Response to Editor Dr. Bettina Schaefli
Comments to the Author:
Dear Authors,
I would like to invite you to implement to changes that you discussed in the public discussion. Even if it is a technical note, I suggest you follow the suggestions of reviewer two to make the paper a bit more accesible for non-specialists.

Response:
Many thanks for giving us an opportunity to revise this manuscript. We have revised the paper according to what we presented in the public discussion. We also explain how we revise in the response to each comment below. We have added more information on the wavelet methods both in Introduction and Theory sections as reviewer #2 suggested to make the paper more accessible to general readers. We have also tried to avoid using abbreviations as much as we can.

Response to Anonymous Referee #1

Comments from Referee #1

In this paper, the authors mainly developed a partial wavelet coherency method, for identifying the relationship between variables. It is an important issue but also a difficult problem for geo-data analysis, and the method developed would be helpful for the data analysis in geosciences. The following comments are suggested to be considered for further improving the paper:

Comment #1:
(1) In lines 108-110: the "sufficient number" should be clarified, as it has a big influence on the uncertainty estimation, that is, what number is sufficient? Furthermore, the reason of using first-order autocorrelation coefficient for MC simulation should be explained and discussed.
Response #1:
Many thanks for your review and positive general comment.
To address the "sufficient number" issue, we added the following sentences "Different combinations of r1 values (i.e., 0.0, 0.5, and 0.9) were used to generate 10 to 10 000 AR(1) series with three, four and five variables. Our results indicate that the noise combination has little impact on the PWC values at the 95% confidence level as also found by Grinsted et al. (2004) for the BWC case (data not shown). The relative difference of PWC at the 95% confidence level compared with that calculated from the 10 000 AR(1) series decreases with the increase in number of AR(1) series. When the number of AR(1) is above 300, a very low maximum relative difference (e.g., <2%) is observed (Fig. S1 of Sect. S3 in the Supplement). Therefore, a repeating number of 300 seems to be sufficient for a significance test. However, if calculation time is not a barrier, a higher repeating number, such as ≥1000, is recommended." at Lines 171-181.

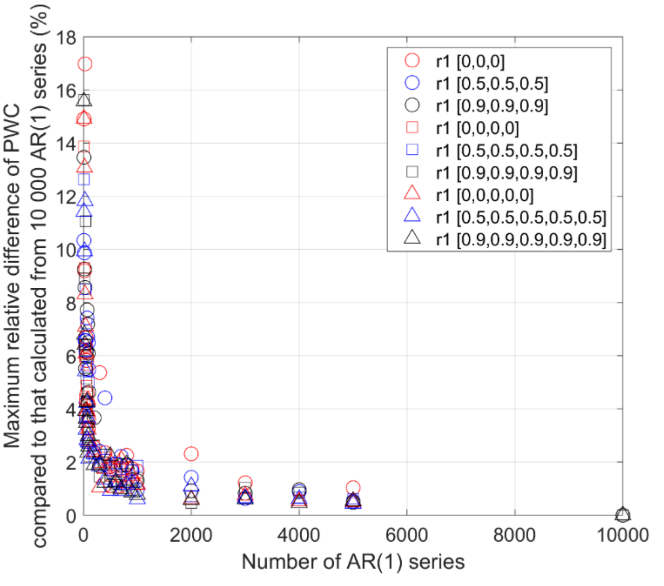

Figure S1. Relationship between maximum relative difference (%) of PWC compared to that calculated from 10 000 AR(1) series (surrogate dataset) versus the number of AR(1) series during the significance test using the Monte Carlo test. Number of scales per octave is 12. The first-order autocorrelation coefficients (r1) in brackets refer to those for the response variable (first), predictor variable (second), and excluding variables (third and onwards).

"The first-order autoregressive model (AR(1)) is chosen because it can be used to simulate most geoscience data very well (Wendroth et al., 1992; Grinsted et al., 2004; Si and Farrell, 2004)" (Lines169-171).

Comment #2:

(2) Lines 121-122, some theoretical lines can be provided to show the difference between Eq. (9) and Eq. (14).

Response #2:

The difference between Eq. (9) and Eq. (14) was explained by derivation of PWC in the case of one excluding variable from Eq. (1).

"When only one variable (e.g., $Z1$) is excluded, Eq.(9) can be written as (see the Supplement (Sect. S2) for the derivation process)

$$\rho^2_{y,x\cdot Z1} = \frac{\left|\gamma_{y,x}(s,\tau)-\gamma_{y,Z1}(s,\tau)\overline{\gamma_{x,Z1}(s,\tau)}\right|^2}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)} \qquad (14)$$ "(Lines 163-165)

In the supplementary (Sect. S2), we added the derivation of Eq. (14) from Eq. (9) as follows:

"S2 Derivation of the PWC in case of one excluding variable (Eq.14) from Eq. (9)

When only one variable (e.g., $Z1$) is excluded, Eq.(9) $\left(\rho^2_{y,x\cdot Z} = \frac{\left|1-R^2_{y,x,Z}(s,\tau)\right|^2 R^2_{y,x}(s,\tau)}{\left(1-R^2_{y,Z}(s,\tau)\right)\left(1-R^2_{x,Z}(s,\tau)\right)}\right)$

can be written as

$$\rho^2_{y,x\cdot Z1} = \frac{\left|1-R^2_{y,x,Z1}(s,\tau)\right|^2 R^2_{y,x}(s,\tau)}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)} \qquad \text{(S8)}$$

Based on Eq. (2),

$$\rho^2_{y,x\cdot Z1} = \frac{\left|1-\frac{\overleftrightarrow{W}^{y,Z1}(s,\tau)\,\overleftrightarrow{W}^{Z1,Z1}(s,\tau)^{-1}\,\overline{\overleftrightarrow{W}^{x,Z1}(s,\tau)}}{\overleftrightarrow{W}^{y,x}(s,\tau)}\right|^2 \frac{\left|\overleftrightarrow{W}^{y,x}(s,\tau)\right|^2}{\overleftrightarrow{W}^{y,y}(s,\tau)\,\overleftrightarrow{W}^{x,x}(s,\tau)}}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)}$$

$$= \frac{\left|\overleftrightarrow{W}^{y,x}(s,\tau)-\frac{\overleftrightarrow{W}^{y,Z1}(s,\tau)\overline{\overleftrightarrow{W}^{x,Z1}(s,\tau)}}{\overleftrightarrow{W}^{Z1,Z1}(s,\tau)}\right|^2}{\overleftrightarrow{W}^{y,y}(s,\tau)\overleftrightarrow{W}^{x,x}(s,\tau)\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)}$$

$$= \frac{\frac{1}{\sqrt{\left(\overleftrightarrow{W}^{y,y}(s,\tau)\overleftrightarrow{W}^{x,x}(s,\tau)\right)}^2}\left|\overleftrightarrow{W}^{y,x}(s,\tau)-\frac{\overleftrightarrow{W}^{y,Z1}(s,\tau)\overline{\overleftrightarrow{W}^{x,Z1}(s,\tau)}}{\left(\sqrt{\overleftrightarrow{W}^{Z1,Z1}(s,\tau)}\right)^2}\right|^2}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)}$$

$$= \frac{\left|\frac{\overleftrightarrow{W}^{y,x}(s,\tau)}{\sqrt{\overleftrightarrow{W}^{y,y}(s,\tau)}\sqrt{\overleftrightarrow{W}^{x,x}(s,\tau)}}-\frac{\overleftrightarrow{W}^{y,Z1}(s,\tau)\,\overline{\overleftrightarrow{W}^{x,Z1}(s,\tau)}}{\sqrt{\overleftrightarrow{W}^{y,y}(s,\tau)}\sqrt{\overleftrightarrow{W}^{x,x}(s,\tau)}\sqrt{\overleftrightarrow{W}^{Z1,Z1}(s,\tau)}\sqrt{\overleftrightarrow{W}^{Z1,Z1}(s,\tau)}}\right|^2}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)}$$

$$= \frac{\left|\frac{\overleftrightarrow{W}^{y,x}(s,\tau)}{\sqrt{\overleftrightarrow{W}^{y,y}(s,\tau)}\sqrt{\overleftrightarrow{W}^{x,x}(s,\tau)}}-\frac{\overleftrightarrow{W}^{y,Z1}(s,\tau)}{\sqrt{\overleftrightarrow{W}^{y,y}(s,\tau)}\sqrt{\overleftrightarrow{W}^{Z1,Z1}(s,\tau)}}\cdot\frac{\overline{\overleftrightarrow{W}^{x,Z1}(s,\tau)}}{\sqrt{\overleftrightarrow{W}^{x,x}(s,\tau)}\sqrt{\overleftrightarrow{W}^{Z1,Z1}(s,\tau)}}\right|^2}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)}$$

$$= \frac{\left|\gamma_{y,x}(s,\tau)-\gamma_{y,Z1}(s,\tau)\overline{\gamma_{x,Z1}(s,\tau)}\right|^2}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)} \qquad \text{(S9)}$$ ”

Later on, we presented the equation for calculating PWC in the classical method and discussed the theoretical differences between two methods in case of one excluding variable at Lines 185-204.

