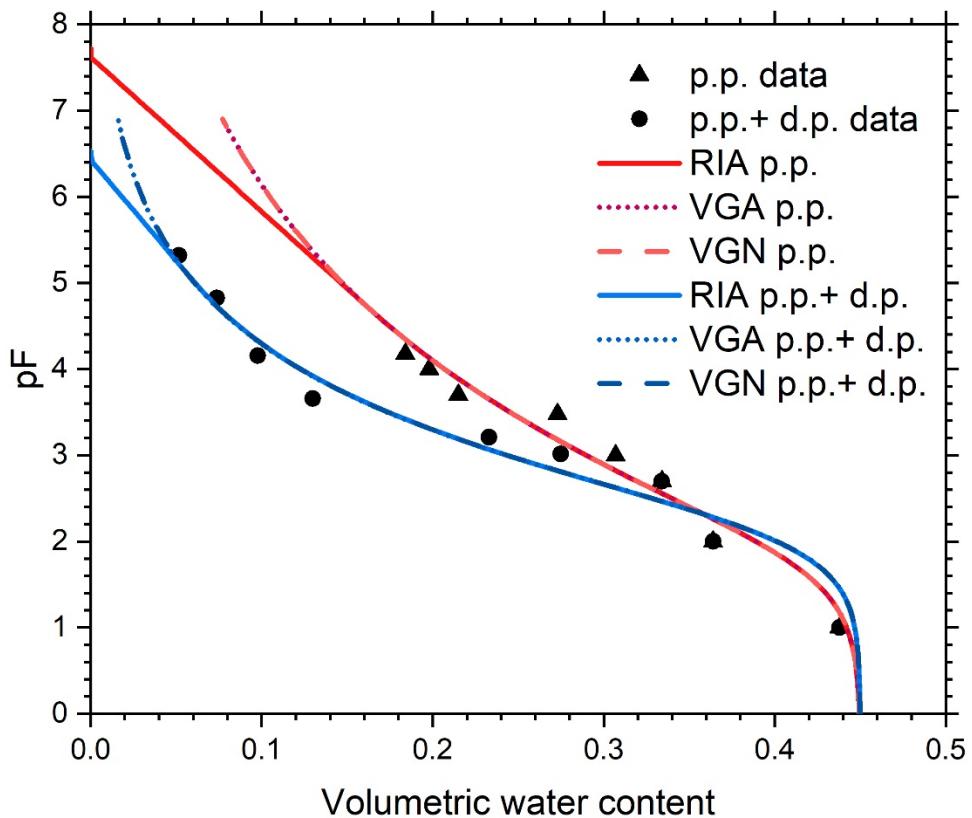
363

364 Table 3. Parameters of the fit to the pressure plate data and a combination of pressure plate  
365 and dew point data of Bittelli and Flury (2009).

Data	Parameter	$\theta_f$	$\theta_s$	$\alpha$ (cm $^{-1}$ )	$n$	$h_{ae}$ (cm)	$h_j$ (cm)
Pressure plate	RIA	–	0.45000	0.019088	1.1479	$-1.072 \times 10^{-4}$	-47876
	VGA	$2.21 \times 10^{-5}$	0.45000	0.019091	1.1479	$-3.105 \times 10^{-3}$	–
	VGN	$6.76 \times 10^{-7}$	0.45000	0.019124	1.1478	–	–
(pF < 3)	RIA	–	0.45000	$7.1162 \times 10^{-3}$	1.3041	$-1.063 \times 10^{-3}$	-99989
	VGA	$5.08 \times 10^{-6}$	0.45000	$7.0471 \times 10^{-3}$	1.3056	$-4.249 \times 10^{-3}$	–
	VGN	$1.37 \times 10^{-6}$	0.45000	$7.0448 \times 10^{-3}$	1.3053	–	–
and dew point							
(pF $\geq$ 3)							

366

367



368  
 369 Figure 3. Soil water retention data obtained by Bittelli and Flury (2009) from samples  
 370 placed on pressure plates (p.p. data) or from a combination of pressure plates for the  
 371 wettest three data points and dew point measurements for the driest six data points (p.p. +  
 372 d.p. data). The curves show the fits of the soil water retention curves according to van  
 373 Genuchten (1989) (VGN), Ippisch et al. (2006) (VGA), and Eq. (5) (RIA).

374  
 375 The hydraulic conductivity curves based on water retention curve parameters and  
 376  $K_s$  very poorly match the data in most cases (Fig. C6–C9), which confirms that it is better to  
 377 fit conductivity parameters to conductivity data instead of relying on values prescribed by  
 378 theoretical models. We note here that all but one (soil 4450, Fig. C8) sets of unsaturated

379 conductivity data were obtained in the field, while all retention data were laboratory data  
380 on drying samples. The reported  $K_s$ -values were probably measured in a separate  
381 experiment (possibly in the laboratory), in which case a mismatch between  $K_s$  and the  
382 unsaturated conductivity data is to be expected. The poor match with field data  
383 notwithstanding, the graphs can be used to study the effect of the parameterization on the  
384 shape of the  $K(\theta)$  curve. The comparison between measured and modeled shapes of the  
385 conductivity curves is inconclusive.

386 In many soils, regardless of texture, RIA's  $K(\theta)$  curve drops off much slower in the  
387 dry range than those of VGN and VGA (Fig. C6–C9), a consequence of the ability of the  
388 underlying SWRC to reach zero water content at a finite matric potential. Near saturation,  
389 RIA's  $K(\theta)$  curve often drops off sharply before leveling off, in stark contrast to that of VGA,  
390 which remains high in the wet range. Given the similarity in the  $\theta(h)$  curves of VGA and RIA,  
391 the difference in their  $K(\theta)$  curves is remarkable. RIA's  $K(\theta)$  curve is the only one that can  
392 drop off sharply near saturation, level out somewhat in the mid-range and drop ever more  
393 sharply in the dry range. It is below many of the other curves in the wet and mid-range, and  
394 above most of them in the dry range.

395 Peters et al. (2011) developed parameter constraints to ensure physically plausible  
396 shapes of the SRWC and the conductivity curve. For FSB, the criterion for non-monotonicity  
397 of the conductivity curve is not met for soil 4450 (Fig. C8), resulting in  $K$  increasing with  
398 decreasing water content near saturation.

399

400 **Model Simulations**

401 In total, 21 out of 36 combinations of soil–climate–parameterization ran to  
402 completion. Runoff did not occur in any of the successful model runs. Convergence was not  
403 achieved for any of the runs for the clay soil with the reduced  $K_s$ –value. For the clay soil  
404 with the high  $K_s$ –value, only RIA ran to completion. The discontinuity of the first derivative  
405 at the air-entry value did not cause numerical problems. In fact, the replacement of any  
406 parametric expression by a look-up table creates a discontinuity of the first derivative at  
407 every point of the look-up table.

408 Table 4 lists the main mean annual fluxes calculated from the simulation study. The  
409 median flux was produced by RIA for all combinations of soil and climate. The mean annual  
410 actual transpiration between the three parameterizations differed by more than 10% from  
411 the median only for the silt loam under a temperate climate. The actual evaporation only  
412 deviated more than 10% for the loamy sand under a semi-arid climate. The amount of  
413 water leaving the soil profile differed substantially between parameterizations for the semi-  
414 arid and temperate climate, especially for VGA (20–46% deviation from the median).

415 The daily data revealed significant differences on smaller time scales that will be  
416 relevant if reactive solute transport is of interest (see the Supplement). The fluxes  
417 generated by the VGA parameterizations responded more quickly and strongly to the  
418 rainfall signal, with VGN and RIA giving a more delayed and smooth response. The SWRCs  
419 (Fig. 1) offer no explanation for this, but Fig. 2 shows that VGA's hydraulic conductivity in  
420 the wet and intermediate water content range for all three soils is considerably higher than  
421 that of VGN and RIA, except for clay, where it is only moderately higher than RIA's and  
422 drops below RIA at a water content of 0.28. Thus, small differences between SWRCs can  
423 have a significant influence on soil water flow simulations through the effect of their

424 parameters on the soil hydraulic conductivity curve, an effect that the reviews by Leij et al.  
425 (1997) and Assouline and Or (2013) took into consideration to some degree, but was not  
426 considered in several other studies that compared different parameterizations (e.g., Rossi  
427 and Nimmo, 1994; Assouline et al., 1998; Cornelis et al., 2005; Khlosi et al., 2008).

428 Table 4. Average of the annual sums of the actual transpiration and evaporation, and of the  
 429 outflow across the lower boundary of the simulated soil profiles. The averages were  
 430 calculated for the final six years of the simulation periods. The values in parentheses are  
 431 scaled with respect to the corresponding value for RIA.

Climate	Soil	Actual transpiration			Actual evaporation			Downward flux at 2 m depth (mm)		
		RIA	VGA	VGN	RIA	VGA	VGN	RIA	VGA	VGN
Monsoon	clay	962	—	—	543	—	—	876	—	—
	silt	1114	1044	1112	541	554	534	711	763	718
	loam		(0.94)	(1.06)		(1.02)	(0.96)		(1.07)	(0.94)
	loamy	1037	976	1024	480	440	461	863	959	896
	sand		(0.94)	(1.05)		(0.92)	(1.05)		(1.11)	(0.93)
Semi-arid	clay	585	—	—	319	—	—	294	—	—
	silt	657	588	653	297	303	287	164	226	177
	loam		(0.90)	(1.11)		(1.02)	(0.95)		(1.37)	(0.78)
	loamy	585	540	572	248	222	234	290	363	319
	sand		(0.92)	(1.06)		(0.89)	(1.06)		(1.25)	(0.88)
Temperate	clay	202	—	—	163	—	—	167	—	—
	silt	281	236	271	150	151	143	99	144	114
	loam		(0.84)	(1.15)		(1.01)	(0.95)		(1.46)	(0.79)
	loamy	224	205	214	130	118	124	174	208	190
	sand		(0.92)	(1.04)		(0.91)	(1.05)		(1.20)	(0.91)

433 **Summary and Conclusions**

434 The improvements incorporated in the RIA parameterization for the first time  
435 remove problems of the popular VGN model in both the wet and the dry range while  
436 retaining the desirable sigmoid shape in the mid-range. This shape allows its multimodal  
437 version to represent SWRCs with multiple humps. RIA offers a wider range of shapes for the  
438 conductivity curve than any other parameterization that does not lead to the unphysical  
439 behavior near saturation that was revealed by Durner (1994) and Ippisch et al. (2006) for  
440 VGN and by Madi et al. (2018) for 14 other parameterizations. RIA also proved to be more  
441 robust during numerical simulations than VGN itself as well as its modification VGA, which  
442 still has a non-physical asymptote at a non-zero residual water content. The deep drainage  
443 generated by RIA was more spread out and smaller than the spiked response to rainfall  
444 produced by VGA, probably because RIA was better able to produce a conductivity curve  
445 with a substantial drop during the early stages of drying. We therefore hope that RIA or its  
446 multimodal version will be adopted for use in numerical simulations of soil water flow. The  
447 catalogue of parameters for 21 soils in Appendix C may be of help for such simulations.

448 **Appendix A. List of Abbreviations**

449

450 BC: Brooks and Corey (1964)

451 BCO: parameterization of the SWRC according to the original Brooks and Corey (1964)

452 equation

453 FSB: parameterization of the SWRC according to the BC-based version of Fayers and

454 Simmons (1995)

455 RIA: parameterization of the SWRC that combines RNA and VGA

456 RNA: parameterization of the SWRC according to the junction model of Rossi and Nimmo  
457 (1994)

458 SWRC: soil water retention curve

459 RMSE: root mean square error

460 UNSODA: unsaturated soil hydraulic properties database

461 VGA: parameterization of the SWRC according to Ippisch et al. (2016)

462 VGN: parameterization of the SWRC according to the original van Genuchten (1980)

463 equation

464

465 **Appendix B. Assessing the near-saturation behavior of recently  
466 developed soil water retention and hydraulic conductivity curves**

467

468 Madi et al. (2018) developed a criterion that needs to be met to avoid unphysical  
469 behavior of the unsaturated soil hydraulic conductivity curve near saturation:

470

471 
$$\lim_{h \uparrow 0} \left( |h|^{-\kappa} \frac{d\theta}{dh} \right) = 0 \quad (B1)$$

472

473 where  $\kappa$  is a fitting parameter ( $> 0$ ) that appears in a several parameterizations of the  
474 unsaturated soil hydraulic conductivity curve based on the capillary bundle  
475 conceptualization.

476 Fredlund and Xing (1994) introduced the following parameterization for the soil  
477 water retention curve:

478

479 
$$\theta(h) = \left[ \frac{-\ln(1 + \frac{|h|}{|h_r|})}{\ln(1 + \frac{b}{|h_r|})} + 1 \right] \frac{\theta_s}{\left\{ \ln \left[ e + \left( \frac{|h|}{a} \right)^n \right] \right\}^m} \quad (B2)$$

480

481 where the subscript 'r' denotes the value when the residual water content is reached, and  $a$   
482  $[L]$ ,  $n$ , and  $m$  are fitting parameters. The first term on the right-hand side is a correction  
483 term that forces the water content to zero for  $h = -b$ , with  $b$  equal to  $10^7$  cm (Fredlund and  
484 Xing, 1994) or  $6.3 \times 10^6$  cm (Wang et al., 2016). Wang et al. (2016) also modified the  
485 correction factor to give

486

$$487 \quad \theta(h) = \left[ \frac{-\ln(1+\frac{c|h|}{|h_r|})}{\ln(1+\frac{bc}{|h_r|})} + 1 \right] \frac{\theta_s}{\left\{ \ln \left[ e + \left( \frac{|h|}{a} \right)^n \right] \right\}^m} \quad (B3)$$

488

489 where  $c$  has to be small and positive. The derivative of Eq. (B3) is

490

$$491 \quad \frac{d\theta}{dh} = \frac{-\theta_s}{\left\{ \ln \left[ e + \left( \frac{|h|}{a} \right)^n \right] \right\}^m} \left\{ \frac{mn|h|^{n-1} \left[ \ln \left( 1 + \frac{bc}{|h_r|} \right) \ln \left( 1 + \frac{c|h|}{|h_r|} \right) - 1 \right]}{a^n \left[ e + \left( \frac{|h|}{a} \right)^n \right] \ln \left[ e + \left( \frac{|h|}{a} \right)^n \right]} - \frac{\ln \left( 1 + \frac{bc}{|h_r|} \right)}{\frac{|h_r|}{c} + |h|} \right\} \quad (B4)$$

492

493 The derivative of Eq. (B2) follows by setting  $c$  equal to 1 in Eq. (B4). Combining Eq.(B4)  
 494 with Eq. (B1) results in the following requirement:

495

$$496 \quad \lim_{h \uparrow 0} \left[ \frac{\theta_s mn}{ea^n} |h|^{-\kappa+n-1} + \frac{c\theta_s \ln(1+\frac{bc}{|h_r|})}{|h_r|} |h|^{-\kappa} \right] = 0 \quad (B5)$$

497

498 The limit goes to zero if and only if the exponents in both terms are positive. Hence,  $\kappa <$   
 499  $n-1$  and  $\kappa < 0$ . The first requirement may be met for some soils, but the second violates  
 500 the physical constraint that  $\kappa$  cannot be negative (Madi et al., 2018). Therefore, neither  
 501 Fredlund and Xing's (1994) nor Wang et al.'s (2016) parameterization lead to unsaturated  
 502 hydraulic conductivity curves that exhibit physically realistic behavior near saturation.  
 503 Wang et al. (2018) added a modification in the dry end of Wang et al. (2016), and  
 504 Rudiyanto et al. (2020) in turn used Wang et al.'s (2018) curves. Because the problem near  
 505 saturation was not resolved, these two hydraulic conductivity models suffer from the same  
 506 problem near saturation.

507 **Appendix C. Fitted Parameters and Root Mean Square Error for Six**  
508 **Parameterizations of the Soil Water Retention Curve applied to Data**  
509 **of 21 Soils**

510

511        The UNSODA soils selected for parameter fitting are grouped in Tables C1–C4  
512        according to their texture classification according to Twarakavi et al. (2010). Sand is a  
513        major constituent of the mineral soil in Tables C1 and C2, silt in Table C3, and clay in Table  
514        C4. Figure C1 shows the textural composition of the soils. For each soil, the parameter  
515        values for six parameterizations are given, resulting in a total of 126 SWRC  
516        parameterizations. The Root Mean Square Errors (RMSE) for all fits are listed in Tables C5–  
517        C8. The saturated hydraulic conductivities of the soils as given by the UNSODA database are  
518        given in Table C9.