“In the case of one excluding variable ($Z = \{Z_1\}$), Mihanović et al. (2009) suggested that the PWC can be calculated by an equation analogous to the traditional partial correlation squared (Kenney and Keeping, 1939) without giving the detailed derivation process. Their equation is the same as Eq. (14). Unfortunately, Ng and Chan (2012a) might have misinterpreted the equation of Mihanović et al. (2009) and developed Matlab code for calculating PWC using the equation expressed as

$$\rho^2_{y,x\cdot Z1} = \frac{\left|R_{y,x}(s,\tau) - R_{y,Z1}(s,\tau)\ R_{x,Z1}(s,\tau)\right|^2}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)} \qquad (15)$$

where $R_{y,x}(s,\tau)$, $R_{y,Z1}(s,\tau)$, and $R_{x,Z1}(s,\tau)$ are the square root of $R^2_{y,x}(s,\tau)$, $R^2_{y,Z1}(s,\tau)$, $R^2_{x,Z1}(s,\tau)$, respectively. $R^2_{y,Z1}(s,\tau)$ and $R^2_{x,Z1}(s,\tau)$ can be calculated from Eq. (10) by replacing y and x with their corresponding variables. Eq. (15) has been widely used to calculate PWC in the case of one excluding variable (Ng and Chan, 2012b; Rathinasamy et al., 2017; Aloui et al., 2018; Altarturi et al., 2018b; Jia et al., 2018; Li et al., 2018; Mutascu and Sokic, 2020; Wu et al., 2020). Note that complex coherence and real coherence are involved in the numerators of Eqs. (14) and (15), respectively, while the denominators are exactly the same. Further comparison indicates that Eq. (15) underestimates PWC value relative to Eq. (14) unless $\gamma_{y,x}(s,\tau)$ and $\gamma_{y,Z1}(s,\tau)\ \overline{\gamma_{x,Z1}(s,\tau)}$ in Eq. (14) are collinear (i.e., their arguments are identical) under which the two equations produce the same PWC values. Differences between Eqs. (14) and (15) will be discussed further using both artificial data and a real dataset. For comparison purposes, we refer to Eqs. (14) and (15) as the new method and the classical method, respectively. ”

The differences in PWC values calculated from the two methods (Eq. 14 and 15) are context-specific. As the Referee #2 mentioned, although the difference between the Mihanovic et al. (2009) model (Eq.15) and the proposed model (Eq.14) are small, i.e., the difference of PWC values is only 0.03 for the artificial data, Eq.14 produces PWC closer to 1.

In addition, the comparison of these two methods using real data indicated that the difference between the two methods can be large. As an example, mean PWC values between E and RH after excluding the effects of T by the new method were consistently higher than the classical method, and the differences ranged from 0.4 to 0.6 around the scale of 1 year. This highlights that the new method produces more accurate results than the classical method.

These have been added to the discussion section at Lines 414-438 as:

“The differences between the new method (Eq.14) and the classical method (Eq. 15) are compared using both the artificial and real datasets. Except for the phase information, the two methods generally produce comparable coherence for the artificial dataset for the case of one excluding variable (Fig. S5 of Sect. S3 in the Supplement). However, the new PWC method produces consistently and slightly higher coherence than the classical method. For example, their mean PWCs between y and y_2 at the scale of 8 after excluding the effect of y_4 are 1.00 and 0.97, respectively. This indicates that the new method produces coherence between y and y_2 at the scale (8) of y_2 closer to 1 as we expect. While the classical method produces similar PWC between E and other meteorological factors in most cases especially for the coherence between E and T after excluding the effects of others (Fig. S6 of Sect. S3 in the Supplement), large differences between these two methods can also be observed. For example, while the new method recognizes the strong coherence between E and RH after excluding the effect of T at scales of around 1 year (Fig. 3d), this coherence was negligible by the classical method (Fig. 5a). Mean PWC values by the new method were consistently higher than the classical method, and the differences ranged from 0.4 to 0.6 around the scale of 1 year (Fig. 5b). Considering the real coherence (Eq.15) rather than complex coherence (Eq.14) between every two variables in the numerators can potentially result in large underestimation of the partial wavelet coherence. Therefore, the ability of the new method to produce more accurate results than the classical method is one of its advantages.

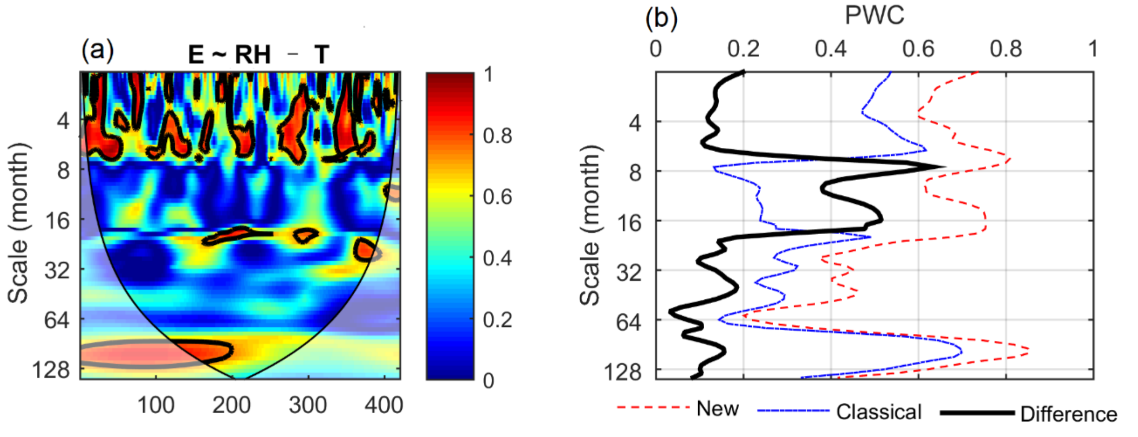

Figure 5.

Partial wavelet coherency (PWC) between evaporation (E) and relative humidity (RH) after excluding the effect of mean temperature (T) using the classical method (Eq. 15) (a) and differences in PWC between the new method (Eq.14) and classical method as a function of scale (b)."

Comment #3:
(3) Regarding the structure, is it more suitable to reorganize the Section 3 and 4, that is, the artificial data and their results are analyzed and discussed in Section 3, while those of real data are analyzed and discussed in Section 4?

Response #3:

Thanks for the good suggestion on paper structure. In the revision, we followed the order of data description, data analysis, results and discussion for each of artificial dataset and real data. To reduce the length of this paper, we have taken the suggestion from Referee #2 to remove the real data related to soil water content by adding more about the introduction of the wavelet methods and in-depth discussion of the advantages and weaknesses of the new method.

Thanks again for your constructive comment.

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Response to Anonymous Referee #2

Comment #1:

Summary *In this technical note, the authors propose a method for identifying relationships between two variables for the case where the two variables are correlated to other variables themselves. They apply their updated partial wavelet coherency' (PWC) method to a synthetic dataset and two real-world applications and show that this updated PWC model shows similar performance as existing PWC models. They conclude that their model outperforms existing models because it provides phase information and allows for excluding several correlated variables from the PWC.*

Response #1:

Many thanks for your comment. We think the new method outperforms the existing one from the three aspects: (1) more accurate results because of the theoretical differences (as explained in the **Response #2** to **Referee #1** above); (2) inclusion of phase information; and (3) any number of excluding variables can be considered.
Below we will respond to each of your comments.

Comment #2:

General remarks *I think that the study addresses a question of interest to the hydrological community, i.e. 'how can we identify the most important driving variables of a certain phenomena at different time scales'. The technical note is generally well structured. However, I think that it lacks a didactical and detailed introduction to the topic, problem, and wavelet analysis. The introduction would significantly benefit from providing examples of when the identification of bivariate relationships are important (i.e. providing a motivation for the study), an in-depth introduction to wavelet analysis (for the readers who are not yet too familiar with the topic), and an introduction to the terminology used. Extending the introduction will increase the length of the note and I suggest removing the practical example number 2 instead. I think it does not provide additional insights regarding the performance of the method proposed compared to the statements* *that were already made based on the synthetic data and the first practical example. Since the new method does not seem to clearly outperform existing methods, I would better explain why adding phase information and excluding several confounding variables is beneficial for the analysis. I would also add a more detailed discussion of model weaknesses, especially the implications of detecting spurious correlations. In addition, the note would profit from careful language editing.*

Response #2:

More detailed information on the general wavelet analysis, PWC, and problem of existing methods were added in the Introduction and Theory sections (see more details below).

The importance of bivariate relationships was explained at Lines 48-57.

An introduction to wavelet analysis in general was added at Lines 41-43.

The original motivation to have both real datasets is to demonstrate that the proposed method can be used for both spatial and temporal data. We agree that more detailed introduction will increase the length of the paper, so we removed the results related to soil water content dataset.

The differences between the new method and the existing method have been explained in the **Response #2** to the **Referee #1** above.

A separate discussion section was added by including a more detailed discussion of model advantages (e.g., the three aspects mentioned in the Response #1) and weaknesses (including spurious correlations and multiple-testing). Please refer to the discussion section at Lines 399-486.

Language has been carefully checked by editors from our publication office.

Please see the details below on how we will address the comments you have made.

Major points

Comment #3:

*1. **Abstract:** The abstract is not very accessible to non-wavelet-specialists. I would provide a short example for when such an analysis would be necessary/beneficial and shortly summarize what wavelet coherence analysis is all about. Please also shortly explain why PWC has been introduced in the first place (l. 12). I would also mention the datasets used for model evaluations (l. 14). I think the statement 'producing more accurate results' (l. 18) needs justification, otherwise it is not very credible. I would exclude lines 21-24 because this is a technical note and specific results regarding the example applications going beyond model performance are in my opinion not of interest here.*

Response #3:

We have added "Bivariate wavelet coherency is a measure of correlation between two spatial (or time) series in the location-scale (or time-frequency) domain. It is particularly suited to geoscience where relationships between multiple variables commonly differ with locations or/and scales because of various processes involved." to explain what is wavelet coherence and when it would benefit (Lines 9-12).

The PWC was introduced "to detect the scale-specific and localized bivariate relationships by excluding the effects of other variables". (Lines 14-15).

The description of dataset used for model evaluations is "Both stationary and non-stationary artificial datasets with the response variable being the sum of five cosine waves at 256 locations are used to test the methods." (Lines 18-19).

Why the new method produces more accurate results was explained by adding "Compared with the previous PWC calculation, the new method produces more accurate results where there is one excluding variable. This is because bivariate real coherence rather than the bivariate complex

coherence was mistakenly used in the previous PWC calculation, which underestimates the PWC" (Lines 22-25).
Lines 21-24 from the previous submission have been removed.

Comment #4:
2. Introduction: *The introduction should in my opinion provide a motivation for the use of PWC methods, also for non-specialists on the topic e.g. by providing examples of important bivariate relationships in the geosciences and why we may be interested in them. In addition, an introduction to wavelet analysis in general and wavelet coherence analysis in particular should be provided. The reader should also be made familiar with the terminology used, e.g. what kind of scales are you talking about and what is an 'excluding variable'. A clear motivation for why excluding variables and including phases matters is required to underline the benefits of the methods later on in the results and conclusions sections (l. 57-58). Currently, the introduction does not very well prepare readers for what they are going to read in the methods and results sections.*
Response #4:

The importance of bivariate relationships was explained by adding "The BWC partitions correlation between two variables into different locations and scales, which are different from the overall relationships at the sampling scale as shown by the traditional correlation coefficient. For example, BWC analysis indicated that soil water content of a hummocky landscape in the Canadian Prairies was negatively correlated to soil organic carbon content at a slope scale (50 m), but they were positively correlated at a watershed scale (120 m) in summer because of the different processes involved at different scales (Hu et al., 2017). Because the positive correlation may cancel out with the negative at different scales and/or locations, the traditional correlation coefficient between soil water content and soil organic carbon content does not differ significantly from zero, which is misleading." (Lines 48-57).

The motivation for the use of PWC method is further explained by adding "Partial correlation analysis is one such method to avoid the misleading relationships resulting from the interdependence between other variables and both predictor and response variables (Kenney and Keeping, 1939)" (Lines 68-70) and "For example, PWC analysis indicated that Southern Oscillation Index and Pacific Decadal Oscillation did not affect precipitation across India, while this was misinterpreted by the BWC analysis because of their interdependence on Niño 3.4 that affects precipitation (Rathinasamy et al., 2017)" (Lines 78-81).

An introduction to wavelet analysis in general was added as "Wavelet analyses are based on wavelet transform using mother wavelet function which expands spatial (or time) series into location-scale (or time-frequency) space for identification of localized intermittent scales (or frequencies)." (Lines 41-43).

When we talk about scale, it can mean spatial or temporal scale depending on if the dataset are spatial series or time series. To avoid repeatedly addressing if this is related to spatial or time scale, we has defined it at the first time by adding "For convenience, we will mainly refer to location and scale irrespective of spatial or time series unless otherwise mentioned.". (Lines 43-45).

Excluding variable refers to "variable that influences the response variable is excluded". (Lines 82-83).

The explanation on the motivation for why excluding variables and including phases matter was added as "The coherence between response and predictor variables can still be misleading if more than one variable is interdependent with the predictor variable. This is especially true if these variables are correlated with the predictor variable at different locations and/or scales. In addition, without phase information, it is hard to tell if the correlation at a location and scale is positive or negative" in the introduction at Lines 84-88.

Comment #5:

3. Theory: *I think that you should start even simpler here and provide a short introduction to wavelet analysis (difference between discrete and complex, terminology) and wavelet coherence analysis. In addition, it is unclear to me what exactly the difference between classical PWC and your proposed method is (l. 74-76). Currently, it is not entirely clear to me how the Monte Carlo experiment was performed (l. 108-110). Could you please slightly expand this section?*

Response #5:

We has added the introduction to wavelet analysis, wavelet coherence analysis and associated equations at start of the Theory section. Here we assume you mean difference between discrete wavelet transform and continuous wavelet transform. These were added at Lines 101-120 as follows:

"Wavelet analysis is based on the calculations of wavelet coefficients using wavelet transform at different locations and scales for each variable involved. Two types of wavelet transform exist including continuous wavelet transform and discrete wavelet transform. While the discrete wavelet transform is mainly used for data compression and noise reduction, the continuous wavelet transform is widely used for extracting scale-specific and localized features, as is the case of this study (Grinsted et al., 2004). For the continuous wavelet transform, the Morlet wavelet is used as a mother wavelet function to transform a spatial (or time) series into location-scale (or time-frequency) domain, which allows us to identify both location-specific amplitude and phase information of wavelet coefficients at different scales (Torrence and Compo, 1998). From wavelet coefficients, auto- and cross-wavelet power spectra for two variables can be calculated as the product of wavelet coefficient and the complex conjugate of itself (auto-wavelet power spectra) or another variable (cross-wavelet power spectra). The BWC is calculated as the ratio of smoothed cross-wavelet power spectra of two variables to the product of their auto-wavelet power spectra (Grinsted et al., 2004). Hu and Si (2016) extended wavelet coherence from two to multiple ($\geq$3) variables and developed MWC. Detailed information on the calculations of wavelet coefficients, auto- and cross-wavelet power spectra, BWC, and MWC based on the continuous wavelet transform can be found elsewhere (Torrence and Compo, 1998; Grinsted et al., 2004; Si and Farrell, 2004; Si, 2008; Hu and Si, 2016; Hu et al., 2017). Here, we will only introduce the theory and calculation that is very relevant to the PWC. "

In addition, the derivation of Eq.(1) in the original submission from equations of complex partial spectrum in frequency domain and bivariate complex coherence from time-frequency domain was added in the supplement as below:

“ **S1 Derivation of the complex PWC Eq.(1)**

Complex partial spectrum from frequency (scale)domain (Makhtar et al., 2014) can be used to define that of time-frequency (location-scale) domain, $\underset{W}{\leftrightarrow}^{y,x\cdot Z}(s,\tau)$, which is expressed as

$$\underset{W}{\leftrightarrow}^{y,x\cdot Z}(s,\tau) = \underset{W}{\leftrightarrow}^{y,x}(s,\tau) - \frac{\underset{W}{\leftrightarrow}^{y,Z}(s,\tau)\overline{\underset{W}{\leftrightarrow}^{x,Z}(s,\tau)}}{\underset{W}{\leftrightarrow}^{Z,Z}(s,\tau)} \qquad \text{(S1)}$$

where $\underset{W}{\leftrightarrow}$ is the smoothed cross spectrum, $\overline{(\cdot)}$ is the complex conjugate operator, y, x, and Z ($Z = \{Z_1, Z_2, \cdots, Z_q\}$) refer to the response variable, predictor variable, and excluding variables, respectively. s and τ refer to scale (frequency) and location (time), respectively.

Given the definition of coherence between two variables y and x, their complex coherence $\gamma_{y,x}(s,\tau)$ (Eq.(5)) can be re-written as

$$\gamma_{y,x}(s,\tau) = \frac{\underset{W}{\leftrightarrow}^{y,x}(s,\tau)}{\sqrt{\underset{W}{\leftrightarrow}^{y,y}(s,\tau)\,\underset{W}{\leftrightarrow}^{x,x}(s,\tau)}} \qquad \text{(S2)}$$

Then we can define complex partial coherence as

$$\gamma_{y,x\cdot Z}(s,\tau) = \frac{\underset{W}{\leftrightarrow}^{y,x\cdot Z}(s,\tau)}{\sqrt{\underset{W}{\leftrightarrow}^{y,y\cdot Z}(s,\tau)\,\underset{W}{\leftrightarrow}^{x,x\cdot Z}(s,\tau)}} \qquad \text{(S3)}$$

Based on Eq. (S1) and Eqs 2, 3, and 4 $\left(R^2_{y,x,Z}(s,\tau) = \frac{\underset{W}{\leftrightarrow}^{y,Z}(s,\tau)\underset{W}{\leftrightarrow}^{Z,Z}(s,\tau)^{-1}\overline{\underset{W}{\leftrightarrow}^{x,Z}(s,\tau)}}{\underset{W}{\leftrightarrow}^{y,x}(s,\tau)}\right.$,

$R^2_{y,Z}(s,\tau) = \frac{\underset{W}{\leftrightarrow}^{y,Z}(s,\tau)\underset{W}{\leftrightarrow}^{Z,Z}(s,\tau)^{-1}\overline{\underset{W}{\leftrightarrow}^{y,Z}(s,\tau)}}{\underset{W}{\leftrightarrow}^{y,y}(s,\tau)}$, and $\left.R^2_{x,Z}(s,\tau) = \frac{\underset{W}{\leftrightarrow}^{x,Z}(s,\tau)\underset{W}{\leftrightarrow}^{Z,Z}(s,\tau)^{-1}\overline{\underset{W}{\leftrightarrow}^{x,Z}(s,\tau)}}{\underset{W}{\leftrightarrow}^{x,x}(s,\tau)}\right)$

we obtain

$$\underset{W}{\leftrightarrow}^{y,x\cdot Z}(s,\tau) = \underset{W}{\leftrightarrow}^{y,x}(s,\tau)\left(1 - \frac{\underset{W}{\leftrightarrow}^{y,Z}(s,\tau)\,\overline{\underset{W}{\leftrightarrow}^{x,Z}(s,\tau)}}{\underset{W}{\leftrightarrow}^{Z,Z}(s,\tau)\,\underset{W}{\leftrightarrow}^{y,x}(s,\tau)}\right) = \underset{W}{\leftrightarrow}^{y,x}(s,\tau)\left(1 - R^2_{y,x,Z}(s,\tau)\right) \qquad \text{(S4)}$$

$$\underset{W}{\leftrightarrow}^{y,y\cdot Z}(s,\tau) = \underset{W}{\leftrightarrow}^{y,y}(s,\tau)\left(1 - \frac{\underset{W}{\leftrightarrow}^{y,Z}(s,\tau)\,\overline{\underset{W}{\leftrightarrow}^{y,Z}(s,\tau)}}{\underset{W}{\leftrightarrow}^{Z,Z}(s,\tau)\,\underset{W}{\leftrightarrow}^{y,y}(s,\tau)}\right) = \underset{W}{\leftrightarrow}^{y,y}(s,\tau)\left(1 - R^2_{y,Z}(s,\tau)\right) \qquad \text{(S5)}$$

$$\underset{W}{\leftrightarrow}^{x,x\cdot Z}(s,\tau) = \underset{W}{\leftrightarrow}^{x,x}(s,\tau)\left(1 - \frac{\underset{W}{\leftrightarrow}^{x,Z}(s,\tau)\,\overline{\underset{W}{\leftrightarrow}^{x,Z}(s,\tau)}}{\underset{W}{\leftrightarrow}^{Z,Z}(s,\tau)\,\underset{W}{\leftrightarrow}^{x,x}(s,\tau)}\right) = \underset{W}{\leftrightarrow}^{x,x}(s,\tau)\left(1 - R^2_{x,Z}(s,\tau)\right) \qquad \text{(S6)}$$

Inserting Eqs S4, S5, and S6 into Eq. (S3), we have

$$\gamma_{y,x\cdot Z}(s,\tau) = \frac{\underset{W}{\leftrightarrow}^{y,x}(s,\tau)\left(1-R^2_{y,x,Z}(s,\tau)\right)}{\sqrt{\underset{W}{\leftrightarrow}^{y,y}(s,\tau)\left(1-R^2_{y,Z}(s,\tau)\right)\underset{W}{\leftrightarrow}^{x,x}(s,\tau)\left(1-R^2_{x,Z}(s,\tau)\right)}} = \frac{\underset{W}{\leftrightarrow}^{y,x}(s,\tau)\left(1-R^2_{y,x,Z}(s,\tau)\right)}{\sqrt{\underset{W}{\leftrightarrow}^{y,y}(s,\tau)\underset{W}{\leftrightarrow}^{x,x}(s,\tau)}\sqrt{\left(1-R^2_{y,Z}(s,\tau)\right)\left(1-R^2_{x,Z}(s,\tau)\right)}} =$$

$$\frac{\left(1-R^2_{y,x,Z}(s,\tau)\right)\gamma_{y,x}(s,\tau)}{\sqrt{\left(1-R^2_{y,Z}(s,\tau)\right)\left(1-R^2_{x,Z}(s,\tau)\right)}} \qquad \text{(S7)}$$

Obviously, Eq. (S7) and Eq. (1) are identical.”