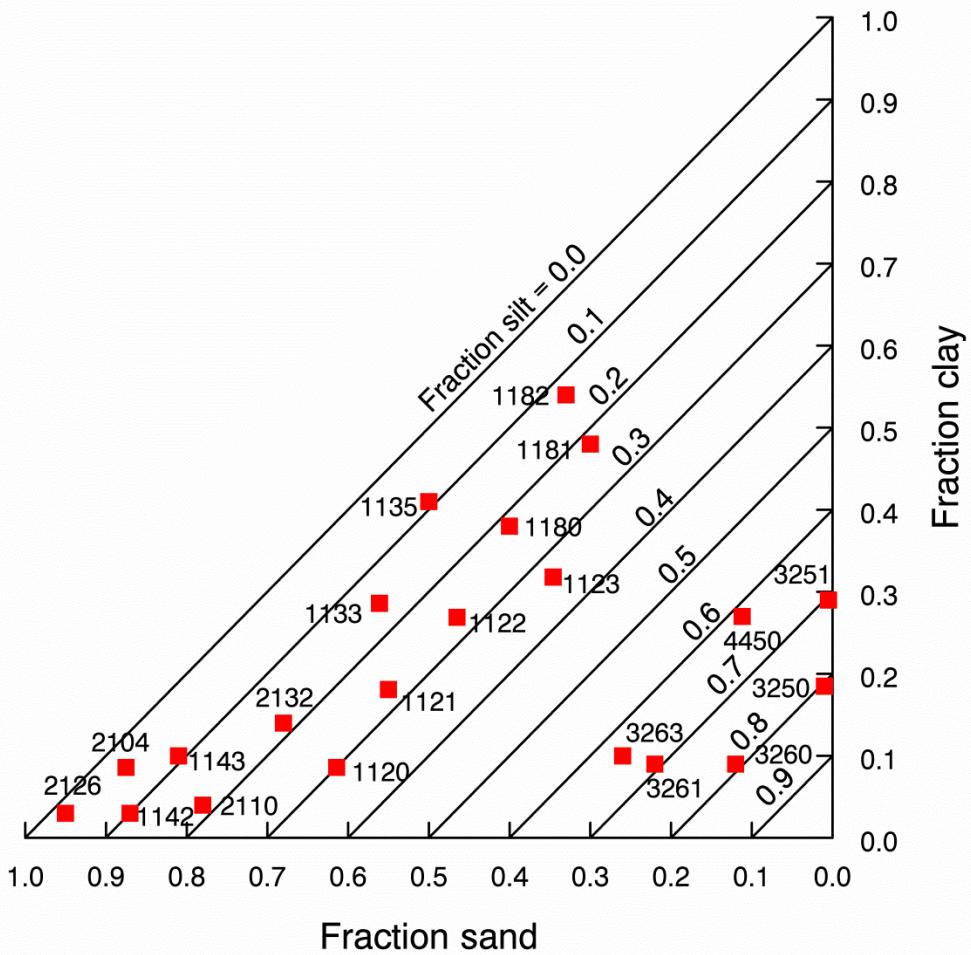
519        Madi et al. (2018) provide an analysis of the underlying functions of all  
520        parameterizations tested here, except RIA. The meaning of all variables except  $\lambda$  is  
521        explained in the main text. Variable  $\lambda$  is the power to which the factor  $(h / h_{ae})$  is raised in  
522        the power-law segments of the SWRCs of BCO, FSB, and RNA. In RNA,  $\lambda$  is expressed as a  
523        function of the fitting parameters, and therefore does not appear in the tables.

524        The SWRCs defined by these parameterizations together with the data points on  
525        which they are based are given in Figs. C2–C5. The hydraulic conductivity curves according  
526        to Mualem (1976) that can be derived from the parameterizations (Eqs. (14a) and (14b))  
527        for RIA, equations for the other parameterizations in Madi et al. (2018)) are plotted in Figs.  
528        C6–C9. We plotted  $K$  as a function of  $\theta$  because this relationship is less hysteretic than  $K(h)$

529 (Koorevaar et al., 1983, p. 141) and some of the UNSODA data only provided  $K(\theta)$  data  
530 points. The conductivity data available in UNSODA are also plotted, but these were not used  
531 to fit the curves to. The maximum pF value for which points on the curves were calculated  
532 was set at 6.8 for VGN and VGA, and to the pF value corresponding to the fitted value of  $h_d$   
533 for RIA.

534 The comparison of theoretical conductivity curves based on water retention data  
535 with the measured values showed a sometimes substantial deviation between hydraulic  
536 conductivities measured at saturation and the  $K_s$  value used to create the theoretical curves.  
537 The latter value was obtained by a query that made the database return a  $K_s$  value for the  
538 soils that met all other criteria also included in the query. The conductivity data displayed  
539 in the plots were separately obtained by requesting the database to return a report of  
540 tabular data for each of the 21 soils. The reasons for the discrepancies between the queried  
541 value and the tabulated may reflect separate experiments for measuring unsaturated and  
542 saturated values of  $K$ .

543 There also are obvious differences between the water content at saturation between  
544 the retention and the conductivity data. In all but one case (soil 4450, Fig. C8) the hydraulic  
545 conductivity observations were made in the field whereas the retention data used were all  
546 obtained from drying experiments in the laboratory. Scale differences, effects of air  
547 enclosure and hysteresis in the field, and differences between different measurement  
548 techniques explain these differences. We can therefore only compare the shape of the  
549 theoretical curves with those of the data clouds.



550

551 Figure C1. The positions in the texture triangle of the selected soils, indicated by their  
 552 UNSODA identifier. (Taken from Madi et al., 2018)

553

554 The RMSE was calculated from the weighted sum of squares of the differences  
555 between calculated and observed water contents and pressure heads. The weights equaled  
556 the estimated scaled standard deviations of the individual water retention observations  
557 (pairs of matric potential and water content values). The standard deviations of the water  
558 content observations were scaled to have an average value of 0.2. The scale factor needed to  
559 arrive at this value was then applied to the standard deviations of the matric potential  
560 values as well. This ensured that the weighting of water contents and matric potential  
561 values was consistent with the original standard deviations of both. The scaling greatly  
562 improved the efficiency of the parameter fitting procedure. The squared difference between  
563 a single observed water content and its fitted value during a given iteration of the  
564 parameter fitting algorithm, taking into account observation errors in both the water  
565 content and the matric potential, is

566

$$567 \left( \frac{\theta_{\text{fit}} - \theta_{\text{obs}}}{\sigma_{\theta, \text{scaled}} + \sigma_{h, \text{scaled}} \left. \frac{d\theta}{dh} \right|_{h_{\text{obs}}}} \right)^2 \quad (C1)$$

568

569 where  $\sigma_{a, \text{scaled}}$  is the scaled standard deviation of the variable  $a$ , subscript 'fit' signifies a  
570 fitted value of the subscripted variable, and subscript 'obs' a measured value (Madi et al.,  
571 2018). The slope of the SWRC in the denominator is estimated from the fitted  
572 parameterization using the parameter values that have been fitted in the iteration that is  
573 currently being tested.

574 The scale factors applied to the observation error standard deviations varied  
575 between soils but not between parameterizations fitted to the same soil. RMSE values of

576 different parameterizations valid for a particular soil can therefore be readily compared.  
577 Comparisons between soils only give a rough indication. When the water contents at which  
578 measurements were taken differ strongly from one soil to another, the comparison of their  
579 RMSE values is less reliable.

580 In case the measured water contents were obtained at hydrostatic equilibrium, the  
581 fitted water content calculated directly from the matric potential could not be compared to  
582 the observed water content. To approximate the soil sample on which the observation was  
583 made, a hypothetical soil slab of the same height as the sample was divided into 20  
584 horizontal layers. If UNSODA did not specify the sample height, it was assumed to be 5.0 cm.  
585 The matric potential in the center of each layer was determined from the given matric  
586 potential, which was assumed to apply to the center of the sample. The water content of the  
587 soil slab was then calculated as the average water content of its 20 layers. This water  
588 content was used to calculate the difference between the observed and the fitted water  
589 content. Figure C10 shows a comparison of retention points calculated for three soils based  
590 directly on the RIA parameterization and based on the same parameterization applied to a  
591 hypothetical sample of 5.0 cm height at hydrostatic equilibrium with the nominal matric  
592 potential valid at the sample center. Deviations are small, even for the loamy sand.

593 Table C1. The fitting parameters and their values for six parameterizations for sandy soils in the A1  
 594 and A2 classifications of Twarakavi et al. (2010) from the UNSODA database (National Agricultural  
 595 Library, 2017; Nemes et al., 2001). The three-character parameterization label is explained in the  
 596 main text.

		Soil (UNSDA identifier and classification according to Twarakavi et al. (2010))		
		2126 A1	1142 A2	2104 A2
Parameterization	Parameter	Unit		
BCO	$\theta_r$	-	1.63E-2	4.92E-5
	$\theta_s$	-	0.377	0.250
	$h_{ae}$	cm	-6.78	-7.00
	$\lambda$	-	0.846	0.210
FSB	$\theta_s$	-	0.377	0.250
	$\theta_a$	-	2.58E-2	6.26E-5
	$h_{ae}$	cm	-6.76	-7.00
	$\lambda$	-	0.861	0.211
RNA	$\theta_s$	-	0.378	0.250
	$h_{ae}$	cm	-6.37	-7.00
	$h_l$	cm	-8.87E4	-9.24E4
	$h_d$	cm	-3.60E5	-1.07E7
VGN	$\theta_r$	-	3.39E-2	9.42E-2
	$\theta_s$	-	0.376	0.242
	$\alpha$	cm <sup>-1</sup>	6.85E-2	1.99E-2
	$n$	-	2.73	2.93
VGA	$\theta_r$	-	3.39E-2	9.64E-2
	$\theta_s$	-	0.376	0.242
	$\alpha$	cm <sup>-1</sup>	6.84E-2	1.98E-2
	$n$	-	2.73	3.05
RIA	$h_{ae}$	cm	-0.101	-0.240
	$\theta_s$	-	0.378	0.245
	$\alpha$	cm <sup>-1</sup>	0.239	2.68E-2
	$n$	-	1.77	1.47
	$h_{ae}$	cm	-5.82	-7.00
	$h_l$	cm	-1.76E6	-4.35E5

598 Table C2. The fitting parameters and their values for six parameterizations for sandy soils in the A3  
 599 and A4 classifications of Twarakavi et al. (2010) from the UNSODA database (National Agricultural  
 600 Library, 2017; Nemes et al., 2001). The three-character parameterization label is explained in the  
 601 main text.