The differences between the new method and the existing method in case of one excluding variable have been explained in the **Response #2** to the **Referee #1** above. By comparing Eq. (14) (new method) and (15) (classical method) , we can conclude that theoretically the classical method underestimates PWC relative to the new one.

Monte Carlo method was explained in more details by adding why we chose AR1 model and how many repeats are needed as we explained in the **Response #1** to the **Referee #1** above.

Comment #6:

4. Data and analysis: *I would recommend removing the 'soil water content' example (section 4.2.2) because as I can see it does not show anything that has not yet been shown by the 'free evaporation example' in terms of the validity of the model. I would rather invest the space in extending the introduction as outlined in more details above. In the figure captions, I would add a reference to the dataset used to generate it. In addition, I am not sure what you would like to show with the cases where the variable of interest is excluded. I would therefore exclude the results referring to this exercise (e.g. Figure 1 last row and see l. 236-237). I also think that the figures would profit a lot from using labels for subfigures, which would facilitate orientation. To me, the difference between the Mihanovic et al. (2009) model and the proposed model are not evident by looking at the Figures presented (a difference of 0.03 does not seem to be a lot, l.293). Therefore, I think the actual advantages of using this new method should better be worked out and explained before a statement such as 'the new method outperforms the Mihanovic et al. method' (l. 293-294) is made. Please also explain why the inclusion of 'phase information' is an advantage of the new method (l. 312-313).*

Response #6:

Thanks for this advice. We have removed the soil water content example.

Reference to the dataset used to generate the figures was added in the figure caption as "All variables were generated by following Yan and Gao (2007) and Hu and Si (2016) and explained in Section 3.1 and are shown in Fig. S2 of Sect. S3 in the Supplement."

The purpose of showing the cases of variable of interest being excluded is to basically show that the PWC values should be theoretically zero in that case. As we have the similar results in the case of two excluding variables (Figure 3 in the original submission and Figure 2 in the current version), we have removed this from Figure 1.

We have added a label for each subfigure in the revision.

As we explained above, theoretical differences exist between these two methods in case of one excluding variable. This has been discussed at Lines 185-204.

In the new discussion section, we have highlighted the advantages and weakness of the new method at Lines 399-486 (Please see the details in the **Response #7** below).

Comment #7:

5. A proper discussion section is missing: *I would add an in-depth discussion of the weaknesses and benefits of the approach and put the new method into perspective by comparing it to existing methods.*

Response #7:
Advantages and weaknesses of the method were added in the discussion section as:

" **5. Discussion on the advantages and weaknesses of the new method**

5.1 Advantages

We extend the partial coherence method from the frequency (scale) domain (Koopmans, 1995) to the time-frequency (location-scale) domain. The new method is an extension of previous work on PWC and MWC (Mihanović et al., 2009; Hu and Si, 2016). The method test and application have verified that it has the advantage of dealing with more than one excluding variable and providing the phase information associated with the PWC. In the case of one excluding variable, Mihanović et al. (2009) has suggested to calculate PWC by using an equation analogous to the traditional partial correlation squared (Eq. 14), which can be derived from our Eq. (9). However, their equation was, unfortunately, widely used by replacing the complex coherence in Eq. (14) with real coherence as expressed in Eq. (15).

The differences between the new method (Eq.14) and the classical method (Eq. 15) are compared using both the artificial and real datasets. Except for the phase information, the two methods generally produce comparable coherence for the artificial dataset for the case of one excluding variable (Fig. S5 of Sect. S3 in the Supplement). However, the new PWC method produces consistently and slightly higher coherence than the classical method. For example, their mean PWCs between y and y_2 at the scale of 8 after excluding the effect of y_4 are 1.00 and 0.97, respectively. This indicates that the new method produces coherence between y and y_2 at the scale (8) of y_2 closer to 1 as we expect. While the classical method produces similar PWC between E and other meteorological factors in most cases especially for the coherence between E and T after excluding the effects of others (Fig. S6 of Sect. S3 in the Supplement), large differences between these two methods can also be observed. For example, while the new method recognizes the strong coherence between E and RH after excluding the effect of T at scales of around 1 year (Fig. 3d), this coherence was negligible by the classical method (Fig. 5a). Mean PWC values by the new method were consistently higher than the classical method, and the differences ranged from 0.4 to 0.6 around the scale of 1 year (Fig. 5b). Considering the real coherence (Eq.15) rather than complex coherence (Eq.14) between every two variables in the numerators can potentially result in large underestimation of the partial wavelet coherence. Therefore, the ability of the new method to produce more accurate results than the classical method is one of its advantages.

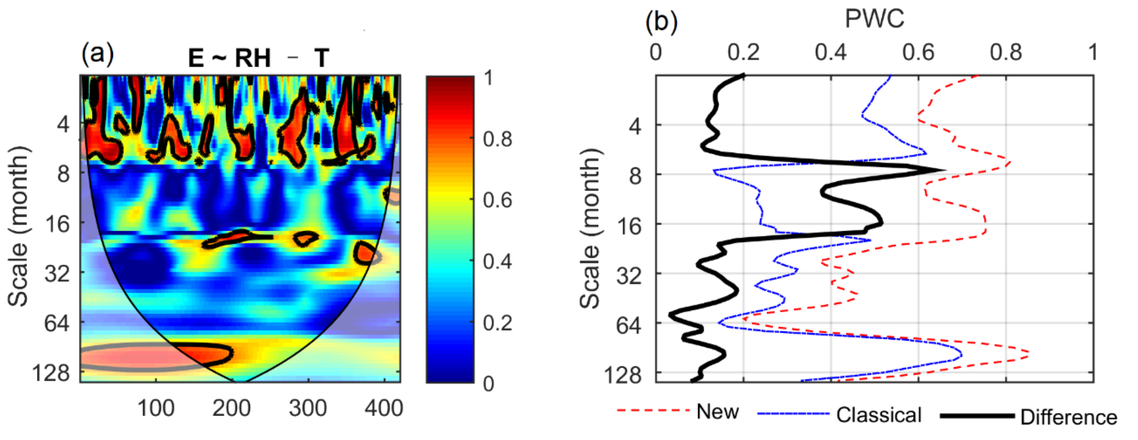

Figure 5.

Partial wavelet coherency (PWC) between evaporation (E) and relative humidity (RH) after excluding the effect of mean temperature (T) using the classical method (Eq. 15) (a) and differences in PWC between the new method (Eq.14) and classical method as a function of scale (b).

Compared with the Mihanović et al. (2009) method, the additional phase information from the new PWC is another advantage of this new method. This is because phase information is directly related to the type of correlation, i.e., in-phase and out-of-phase indicating positive and negative correlation, respectively. Different types of correlations were usually found at different locations and scales (Hu et al., 2017). The phase information helps understand the differences in associated mechanisms or processes at different locations and scales. In addition, the phase information will allow us to detect the changes in not only the degree of correlation (i.e., coherence) but also the type of correlation after excluding the effect of other variables. For example, E and RH were positively correlated at the 1-year cycle (8–16 months) from year 1979 to 1995. This is because higher evaporation usually occurs in summer when high T coincides with high RH as influenced by the monsoon climate in the study area (Fig. S4 of Sect. S3 in the Supplement). Interestingly, after excluding the effect of T, E was negatively correlated with RH at the scale of 1-year as we expect (Fig. 3d).

Moreover, our new PWC method applies to cases with more than one excluding variable, which is a knowledge gap. When multiple variables are correlated with both the predictor and response variables, the correlations between predictor and response variables may be misleading if the effects of all these multiple variable were not removed. For example, at the dominant scale (i.e., 1-year) of E variation, the effects of RH on E existed after excluding the effects of T or SH. However, their contrasting correlations (Fig. 3d-e) resulted in negligible effects of RH on E at this scale after the effects of all other variables were excluded (Fig. 4b). In this case, the dominant role of mean temperature in driving free water evaporation was proved at the 1-year cycle (Fig. 4a). This also further verifies the suitability of the Hargreaves model (only air temperature and incident solar radiation required) (Hargreaves, 1989) for estimating potential evapotranspiration on the Chinese Loess Plateau (Li, 2012).

5.2 Weaknesses

Similar to the Mihanović et al. (2009) method, the new method has the risk to produce spurious high correlations after excluding the effect from other variables. Take the artificial dataset for example, at a scale of 32, PWC values between y and y_2 after excluding y_4 are not significant, but relatively high, partly because of small octaves per scale (octave refers to the scaled distance between two scales with one scale being twice or half of the other, default of 1/12). This spurious unexpected high PWC is caused by low values in both the numerator (partly associated with the low coherence between response y and predictor variables y_2 at scale of 32) and denominator (partly associated with the high coherence between response y and excluding variable y_4 at a scale of 32) in Eq. (9). The same problem also exists in the classical method (Fig. S5 of Sect. S3 in the Supplement). So, caution should be taken to interpret those results. However, it seems that the domain with spurious correlation calculated by the new method is very limited and it is located mainly outside of the cones of influence. Moreover, the unexpected results can be easily ruled out with knowledge of BWC between response and predictor variables. It is expected that the correlation between two variables should not increase after excluding one or more variables. Therefore, BWC analysis is suggested for better interpretation of the PWC results.

Similar to BWC and MWC, the confidence level of PWC calculated from the Monte Carlo simulation is based on a single hypothesis testing. But in reality, the confidence level of PWC values at all locations and scales needs to be tested simultaneously. Therefore, the significance test has the multiple-testing problem (Schaefli et al., 2007; Schulte et al., 2015). The new method may benefit from a better statistical significance testing method. Options for multiple-testing can be the Bonferroni adjusted p test (Westfall and Young, 1993) or false discovery rate (Abramovich and Benjamini, 1996; Shen et al., 2002) which is less stringent than the former. ”

Comment #8:

6. Conclusions: *Given the evidence provided in the results section, statements such as 'the new method produces slightly more accurate coherence' do not seem to be justified. As mentioned earlier the benefits of including phase information and excluding several variables need to be better explained. Some of the material presented in this section could be moved to the new discussion section.*

Response #8:
As we replied above, we think 'the new method produces more accurate coherence' is justified by considering both the theoretical differences and the example of real data (Figure 5) explained above. The benefits of including phase information and excluding several variables were discussed in the new discussion section as we explained in the Response #7.
Yes, a large part from the conclusions part was moved to the Discussion section as shown in Response #7.

Comment #9:

7. ***Code availability:*** *I would provide the Matlab code via a data/file repository such as HydroShare or Zenodo instead of the supplement (l.27). This would be very helpful for the community and potential users.*

Response #9:
We have provided the Matlab code to the figshare (https://figshare.com/s/bc97956f43fe5734c784). Meanwhile, we have also put the updated codes for multiple wavelet coherence (MWC) which is necessary for calculating PWC in the same repository. We have improved the calculation time for MWC.

Minor points

Comment #10:
L. 31: please explain what you mean by 'time and space localization'.
Response #10:
We have added an example to show the localization "For example, time series of air temperature usually fluctuates periodically at different scales (e.g., daily and yearly), but abrupt changes in air temperature (e.g., extremely high or low) may occur at certain time points as a result of extreme weather and climate events (e.g., heat and rain)." (Lines 35-38).

Comment #11:

L.34: 'among these methods'

Transition from l. 42 to l. 43: very sharp transition from bivariate relationships to prediction. I would try to establish a clear link between the two things.

Response #11:
We have changed "Among which" to "Among these wavelet methods". (Line 45).

We're sorry that we are not sure we understood this comment. But we end up with the wide application of multiple wavelet coherence (MWC) method in the previous graph, and the next paragraph we start with what the MWC application has told us. Namely more predictor variables does not necessarily explain more variations in the response variable because predictor variables are usually cross-correlated. Because of the same reason, bivariate relationships can be misleading. Then we call the need to develop partial wavelet coherence (PWC). Now in the revision, we have put them in the same paragraph.

Comment #12:

L. 48: what do you mean by 'this issue'?

Response #12:
We mean "the misleading relationships resulting from the interdependence between other variables and both predictor and response variables". (Lines 68-70).

Comment #13:
L. 50: what kind of scales? Temporal or spatial?
Response #13:
We mean either temporal or spatial scales depending on if the dataset are time series or spatial series. For avoiding repeatedly saying this, we has clarified this at the first time by adding "For convenience, we will mainly refer to location and scale irrespective of spatial or time series unless otherwise mentioned". (Lines 43-45).

Comment #14:
L. 53-54: would combine greenhouse gas emissions and climate in one category.

Response #14:
Actually we mean different things. We mean precipitation by climate, so we changed climate to meteorology for avoiding confusing.

Comment #15:
L. 61: information 'which will allow to….'

Response #15:
We changed the whole sentence to "this paper aims to develop a PWC method that considers more than one excluding variable and presents phase information. This method reveals the magnitude and type of bivariate relationships after removing the effects from all potentially interdependent variables." at Lines 89-92.

Comment #16:
L. 61: what do you mean by 'analogy' in this context. I think that rephrasing may be required.

Response #16:
We have changed "in analogy with" simply to "from".

Comment #17:
L. 62: Be specific with what you mean by 'it': 'the proposed method'.

Response #17:
We have changed it to "The proposed method".

Comment #18:
L. 76: Please explain to the reader what you mean by 'scale' and 'location'.

Response #18:
Scale and location for spatial series correspond to frequency (periodicity) and time, respectively. As mentioned above, we have added "For convenience, we will mainly refer to location and scale irrespective of spatial or time series unless otherwise mentioned". (Lines 43-45).

Comment #19:
L. 99: same for 'phase angle'.

Response #19:
We have added its explanation in the bracket as "(i.e., angle between two complex numbers)" at Line 153.

Comment #20:
L. 184-185: can in my opinion be removed.

Response #20:
We have removed this sentence.

Comment #21:
L. 191: what does data refer to? Soil water content?

Response #21:
It refers to soil water datasets. Now removed as you suggested.

Comment #22:
L. 214: 'significance band'.

Response #22:
We have changed it to significance band.

Comment #23:
L. 215-216: is this statement underlined by any analysis performed?

Response #23:
Yes. The number is obtained from calculation.

Comment #24:
L. 247: what is the purpose of replacing half of the time series by 0?

Response #24:
As we highlighted in Section 3.1, "second half of the original series of y2 (or z2) are replaced by 0 to simulate abrupt changes (i.e., transient and localized feature) of the spatial series". (Lines 227-228).

Comment #25:
L. 261-263: Which feature in the plots actually indicates these 'abrupt changes'?

Response #25:
The abrupt changes were captured by the abrupt transition from coherence of 0 to coherence of 1 as shown in figure 1i and 1m of current version (top 2 at the left hand side of figure 2 in the original submission).

Comment #26:
L. 266: I can only see one wavelet band of high significance in Figure 3. Where is the second one you mention here?

Response #26:
We did not show the results here, but it was shown in Fig. 2 of our previous paper (Hu and Si, 2016). For this reason, the citation of "(Hu and Si, 2016)" was added here.

Comment #27:
L. 298: introduce term 'octave'.

Response #27:
We have added the explanation "octave refers to the scaled distance between two scales with one scale being twice or half of the other." (Lines 466-467).

Comment #28:
L. 363-366: would move this sentence to discussion section.

Response #28:
Yes, we have moved this sentence to the discussion section.

Thanks again for your constructive comment.

Response to Anonymous Referee #3

Anonymous Referee #3

Comment #1:

In this paper, the authors presented an improved variant of PWC for identifying the relationship between variables. This should be reflected in the title (like Improved PWC etc to be included in the title) to convey novel contribution. Also at present it is misleading like the authors proposes PWC concept.

Response #1:

Many thanks for your comments. We have changed the title to "Technical Note: Improved partial wavelet coherency for understanding scale-specific and localized bivariate relationships in geosciences".

Overall the paper is well written. I recommend for minor revision.

Comment #2:

Line 18– and producing more accurate results.- pl give quantitative statements

Response #2:
As the two methods in case of one excluding variables have theoretical differences, the outperformance is obvious. However, the degree of outperformance depends, in the case of our artificial dataset, the new method produces PWC values more close to 1 than the existing method as we expect although the difference is not big (e.g., PWC value of 1.0 versus 0.97 between y and y_2 at the scale of 8 after excluding the effect of y_4). However, the comparison of these two methods using real data indicated that the difference between the two methods can be large. For example, the differences in PWC between evaporation (E) and relative humidity (RH) after excluding the effect of mean temperature (T) can be 0.4-0.6 at the scales of about 1 year. For this reason, rather than giving quantitative statements, we have pointed out why the proposed method produces more accurate results by changing the sentence to "Compared with the previous PWC calculation, the new method produces more accurate results where there is one excluding variable. This is because bivariate real coherence rather than the bivariate complex coherence was mistakenly used in the previous PWC calculation, which underestimates the PWC.". (Lines 22-25).

Comment #3:
Line 31- provide the developments in chronological order – should be checked at all places
What is the real advantage in bringing the phase information in practical cases? this should be mentioned in the introduction section

Response #3:
All citations were changed in a chronological order.
The importance of phase information have been explained by adding "without phase information, it is hard to tell if the correlation at a location and scale is positive or negative." (Lines 87-88)

Comment #4:

Line 109 .. sufficient number of times using : : :pl make it clear
Response #4:

Discussion on the sufficient number of times was added as we explained in the **Response #1** to the **Referee #1** above.

Comment #5:
Line 214- significance band
Response #5:

We have changed it to significance band.

Comment #6:
Conclusion: Avoid the statements like – 'this new method produces slightly more accurate coherence'

Response #6:

We have changed it to "Compared with the previous PWC method, the new PWC method has the advantage of dealing with more than one excluding variable and providing the phase information (i.e., correlation type) associated with the PWC. In the case of one excluding variable, this new method produces more accurate coherence than the previous PWC method because the former considers complex coherence between every two variables, while the latter only considers the real coherence "(Lines 492-497).

Comment #7:

Line 450-455 should be explained better ; how can you overcome such problems ? I think better to provide a discussion section before conclusion where such references and unfamiliar terms can be explained in a better way. Then conclusion section should be presented as more specific

Response #7:

New discussion section was be added by moving this part to the discussion section. In terms of spurious correlations and multiple-testing problem, we have put it to a new section 5.2 weaknesses. Meanwhile, the advantages was mentioned in section 5.1. Please see the detailed revision at Lines 399-486 which has also shown above.

Thanks again for your constructive comment.