		Soil (UNSOADA identifier and classification according to Twarakavi et al. (2010))					
		1120 A3	1143 A3	2110 A3	2132 A3	1121 A4	1133 A4
Para-meteri-zation	Para-me-ter	Unit					
BCO	$\theta_r$	-	6.15E-6	2.71E-5	0.103	4.10E-5	2.64E-5
	$\theta_s$	-	0.311	0.279	0.348	0.303	0.350
	$h_{ae}$	cm	-10.0	-7.00	-25.5	-8.00	-10.0
	$\lambda$	-	0.204	0.169	0.537	0.107	0.118
FSB	$\theta_s$	-	0.311	0.279	0.348	0.308	0.346
	$\theta_a$	-	4.33E-5	4.26E-4	0.213	0.298	0.324
	$h_{ae}$	cm	-10.0	-7.00	-25.7	-3.24	-10.0
	$\lambda$	-	0.204	0.169	0.763	0.422	0.377
RNA	$\theta_s$	-	0.311	0.279	0.351	0.303	0.352
	$h_{ae}$	cm	-10.0	-7.00	-20.3	-8.00	-10.0
	$h_j$	cm	-7.32E4	-8.46E4	-6.44E4	-7.18E4	-8.73E4
	$h_d$	cm	-9.95E6	-3.18E7	-2.35E6	-8.06E8	-8.68E8
VGN	$\theta_r$	-	7.22E-2	9.24E-2	0.126	1.25E-4	1.70E-5
	$\theta_s$	-	0.305	0.278	0.360	0.305	0.339
	$\alpha$	cm <sup>-1</sup>	1.72E-2	4.66E-2	2.63E-2	5.73E-2	7.21E-3
	$n$	-	1.69	1.49	1.84	1.14	1.26
VGA	$\theta_r$	-	7.37E-2	0.112	0.106	6.61E-4	1.47E-5
	$\theta_s$	-	0.303	0.276	0.348	0.306	0.339
	$\alpha$	cm <sup>-1</sup>	1.72E-2	4.41E-2	0.230	6.06E-2	7.16E-3
	$n$	-	1.71	1.66	1.56	1.14	1.27
	$h_{ae}$	cm	-9.37	-6.81	-25.3	-8.47E-4	-2.73E-2
RIA	$\theta_s$	-	0.308	0.280	0.360	0.306	0.339
	$\alpha$	cm <sup>-1</sup>	3.01E-2	6.39E-2	4.18E-2	6.08E-2	7.13E-2
	$n$	-	1.29	1.23	1.33	1.14	1.27
	$h_{ae}$	cm	-1.24E-3	-4.96E-4	-7.85	-7.44E-4	-9.11E-4
	$h_j$	cm	-1.71E6	-7.01E6	-9.37E6	-7.81E6	-2.04E6

602 Table C3. The fitting parameters and their values for six parameterizations for silty soils from the  
 603 UNSODA database (National Agricultural Library, 2017; Nemes et al., 2001). The three-character  
 604 parameterization label is explained in the main text.

		Soil (UNSOADA identifier and classification according to Twarakavi et al. (2010))						
		3260 B2	3261 B2	3263 B2	3250 B4	3251 B4	4450 B4	
Para-meteri-zation	Para-me-ter	Unit						
BCO	$\theta_r$	-	2.85E-6	3.42E-6	3.85E-7	3.85E-6	1.94E-6	2.77E-5
	$\theta_s$	-	0.470	0.499	0.460	0.540	0.500	0.380
	$h_{ae}$	cm	-28.6	-13.5	-28.8	-30.5	-18.2	-4.80
	$\lambda$	-	0.281	0.256	0.255	0.183	9.56E-2	9.51E-2
FSB	$\theta_s$	-	0.470	0.499	0.460	0.540	0.500	0.380
	$\theta_a$	-	1.33E-5	6.49E-5	1.33E-5	0.173	0.431	0.320
	$h_{ae}$	cm	-28.6	-13.5	-28.8	-30.0	-10.9	-0.882
	$\lambda$	-	0.281	0.256	0.255	0.241	0.197	0.196
RNA	$\theta_s$	-	0.470	0.499	0.460	0.540	0.500	0.380
	$h_{ae}$	cm	-28.6	-13.5	-28.8	-30.5	-18.2	-4.81
	$h_j$	cm	-9.23E4	-9.43E4	-7.48E4	-7.87E4	-1.97E4	-2.63E4
	$h_d$	cm	-3.23E6	-4.96E6	-3.75E6	-6.86E6	-7.66E8	-9.69E8
VGN	$\theta_r$	-	5.25E-2	4.88E-2	4.48E-2	3.10E-2	1.60E-5	1.16E-6
	$\theta_s$	-	0.472	0.491	0.461	0.540	0.501	0.379
	$\alpha$	cm <sup>-1</sup>	1.62E-2	1.84E-2	1.53E-2	1.21E-2	2.62E-2	0.164
	$n$	-	1.47	1.51	1.41	1.28	1.11	1.10
VGA	$\theta_r$	-	5.27E-2	4.89E-2	6.12E-2	3.80E-4	7.21E-5	2.80E-5
	$\theta_s$	-	0.472	0.491	0.457	0.540	0.500	0.379
	$\alpha$	cm <sup>-1</sup>	1.62E-2	1.84E-2	1.49E-2	1.33E-2	3.66E-2	1.26
	$n$	-	1.47	1.52	1.46	1.25	1.11	1.10
	$h_{ae}$	cm	-3.26E-3	-2.08E-3	-15.1	-4.87	-7.31	-4.54
RIA	$\theta_s$	-	0.474	0.493	0.463	0.540	0.500	0.379
	$\alpha$	cm <sup>-1</sup>	2.04E-2	2.33E-2	1.86E-2	1.33E-2	3.57E-2	0.164
	$n$	-	1.33	1.37	1.31	1.25	1.11	1.10
	$h_{ae}$	cm	-5.95E-3	-2.36E-3	-2.43E-3	-4.80	-7.12	-1.21E-3
	$h_j$	cm	-1.49E6	-1.07E6	-8.62E6	-8.02E6	-8.33E6	-9.89E6

605

606 Table C4. The fitting parameters and their values for six parameterizations for clayey soils from the  
 607 UNSODA database (National Agricultural Library, 2017; Nemes et al., 2001). The three-character  
 608 parameterization label is explained in the main text.

		Soil (UNSOADA identifier and classification according to Twarakavi et al. (2010))						
			1135 C2	1182 C2	1122 C4	1123 C4	1180 C4	1181 C4
Para-meteri-zation	Para-meter	Unit						
BCO	$\theta_r$	-	3.94E-4	1.79E-4	2.64E-4	2.13E-4	5.22E-4	1.24E-5
	$\theta_s$	-	0.420	0.549	0.362	0.358	0.497	0.456
	$h_{ae}$	cm	-106	-0.977	-10.0	-10.0	-11.1	-5.17
	$\lambda$	-	7.85E-2	4.41E-2	3.37E-2	2.69E-2	5.65E-2	5.40E-2
FSB	$\theta_s$	-	0.420	0.548	0.360	0.356	0.495	0.456
	$\theta_a$	-	0.400	0.307	0.350	0.340	0.491	0.345
	$h_{ae}$	cm	-106	-0.230	-5.75	-10.0	-8.58	-13.2
	$\lambda$	-	0.172	5.64E-2	6.60E-2	5.69E-2	100	8.08E-2
RNA	$\theta_s$	-	0.420	0.549	0.370	0.370	0.497	0.456
	$h_{ae}$	cm	-106	-3.61	-9.99	-9.99	-0.150	-7.66
	$h_j$	cm	-107	-12.2	-10.5	-10.7	-24.1	-22.1
	$h_d$	cm	-1.64E8	-1.00E9	-1.00E9	-1.00E9	-1.00E9	-1.00E9
VGN	$\theta_r$	-	0.263	8.94E-6	1.16E-4	0.210	0.255	3.34E-6
	$\theta_s$	-	0.413	0.548	0.359	0.354	0.496	0.456
	$\alpha$	cm <sup>-1</sup>	1.02E-3	0.753	1.37E-2	2.92E-3	0.805	0.143
	$n$	-	2.37	1.05	1.05	1.21	1.26	1.06
VGA	$\theta_r$	-	0.263	1.19E-5	5.78E-2	0.188	2.63E-2	2.12E-6
	$\theta_s$	-	0.413	0.548	0.359	0.354	0.498	0.456
	$\alpha$	cm <sup>-1</sup>	1.02E-3	1.21	1.37E-2	3.22E-3	10.1	1.70
	$n$	-	2.37	1.05	1.07	1.17	1.06	1.05
	$h_{ae}$	cm	-1.03	-0.467	-4.18E-2	-10.0	-1.11E-2	-4.83
RIA	$\theta_s$	-	0.416	0.548	0.359	0.354	0.497	0.457
	$\alpha$	cm <sup>-1</sup>	1.86E-3	1.31	1.39E-2	4.00E-3	14.2	0.702
	$n$	-	1.16	1.05	1.05	1.06	1.06	1.05
	$h_{ae}$	cm	-106	-0.525	-3.80E-3	-9.99	-4.41E-2	-4.25
	$h_j$	cm	-7.57E6	-9.97E6	-9.56E4	-4.43E6	-415	-206