Technical Note: Improved ~~P~~partial wavelet coherency for ~~improved~~ understanding ~~of~~ scale-specific and localized bivariate relationships in geosciences

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Abstract

Bivariate wavelet coherency is ~~widely used to untangle the scale-specific and localized bivariate relationships in geosciences.~~a measure of correlation between two spatial (or time) series in the location-scale (or time-frequency) domain. It is particularly suited to geoscience where relationships between multiple variables commonly differ with locations or/and scales because of various processes involved. ~~–~~However, it is well-known that bivariate relationships can be misleading when both variables are dependent~~correlated~~ ~~to~~ on other variables. Partial wavelet coherency (PWC) has been proposed to detect the scale-specific and localized bivariate relationships by excluding the effects of other variables, but is limited to one excluding variable and presents no phase information. We aim to develop a new PWC method that can deal with multiple excluding variables and presents phase

information~~ for the PWC~~. Both stationary and non-stationary artificial datasets with the response variable being the sum of five cosine waves at 256 locations are used to test the method~~Test~~s. The new method was also applied to a free water evaporation dataset. Our results~~ with both stationary and non-stationary artificial datasets~~ verified the advantages of the new method in capturing phase information and dealing with multiple excluding variables. ~~–~~C~~known scale- and localized bivariate relationships after eliminating the effects of other variables. C~~ompared with the previous PWC ~~method~~calculation~~–~~, t~~the ~~he new method ~~has the advantages of capturing phase information, dealing with multiple excluding variables, and producing ~~produces more accurate results where there is one excluding variable. This is because bivariate real coherence rather than the bivariate complex coherence was mistakenly used in the previous PWC calculation, which underestimates the PWC. ~~The new method was also applied to two field measured datasets. Results showed that the coherency between response and predictor variables was usually less affected by excluding variables when predictor variables had higher correlation with the response variable. Application of the new method also confirmed the best predictor variables for explaining temporal variations in free water evaporation at Changwu site in China and spatial variations in soil water content in a hummocky landscape in Saskatchewan Canada.~~ We suggest the PWC method to be used in combination with previous wavelet methods to untangle the scale-specific and localized multivariate relationships in geosciences. The PWC calculations were coded with Matlab and are freely accessible~~vailable in the supplement~~ (https://figshare.com/s/bc97956f43fe5734c784).

1. Introduction

Geoscience data, such as spatial distribution of soil moisture in undulating terrains and ~~temporal~~ time series of climatic variables, usually consist of a variety of transient processes with different scales or ~~frequencies~~ frequencies~~(scales)~~ that may be localized in ~~time~~ space or time ~~space~~(Torrence and Compo, 1998; Si, 2008; Graf et al., 2014). For example, time series of air temperature usually fluctuates periodically at different scales (e.g., daily and yearly), but abrupt changes in air temperature (e.g., extremely high or low) may occur at certain time points as a result of extreme weather and climate events (e.g., heat and rain). ~~(Torrence and Compo, 1998; Si, 2008; Graf et al., 2014).~~ Wavelet methods are widely used to detect scale-specific and localized features of geoscience data irrespective of whether they are stationary or non-stationary.

Wavelet analyses are based on wavelet transform using mother wavelet function which expands spatial (or time) series into location-scale (or time-frequency) space for identification of localized intermittent scales (or frequencies). For convenience, we will mainly refer to location and scale irrespective of spatial or time series unless otherwise mentioned. Among ~~which~~these wavelet methods, bivariate wavelet coherency (BWC) is widely accepted as a tool for detecting scale-specific and localized bivariate relationships in a range of areas in geoscience (Lakshmi et al., 2004; Si and Zeleke, 2005; Das and Mohanty, 2008; Polansky et al., 2010; Biswas and Si, 2011). The BWC partitions correlation between two variables into different locations and scales, which are different from the overall relationships at the sampling scale as shown by the traditional correlation

Formatted: Font: (Default) Times New Roman, 12 pt, Font color: Text 1 coefficient. For example, BWC analysis indicated that soil water content of a hummocky landscape in the Canadian Prairies was negatively correlated to soil organic carbon content at a slope scale (50 m), but they were positively correlated at a watershed scale (120 m) in summer because of the different processes involved at different scales (Hu et al., 2017). Because the positive correlation may cancel out with the negative at different scales and/or locations, the traditional correlation coefficient between soil water content and soil organic carbon content does not differ significantly from zero, which is misleading.

Recently, Hu and Si (2016) have extended the BWC to multiple wavelet coherence (MWC) that can be used to untangle multivariate (≥3 variables) relationships in multiple location-scale ~~location~~ domains. This method has been successfully used in hydrology (Hu et al., 2017; Nalley et al., 2019; Su et al., 2019; Gu et al., 2020; Mares et al., 2020) and other areas such as soil science (Centeno et al., 2020), environmental science (Zhao et al., 2018), ~~climate~~ meteorology (Song et al., 2020), and economics (Sen et al., 2019).

– The MWC application has shown that an increased number of predictor variables does not necessarily explain more variations in the response variable, partly because predictor variables are usually cross-correlated (Hu and Si, 2016). For the same reason, bivariate relationships can be misleading if the predictor variable is correlated with other variables that control the response variable. Partial correlation analysis is one such method to ~~deal with~~avoid th~~is issue~~e misleading relationships resulting from the interdependence between other variables and both predictor and response variables (Kenney and Keeping, 1939), but the extension of partial correlation to the multiple location-scale ~~location~~ domain is limited.

In order to better understand the bivariate relationships at multiple scales and locations, the BWC needs to be extended to partial wavelet coherency (PWC) by eliminating the effects of other variables.

The BWC was extended to PWC by Mihanović et al. (2009). Their method has been widely ~~employed~~ used in the areas of marine science (Ng and Chan, 2012a, b), meteorology ~~climate~~ (Tan et al., 2016; Rathinasamy et al., 2017), and economics (Aloui et al., 2018; Altarturi et al., 2018a; Wu et al., 2020), as well as in the study of greenhouse gas emissions (Jia et al., 2018; Li et al., 2018; Mutascu and Sokic, 2020), ~~and economics (Aloui et al., 2018; Altarturi et al., 2018a; Wu et al., 2020),~~ among others. For example, PWC analysis indicated that Southern Oscillation Index and Pacific Decadal Oscillation did not affect precipitation across India~~(Ng and Chan, 2012a)~~, while this was misinterpreted by the BWC analysis because of their interdependence on Niño 3.4 that affects precipitation (Rathinasamy et al., 2017). However, Mihanović et al. (2009) considered one excluding variable (i.e., variable that influences the response variable is excluded) –only and did not include the phase angle difference between response and predictor variables. The coherence between response and predictor variables can still be misleading if more than one variable is interdependent with the predictor variable. This is especially true if these variables are correlated with the predictor variable at different locations and/or scales. In addition, without phase information, it is hard to tell if the correlation at a location and scale is positive or negative.

Field Code Changed

Field Code Changed

As an extension of previous studies (Mihanović et al., 2009; Hu and Si, 2016), this paper aims to develop a PWC method that considers more than one excluding variable and presents phase information. This method reveals the magnitude and type of bivariate relationships after removing the effects from all potentially interdependent variables.~~.~~ The new method is an extension ~~developed in analogy with~~from the ~~partial coherency in the multiple~~ multi-variate ~~spectral~~ partial coherency in ~~case~~the frequency (scale) domain (Koopmans, 1995). ~~It~~ The proposed method is first tested with artificial datasets following Yan and Gao (2007) and Hu and Si (2016) to demonstrate its capability of capturing the known relationships of the artificial data. ~~Next, the new method is compared with the Mihanović et al. (2009) method.~~ Then it is applied to ~~two~~ a real ~~(i.e., field measured)~~ dataset, i.e.,~~s in geosciences including~~ ~~temporal~~ time series of free water evaporation at the Changwu site in China (Hu and Si, 2016)~~ and spatial series of soil water content from a transect in the hummocky landscape in Saskatchewan, Canada (Biswas and Si, 2011a; Hu et al., 2017)~~. ~~These two datasets are chosen because the MWC results previously presented (Hu and Si, 2016) can be used to assess the new method.~~Finally, the advantages and weaknesses of the new method are discussed by comparing it with the previous PWC method.

2. Theory

—Wavelet analysis is based on the calculations of wavelet coefficients using wavelet transform at different locations and scales for each variable involved. Two types of wavelet transform exist including continuous wavelet transform and discrete wavelet transform. While the discrete wavelet transform is mainly used for data compression and noise

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reduction, the continuous wavelet transform is widely used for extracting scale-specific and localized features, as is the case of this study (Grinsted et al., 2004). For the continuous wavelet transform, the Morlet wavelet is used as a mother wavelet function to transform a spatial (or time) series into location-scale (or time-frequency) domain, which allows us to identify both location-specific amplitude and phase information of wavelet coefficients at different scales (Torrence and Compo, 1998). From wavelet coefficients, auto- and cross-wavelet power spectra for two variables can be calculated as the product of wavelet coefficient and the complex conjugate of itself (auto-wavelet power spectra) or another variable (cross-wavelet power spectra). The BWC is calculated as the ratio of smoothed cross-wavelet power spectra of two variables to the product of their auto-wavelet power spectra (Grinsted et al., 2004). Hu and Si (2016) extended wavelet coherence from two to multiple (≥3) variables and developed MWC. Detailed information on the calculations of wavelet coefficients, auto- and cross-wavelet power spectra, BWC, and MWC based on the continuous wavelet transform can be found elsewhere (Torrence and Compo, 1998; Grinsted et al., 2004; Si and Farrell, 2004; Si, 2008; Hu and Si, 2016; Hu et al., 2017). Here, we will only introduce the theory and calculation that is very relevant to the PWC.