609

610 Table C5. Root mean square errors of the parameter fits for the sandy or loamy soils (A1 and A2  
611 soils according to Twarakavi et al., 2010)

Parameterization	Soil (UNSOADA identifier and classification according to Twarakavi et al. (2010))		
	2126 A1	1142 A2	2104 A2
BCO	0.0620	0.0990	0.0481
FSB	0.0626	0.0990	0.0517
RNA	0.0659	0.0989	0.0553
VGN	0.0330	0.0252	0.0278
VGA	0.0330	0.0250	0.0278
RIA	0.0652	0.0504	0.0542

612

613

614 Table C6. Root mean square errors of the parameter fits for the sandy soils (A3 and A4 soils  
 615 according to Twarakavi et al., 2010)

Parameterization	Soil (UNSOADA identifier and classification according to Twarakavi et al. (2010))					
	1120 A3	1143 A3	2110 A3	2132 A3	1121 A4	1133 A4
BCO	0.0926	0.0501	0.0445	0.0356	0.1288	0.0803
FSB	0.0926	0.0500	0.0445	0.0292	0.1054	0.0700
RNA	0.0926	0.0500	0.0457	0.0356	0.1286	0.0775
VGN	0.0446	0.0333	0.0378	0.0204	0.0720	0.0175
VGA	0.0489	0.0396	0.0445	0.0203	0.0720	0.0175
RIA	0.0643	0.0346	0.0491	0.0203	0.0720	0.0530

616

617

618 Table C7. Root mean square errors of the parameter fits for the silty soils.

Parameterization	Soil (UNSOADA identifier and classification according to Twarakavi et al. (2010))					
	3260	3261	3263	3250	3251	4450
	B2	B2	B2	B4	B4	B4
BCO	0.0793	0.1316	0.0973	0.0822	0.0551	0.0499
FSB	0.0794	0.1316	0.0973	0.0815	0.0395	0.0445
RNA	0.0793	0.1316	0.0973	0.0822	0.0551	0.0499
VGN	0.0456	0.0607	0.0638	0.0413	0.0474	0.0485
VGA	0.0455	0.0607	0.0769	0.0412	0.0466	0.0497
RIA	0.0543	0.0698	0.0668	0.0412	0.0466	0.0485

619

620

621 Table C8. Root mean square errors of the parameter fits for the clayey soils.

Parameterization	Soil (UNSOADA identifier and classification according to Twarakavi et al. (2010))					
	1135	1182	1122	1123	1180	1181
	C2	C2	C4	C4	C4	C4
BCO	0.0913	0.0494	0.0349	0.0489	0.0187	0.0428
FSB	0.0721	0.0441	0.0212	0.0321	0.1196	0.0360
RNA	0.0812	0.0913	0.1235	0.1501	0.0347	0.0570
VGN	0.0208	0.0488	0.0197	0.0244	0.0411	0.0433
VGA	0.0208	0.0485	0.0198	0.0243	0.0197	0.0429
RIA	0.0519	0.0485	0.0197	0.0244	0.0180	0.0391

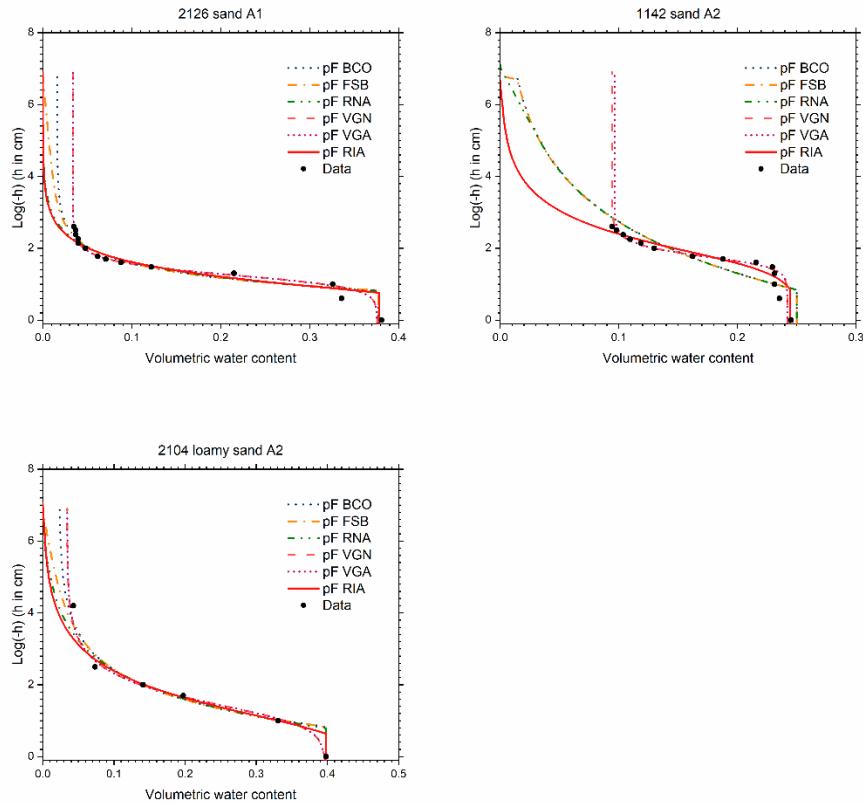
622

623

624 Table C9. Values for the hydraulic conductivity at saturation ( $K_s$ ) for the selected soils from  
625 the UNSODA database. The soils are identified by their UNSODA identifier. Their classification  
626 according to Twarakavi et al. (2010) is also given.

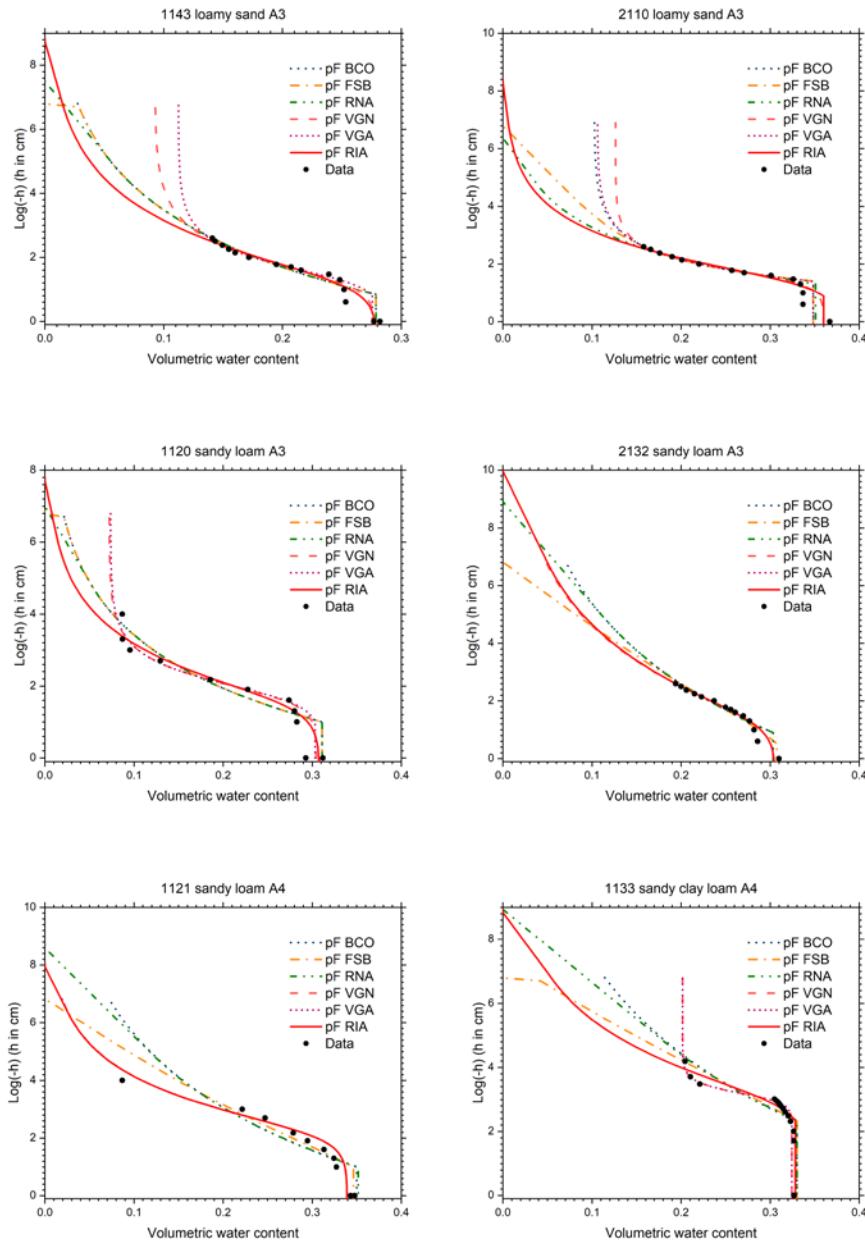
UNSOADA identifier	Texture classification	$K_s$ (cm d <sup>-1</sup> )
2126	A1	1.10E3
1142	A2	13.4
2104	A2	553
1120	A3	37.9
1143	A3	23.5
2110	A3	16.3
2132	A3	5.52
1121	A4	7.13
1133	A4	7.13
3260	B2	10.8
3261	B2	32.0
3263	B2	54.0
3250	B4	1.51
3251	B4	2.74
4450	B4	1.20
1135	C2	0.142
1182	C2	1.25
1122	C4	2.92
1123	C4	0.740
1180	C4	215
1181	C4	178