Similar to BWC and MWC, PWC is calculated from auto- and cross-wavelet power spectra, for the response variable y, predictor variable x, and excluding variables Z ($Z = \{Z_1, Z_2, \cdots, Z_q\}$). Koopmans (1995) developed the multivariate complex PWC in the frequency (scale) domain~~In analogy with the partial coherency in the multivariate spectral case (Koopmans, 1995),~~. Here, we extend the Koopmans (1995) method from the frequency (scale) domain to the time-frequency (location-scale) domain. Therefore, the complex PWC

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Formatted: Font: (Default) Times New Roman, 12 pt between y and x after excluding variables Z at scale s and location τ, $\gamma_{y,x\cdot Z}(s,\tau)$ $\cancel{\gamma_{\overline{y,x\cdot Z}}}$, can be written as~~:~~

$$\gamma_{y,x\cdot Z}(s,\tau)\,\cancel{\gamma_{\overline{y,x\cdot Z}}(s,\tau)} = \frac{\left(1 - R^2_{y,x,Z}(s,\tau)\right)\cancel{\gamma_{\overline{y,x}}(s,\tau)}\,\gamma_{y,x}(s,\tau)}{\cancel{\left(\left(1 - R^2_{y,Z}(s,\tau)\right)\left(1 - R^2_{x,Z}(s,\tau)\right)\right)^{1/2}}\sqrt{\left(1 - R^2_{y,Z}(s,\tau)\right)\left(1 - R^2_{x,Z}(s,\tau)\right)}} \qquad (1)$$

where $R^2_{y,x,Z}(s,\tau)$, $R^2_{y,Z}(s,\tau)$, and $R^2_{x,Z}(s,\tau)$ can be calculated by following Hu and Si (2016) as

$$R^2_{y,x,Z}(s,\tau) = \frac{\overleftrightarrow{W}^{y,Z}(s,\tau)\,\overleftrightarrow{W}^{Z,Z}(s,\tau)^{-1}\,\overline{\overleftrightarrow{W}^{x,Z}(s,\tau)}}{\overleftrightarrow{W}^{y,x}(s,\tau)} \qquad (2)$$

$$R^2_{y,Z}(s,\tau) = \frac{\overleftrightarrow{W}^{y,Z}(s,\tau)\,\overleftrightarrow{W}^{Z,Z}(s,\tau)^{-1}\,\overline{\overleftrightarrow{W}^{y,Z}(s,\tau)}}{\overleftrightarrow{W}^{y,y}(s,\tau)} \qquad (3)$$

$$R^2_{x,Z}(s,\tau) = \frac{\overleftrightarrow{W}^{x,Z}(s,\tau)\,\overleftrightarrow{W}^{Z,Z}(s,\tau)^{-1}\,\overline{\overleftrightarrow{W}^{x,Z}(s,\tau)}}{\overleftrightarrow{W}^{x,x}(s,\tau)} \qquad (4)$$

Eq. (1) can be also derived analogously from the complex partial spectrum for the frequency domain and the definition of complex coherence between two variables in the time-frequency domain (see the Supplement (Sect. S1) for the derivation process). Note that $R^2_{y,x\cdot Z}(s,\tau)$ is a matrix with complex values while $R^2_{y,Z}(s,\tau)$ and $R^2_{x,Z}(s,\tau)$ are matrices with real numbers.

$\gamma_{y,x}(s,\tau)$ $\cancel{\gamma_{\overline{y,x}}(s,\tau)}$ is the complex wavelet coherence between y and x, which can be written as

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$$\gamma_{y,x}(s,\tau)\ \cancel{\gamma_{\overline{y,x}}(s,\tau)} = \frac{\overleftrightarrow{W}^{y,x}(s,\tau)}{\left(\overleftrightarrow{W}^{y,y}(s,\tau)\overleftrightarrow{W}^{x,x}(s,\tau)\right)^{1/2}} \tag{5}$$

where $\underset{(\cdot)}{\leftrightarrow}$ is the smoothing operator, $\overline{(\cdot)}$ is the complex conjugate operator, $(\cdot)^{-1}$ indicates the inverse of the matrix, and

$$\overleftrightarrow{W}^{y,Z}(s,\tau) = \left[\overleftrightarrow{W}^{y,Z_1}(s,\tau)\ \overleftrightarrow{W}^{y,Z_2}(s,\tau) \cdots \overleftrightarrow{W}^{y,Z_q}(s,\tau)\right] \tag{6}$$

$$\overleftrightarrow{W}^{x,Z}(s,\tau) = \left[\overleftrightarrow{W}^{x,Z_1}(s,\tau)\ \overleftrightarrow{W}^{x,Z_2}(s,\tau) \cdots \overleftrightarrow{W}^{x,Z_q}(s,\tau)\right] \tag{7}$$

$$\overleftrightarrow{W}^{Z,Z}(s,\tau) = \begin{bmatrix} \overleftrightarrow{W}^{Z_1,Z_1}(s,\tau) & \cdots & \overleftrightarrow{W}^{Z_1,Z_q}(s,\tau) \\ \vdots & \ddots & \vdots \\ \overleftrightarrow{W}^{Z_q,Z_1}(s,\tau) & \cdots & \overleftrightarrow{W}^{Z_q,Z_q}(s,\tau) \end{bmatrix} \tag{8}$$

where $\overleftrightarrow{W}^{A,B}(s,\tau)$ is the smoothed auto-wavelet power spectra (when A=B) or cross-wavelet power spectra (when $A \neq B$) at scale s and location τ, respectively. ~~Please refer to previous publications for detailed calculation of smoothed auto- and cross-wavelet power spectra (Grinsted et al., 2004; Hu and Si, 2016).~~

The squared PWC (hereinafter referred to as PWC) at scale s and location τ, $\rho^2_{y,x\cdot Z}$, can be written as

$$\rho^2_{y,x\cdot Z} = \frac{\left|1-R^2_{y,x,Z}(s,\tau)\right|^2 R^2_{y,x}(s,\tau)}{\left(1-R^2_{y,Z}(s,\tau)\right)\left(1-R^2_{x,Z}(s,\tau)\right)} \tag{9}$$

where $R^2_{y,x}(s,\tau)$ is squared BWC between y and x, which can be expressed as

$$R^2_{y,x}(s,\tau) = \frac{\overleftrightarrow{W}^{y,x}(s,\tau)\overline{\overleftrightarrow{W}^{y,x}(s,\tau)}}{\overleftrightarrow{W}^{y,y}(s,\tau)\overleftrightarrow{W}^{x,x}(s,\tau)} \tag{10}$$

The phase angle (i.e., angle between two complex numbers) between y and x after excluding effect of Z is

$$\cancel{\vartheta_{\overline{y,x\cdot Z}}(s,\tau)}\vartheta_{y,x\cdot Z}(s,\tau) = \varphi_{y,x\cdot Z}(s,\tau) + \vartheta_{y,x}(s,\tau)\cancel{\varphi_{\overline{y,x\cdot Z}}(s,\tau) + \vartheta_{\overline{y,x}}(s,\tau)} \qquad (11)$$

where

$$\varphi_{y,x\cdot Z}(s,\tau)\cancel{\varphi_{\overline{y,x\cdot Z}}(s,\tau)} = \arg\left(1 - R^2_{y,x\cdot Z}(s,\tau)\right) \qquad (12)$$

and ~~$\vartheta_{\overline{y,x}}(s,\tau)$~~ $\vartheta_{y,x}(s,\tau)$ is the wavelet phase between y and x, which can be expressed as

$$\vartheta_{y,x}(s,\tau)\ \cancel{\vartheta_{\overline{y,x}}(s,\tau)} = \tan^{-1}\left(\mathrm{Im}\left(W^{y,x}(s,\tau)\right)/\mathrm{Re}\left(W^{y,x}(s,\tau)\right)\right) \qquad (13)$$

where arg denotes the argument of the complex number, $W^{y,x}(s,\tau)$ is the cross-wavelet power spectrum between y and x at scale s and location τ; Im and Re denote the imaginary and real part of $W^{y,x}(s,\tau)$, respectively.

When only one variable (e.g., $Z1$) is excluded, Eq.(9) can be written as (see the Supplement (Sect. S2) for the derivation process)

$$\rho^2_{y,x\cdot Z1} = \frac{\left|\gamma_{y,x}(s,\tau) - \gamma_{y,Z1}(s,\tau)\overline{\gamma_{x,Z1}(s,\tau)}\right|^2}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)} \qquad (14)$$

The widely used Monte Carlo method (Torrence and Compo, 1998; Grinsted et al., 2004; Si and Farrell, 2004) is ~~used~~ used to calculate PWC at the 95% confidence level. In brief, the ~~calculation of~~ PWC calculation is repeated for a sufficient number of times using data

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generated by Monte Carlo simulations ~~–~~based on the first-order autocorrelation coefficient (r1). ~~–~~The first-order autoregressive model (AR(1)) is chosen because it can be used to simulate most geoscience data very well (Wendroth et al., 1992; Grinsted et al., 2004; Si and Farrell, 2004). Different combinations of r1 values (i.e., 0.0, 0.5, and 0.9) were used to generate 10 to 10 000 AR(1) series with three, four and five variables. Our results indicate that the noise combination has little impact on the PWC values at the 95% confidence level as also found by Grinsted et al. (2004) for the BWC case (data not shown). The relative difference of PWC at the 95% confidence level compared with that calculated from the 10 000 AR(1) series decreases with the increase in number of AR(1) series. When the number of AR(1) is above 300, a very low maximum relative difference (e.g., <2%) is observed (Fig. S1 of Sect. S3 in the Supplement). Therefore, a repeating number of 300 seems to be sufficient for a significance test. However, if calculation time is not a barrier, a higher repeating number, such as ≥1000, ~~Grinsted et al. (2004)~~is recommended. The 95th percentile of PWCs of all simulations at each scale represents the PWC at the 95% confidence level. The average PWC, percent area of significant coherence (PASC) relative to the whole wavelet location–scale ~~–location~~ domain, and average value of significant PWC (PWC_{sig}) are also calculated for different location–scale ~~scale-location~~ domains. ~~The Matlab codes for calculating PWC and significance level are provided in the Supplement (Sect. S1–S3).~~

~~The new method is compared with the method of Mihanović et al. (2009) in~~ In the case of one excluding variable ($Z = \{Z_1\}$~~).~~), Mihanović et al. (2009) suggested that the PWC can be calculated by an equation analogous to the traditional partial correlation squared (Kenney and Keeping, 1939) without giving the detailed derivation process. Their equation