627



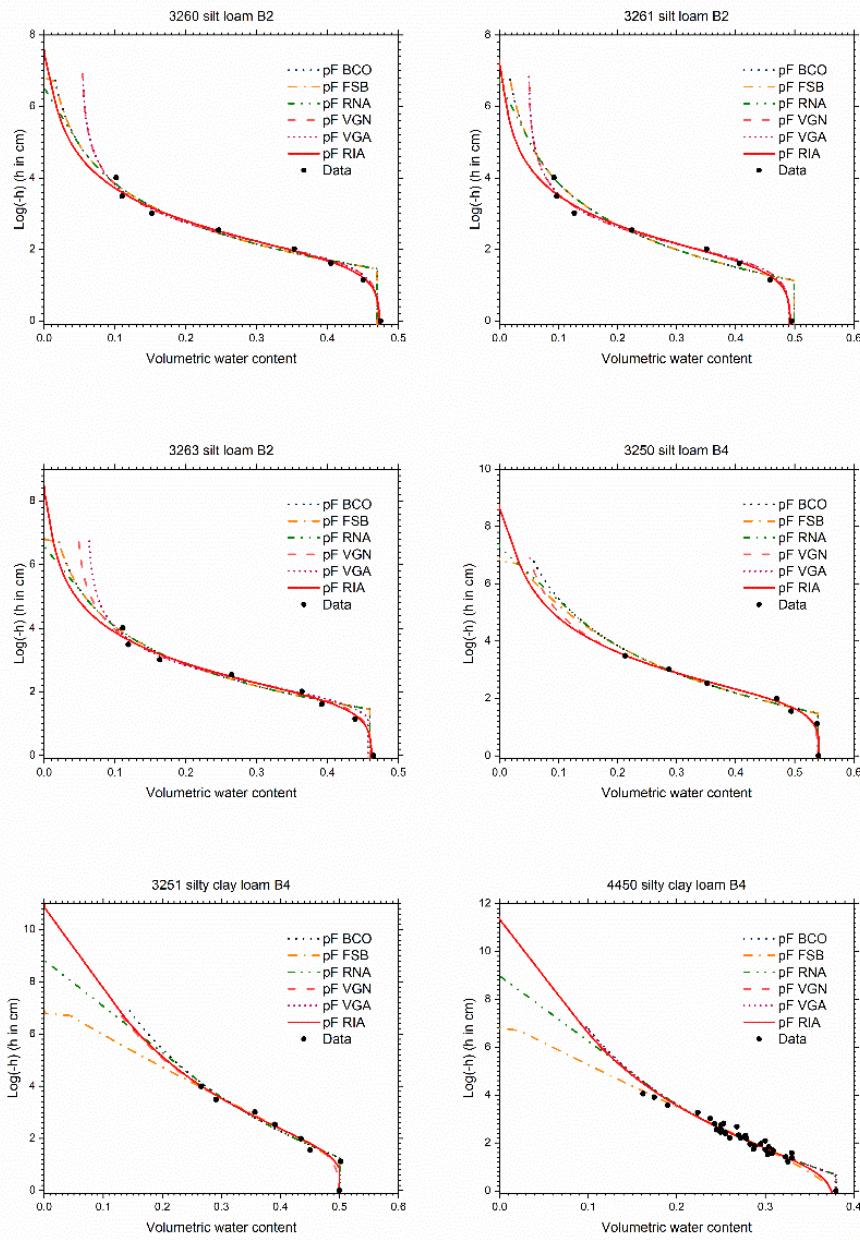
628

629 Figure C2. Retention data and fitted soil water retention curves according to six  
 630 parameterizations for selected UNSODA soils with Twarakavi et al.'s (2010) A1 or A2  
 631 classification. The parameterizations are explained in the text.



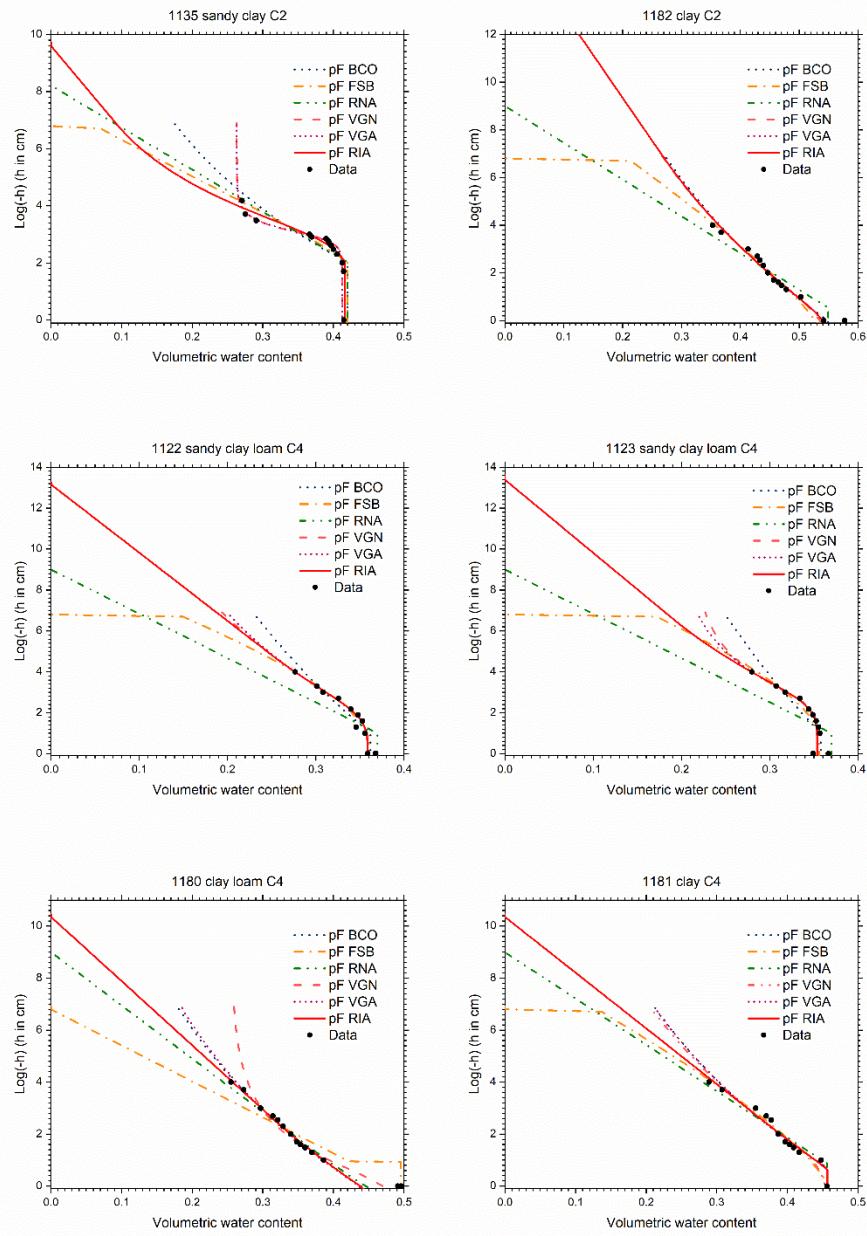
632

633 Figure C3. Retention data and fitted soil water retention curves according to six  
 634 parameterizations for selected UNSODA soils with Twarakavi et al.'s (2010) A3 or A4  
 635 classification. The parameterizations are explained in the text.



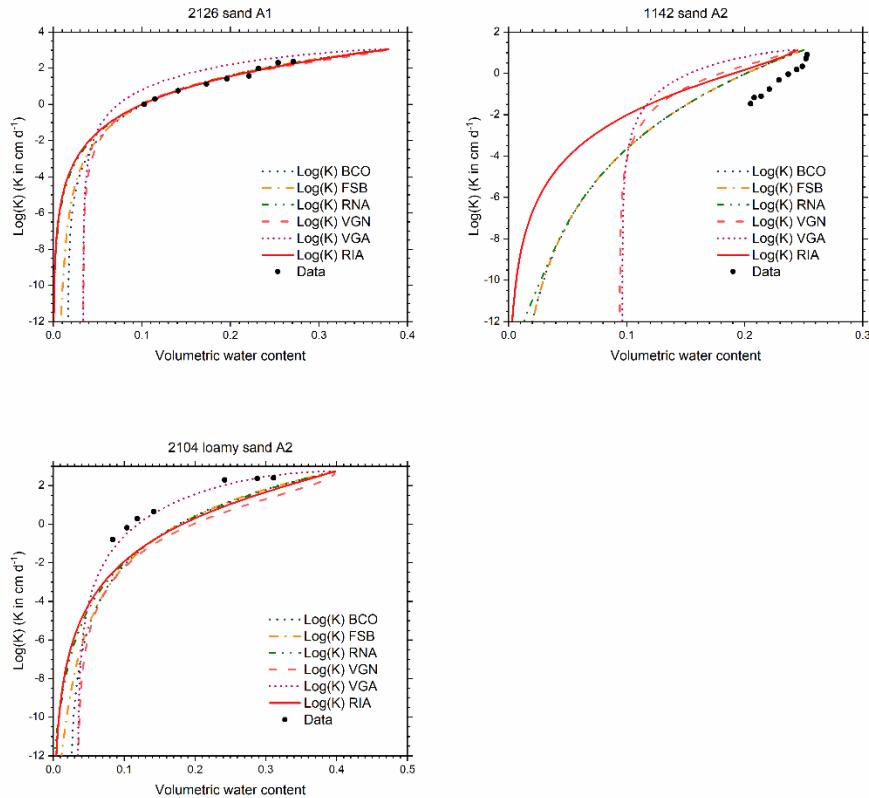
636

637 Figure C4. Retention data and fitted soil water retention curves according to six  
 638 parameterizations for selected UNSODA soils with Twarakavi et al.'s (2010) B2 or B4  
 639 classification. The parameterizations are explained in the text.



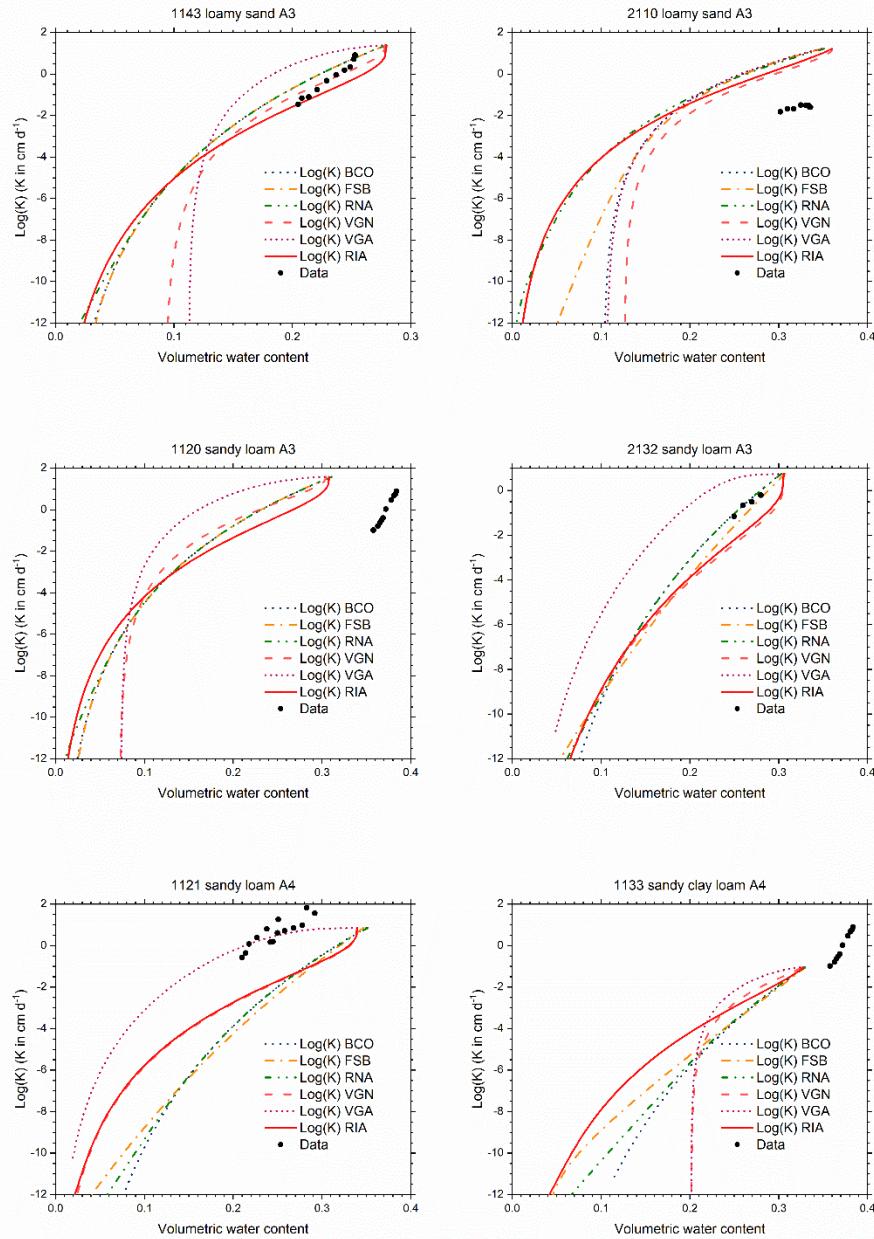
640

641 Figure C5. Retention data and fitted soil water retention curves according to six  
 642 parameterizations for selected UNSODA soils with Twarakavi et al.'s (2010) C2 or C4  
 643 classification. The parameterizations are explained in the text.



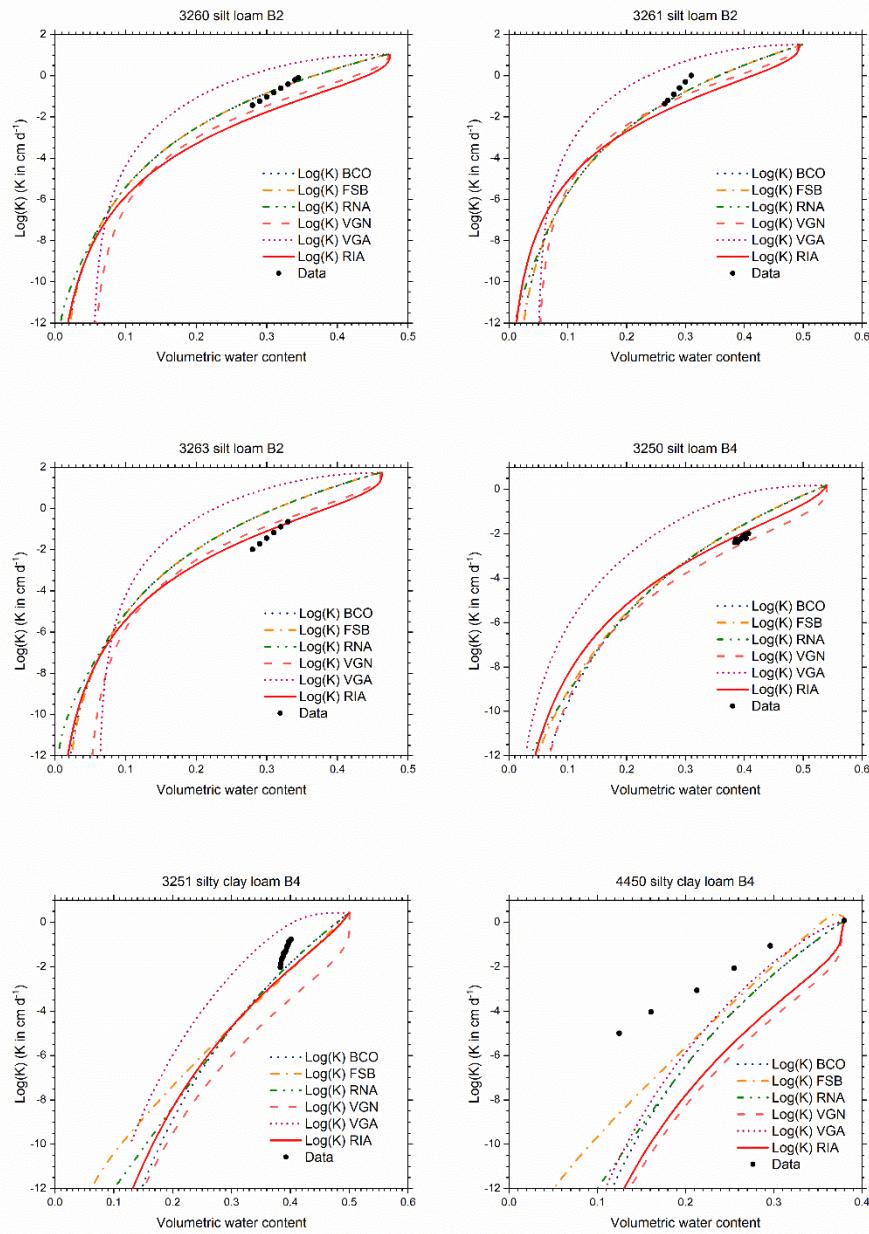
644

645 Figure C6. Conductivity data and conductivity curves derived from the retention curves of  
 646 Fig. C2 according to six parameterizations for selected UNSODA soils with Twarakavi et al.'s  
 647 (2010) A1 or A2 classification.



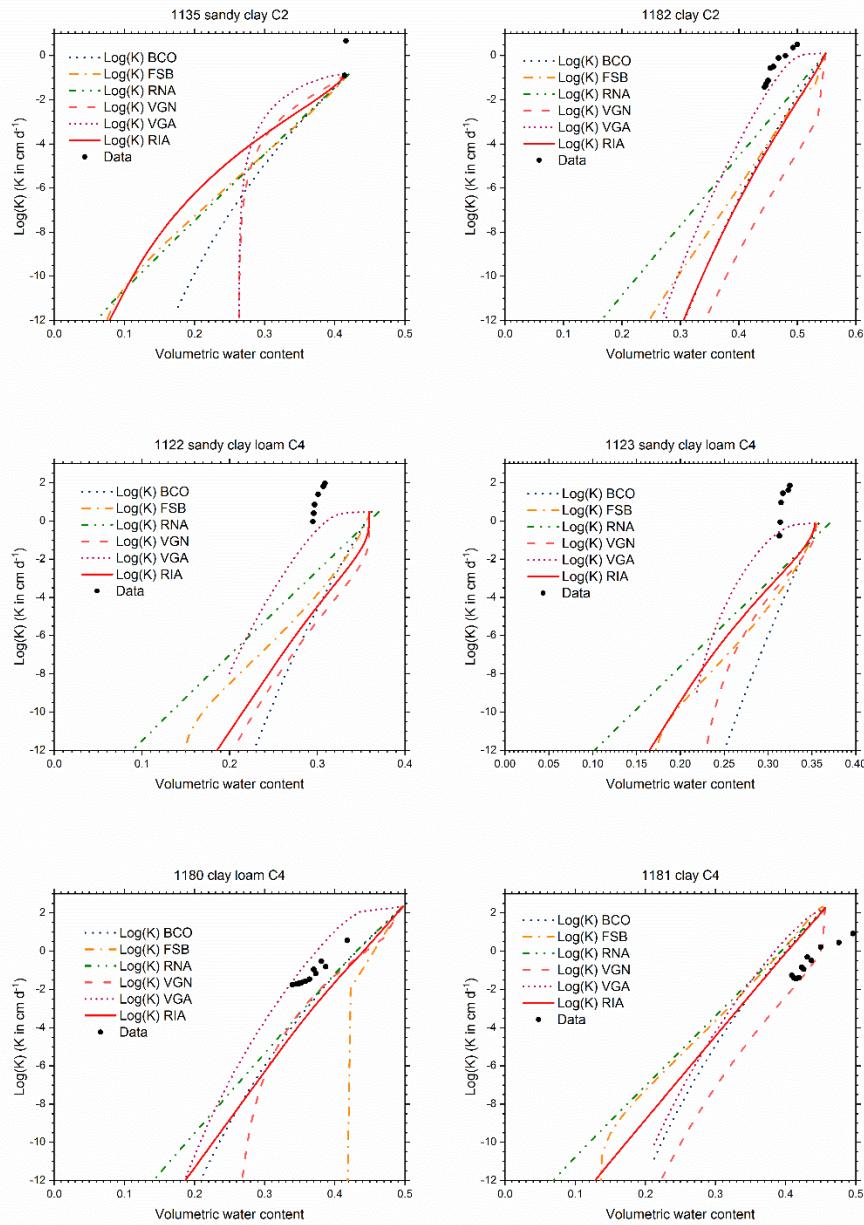
648

649 Figure C7. Conductivity data and conductivity curves derived from the retention curves of  
 650 Fig. C3 according to six parameterizations for selected UNSODA soils with Twarakavi et al.'s  
 651 (2010) A3 or A4 classification.



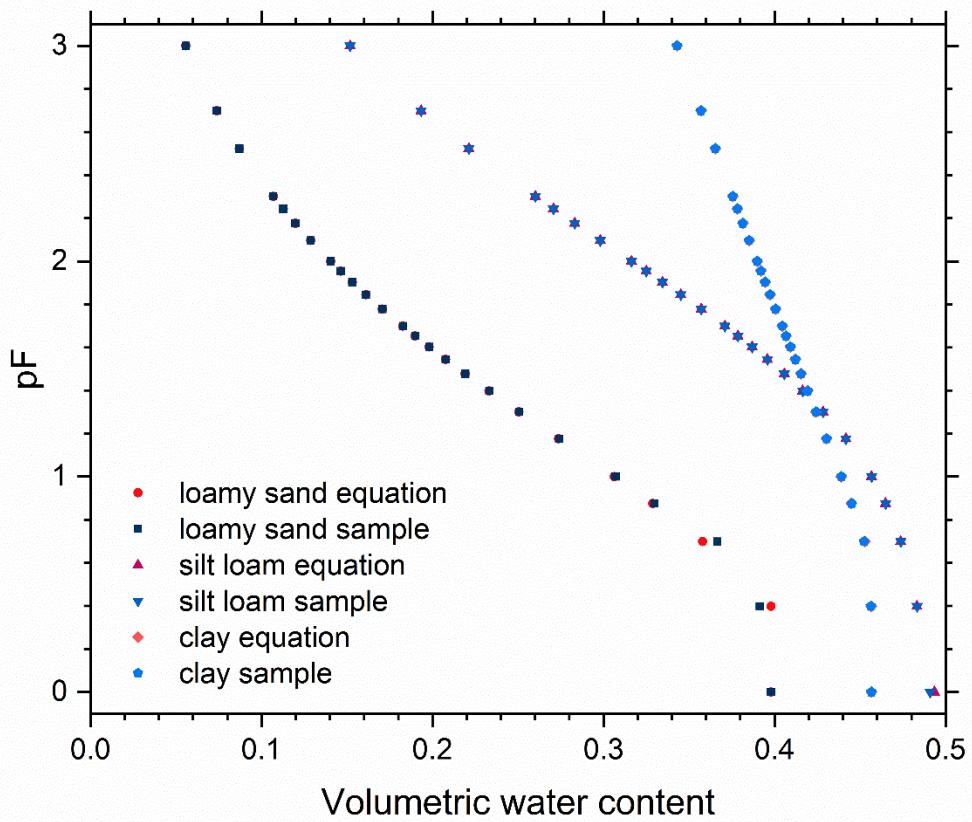
652

653 Figure C8. Conductivity data and conductivity curves derived from the retention curves of  
 654 Fig. C4 according to six parameterizations for selected UNSODA soils with Twarakavi et al.'s  
 655 (2010) B2 or B4 classification.



656

657 Figure C9. Conductivity data and conductivity curves derived from the retention curves of  
 658 Fig. C5 according to six parameterizations for selected UNSODA soils with Twarakavi et al.'s  
 659 (2010) C2 or AC4 classification.



660

661 Figure C10. Soil water retention points calculated from RIA parameterizations for loamy  
 662 sand (UNSOADA identifier 2104, classification according to Twarakavi et al. (2010) A4), silt  
 663 loam (3261, B2), and clay (1181, C4). The points were either calculated directly for the  
 664 given pF value ('equation'), or by calculating the average water content in a sample of 5.0  
 665 cm height at hydrostatic equilibrium, with the matric potential at the center of the sample  
 666 corresponding to the indicated pF value ('sample'). N.B. The data points for zero matric  
 667 potential were plotted at pF = 0 instead of pF =  $-\infty$ .

668

669 **Author contributions**

670

671 GdR developed the new parameterization, assisted by RM. GdR identified the 21 soils from  
672 the UNSODA database and designed the model test. JM developed the SCE code for  
673 parameter identification. GdR coded the shell around the SCE core to make it suitable for  
674 fitting various SWRC parameterizations. GdR collected the weather data and generated the  
675 weather records. GdR and RM carried out the parameter fitting and the Hydrus-1D  
676 simulations. GdR wrote the manuscript with contributions from JM and RM.

677

678 **Competing interests**

679

680 GdR is a member of the HESS Editorial Board.

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