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is the same as Eq. (14). Unfortunately, Ng and Chan (2012a) might have misinterpreted the equation of Mihanović et al. (2009) and developed Matlab code for calculating PWC using the equation expressed as ~~The Mihanović et al. (2009) method was developed directly from the traditional partial correlation analysis (Kenney and Keeping, 1939), and therefore has a similar equation for calculating PWC, which can be expressed as~~

$$\rho^2_{y,x\cdot Z1} = \frac{\left|\cancel{R_{\overline{y,x}}}R_{y,x}(s,\tau)-R_{y,Z1}(s,\tau)\cancel{R_{\overline{y,Z1}}(s,\tau)}\,R_{x,Z1}(s,\tau)\cancel{R_{\overline{x,Z1}}(s,\tau)}\right|^2}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)}$$

(~~14~~15)

where $R_{y,x}(s,\tau)$, $R_{y,Z1}(s,\tau)$, and $R_{x,Z1}(s,\tau)$ are the square root of $R^2_{y,x}(s,\tau)$, $R^2_{y,Z1}(s,\tau)$, $R^2_{x,Z1}(s,\tau)$, respectively. $R^2_{y,Z1}(s,\tau)$ and $R^2_{x,Z1}(s,\tau)$ can be calculated from Eq. (10) by replacing y and x with their corresponding variables. Eq. (15) has been widely used to calculate PWC in the case of one excluding variable (Ng and Chan, 2012b; Rathinasamy et al., 2017; Aloui et al., 2018; Altarturi et al., 2018b; Jia et al., 2018; Li et al., 2018; Mutascu and Sokic, 2020; Wu et al., 2020). Note that complex coherence and real coherence are involved in the numerators of Eqs. (14) and (15), respectively, while the denominators are exactly the same. ~~In the case of one excluding variable, the numerators between Eqs. (9) and (14) differ, but the denominators remain the same.~~Further comparison indicates that Eq. (15) underestimates PWC value relative to Eq. (14) unless $\gamma_{y,x}(s,\tau)$ and $\gamma_{y,Z1}(s,\tau)\overline{\gamma_{x,Z1}(s,\tau)}$ in Eq. (14) are collinear (i.e., their arguments are identical) under which the two equations produce the same PWC values. Differences between Eqs. (14) and (15) will be discussed further using both artificial data and a real dataset. For comparison purposes, we refer to Eqs. (14) and (15) as the new method and the classical method, respectively.

3. Method test using artificial data ~~Data and analysis~~

3.1 Artificial data and analysis ~~for method test~~

The PWC is first tested using the cosine-like artificial dataset produced following Yan and Gao (2007). The cosine-like artificial datasets are suitable for testing the new method because they mimic many spatial or temporal series in geoscience such as climatic variables, hydrologic fluxes, seismic signals, El Niño-Southern Oscillation, land surface topography, ocean waves, and soil moisture. The procedures to test the PWC is largely based on Hu and Si (2016), where the same dataset has been used to test the MWC method~~. Please~~ (refer to Hu and Si (2016) for ~~the~~ a detailed description of the artificial dataset). T~~In brief, t~~he response variable (y and z for the stationary and non-stationary case, respectively) is the sum of five cosine waves (y_1 to y_5 and z_1 to z_5 for the stationary and non-stationary case, respectively) at 256 locations (Hu and Si, 2016). For y_1, y_2, y_3, y_4, and y_5, they have consistent dimensionless scales of 4, 8, 16, 32, and 64, respectively, across the series. For z_1, z_2, z_3, z_4, and z_5, the dimensionless scales gradually change with location, with the maximum dimensionless scales of 4, 8, 16, 32, and 64, respectively. The variance of the response variable y and z is 2.5. All other variables (y_1 to y_5 or z_1 to z_5) are orthogonal to each other with equal variance of 0.5. The predictor and excluding variables (Fig. S1 of Sect. S4 in the Supplement) are selected from the five cosine waves (e.g., y_1 to y_5 or z_1 to z_5) or their derivatives. The exact variables and procedures to test the new PWC method are explained ~~later on~~below.

The PWC between response variable y (or z) and predictor variable, i.e., y_2 (or z_2), is first calculated after excluding the effect of one variable. Four types of excluding variable are involved (Fig. ~~S1~~ S2 of Sect. S4 in the Supplement): (a) original series of y_2 (or z_2) or y_4 (or z_4); (b) second half of the original series of y_2 (or z_2) are replaced by 0 to simulate abrupt changes (i.e., transient and localized feature) of the spatial series. They are referred to as y_2h_0 (*or* z_2h_0); (c) white noises with zero-mean and standard deviations of 0.3 (weak noise), 1 (moderate noise), and 4 (high noise) are added to y_2 (or z_2) as suggested by Hu and Si (2016) to simulate non-perfect cyclic patterns of the excluding variables. They are referred to as y_2wn (*or* z_2wn), y_2mn (*or* z_2mn), and y_2sn (*or* z_2sn), respectively; and (d) a combination of type b and type c. They are referred to as y_2wnh_0 (*or* z_2wnh_0), y_2mnh_0 (*or* z_2mnh_0), and y_2snh_0 (*or* z_2snh_0), respectively. ~~The same data are also analyzed using the Mihanović et al. (2009) method for comparison.~~

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The PWC between response variable y (or z) and predictor variable, i.e., y_2y_4 (sum of y_2 and y_4) for the stationary case or z_2z_4 (sum of z_2 and z_4) for the non-stationary case, is calculated with two excluding variables, which is a combination of y_4 (or z_4) and y_2 (or z_2) or its noised series (y_2wn *or* z_2wn, y_2mn *or* z_2mn, and y_2sn *or* z_2sn). Note that PWC between y (or z) and other predictor variables (e.g., y_4 or z_4) after excluding y_2 or z_2 and their equivalent derivative variables (i.e., noised variables or variables with 0) are also calculated. The related results are not shown because they are analogous to those in case of predictor variable of y_2 (or z_2).

The merit of the artificial data is that we know the exact scale-specific and localized bivariate relationships after the effect of excluding variables is removed. Theoretically, we expect (a) PWC is 1 at scales corresponding to scale difference of excluding variables from predictor variable, and 0 at other scales. For example, PWC between y and y_2y_4 after excluding the effect of y_4 is expected to be 1 at the scale of 8, which is the difference of y_4 (32) from y_2y_4 (8 and 32), and 0 at other scales (e.g., 32); (b) PWC remains 1 at the second half of series where spatial series is replaced by 0, and 0 at the first half of the original series. For example, PWC between y and y_2 after excluding the effect of y_2h_0 is expected to be 0 and 1 at the first and second half of series, respectively, at the scale of 8; and (c) PWC increases as more noises are included in the excluding variables. For example, PWC between y and y_2 after excluding the effect of noised series of y_2 is expected to increase with increasing noises in an order of $y_2sn>y_2mn>y_2wn$ at the scale of 8.

~~3.2~~1.1 ~~Real data for application~~

~~3.2.1~~1.1.1 ~~Free water evaporation~~

~~The free water evaporation dataset has been used to test the MWC (Hu and Si, 2016). In brief, this dataset includes monthly free water evaporation (E), mean temperature (T), relative humidity (RH), sun hours (SH), and wind speed (WS) between January 1979 and December 2013 at Changwu site in Shaanxi province provided by the China Meteorological Administration. During this period, the average daily temperature was 9.4 °C, the average annual rainfall was 571 mm and annual ET_p was 883 mm. Being located in the transition between semi-arid and subhumid climates, agricultural production at the Changwu site is constrained by water availability. The PWC between E and each meteorological variable is~~

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~~calculated by excluding the effect of each or all of the other meteorological variables. Results of wavelet power spectrum of E and BWC between every two variables are shown in Fig. S2 and Fig. S3 (Sect. S5 in the Supplement), respectively.~~

~~3.2.2~~1.1.1 ~~Soil water content~~

~~Soil water datasets were obtained from the hummocky landscape of Canadian Prairies (Biswas and Si, 2011b; Hu et al., 2017). The sampling site is characterized by a subhumid continental climate with Dark Brown Chernozem soils. Data were collected from 128 locations with equal intervals (4.5 m) along a 576 m long transect. Soil water contents of top layer (0–0.2 m) were measured by a portable Tektronix TDR in spring (May 2, 2008) and summer (August 23, 2008). Other environmental variables measured were clay content, sand content, soil organic carbon content (SOC), bulk density (BD) of 0–0.2 m, depth to $CaCO_3$ layer (vertical distance between surface and the layer of first presence of $CaCO_3$), elevation, slope, aspect (calculated as cos(aspect)), and wetness index. Please refer to previous studies for detailed information on this dataset~~ (Biswas and Si, 2011a, b; Biswas et al., 2012)~~.~~

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~~The PWC between SWC and each environmental variable is calculated by excluding the effect of another environmental factor. The BWC between SWC and each environmental factor (Fig. S4 and S5 of Sect. S5 in the Supplement), BWC between environmental factors (Fig. S6 of Sect. S5 in the Supplement), and MWC between SWC and environmental factors have been previously analyzed (Biswas and Si, 2011a; Hu et al., 2017).~~

~~4. Results and discussion~~

~~4.1~~3.2 PWC with artificial data

~~4.1.1~~3.2.1 PWC with one excluding variable using the new method

Fig. 1~~a~~ shows PWC between dependent variable y (or z) and predictor variable y_2 (or z_2) by excluding one variable. For the stationary case, there is one horizontal band (red color) representing an in-phase high PWC value at scales around 8 for all locations after eliminating the effect of y_4 (Fig. 1a). Note that the PWC values between y and y_2 after excluding the effect of y_4 are not exactly 1 as would be expected at all location-scale~~-location~~ domains, because of the effect of smoothing along locations ~~scales~~ and scales ~~locations~~. However, the PWC values at the center of the ~~significant~~ significance band, which correspond~~sing~~ to ~~the exact scale (8) of~~ the predictor variable y_2 at exactly the scale of 8, are very close to 1 (0.996), and the mean PWC_{sig} values are very high (i.e., 0.96). The result is similar to the BWC between y and y_2. This is understandable because y_4 is orthogonal to y_2, and excluding the effect of y_4 does not affect the relationship between y and y_2 at all.

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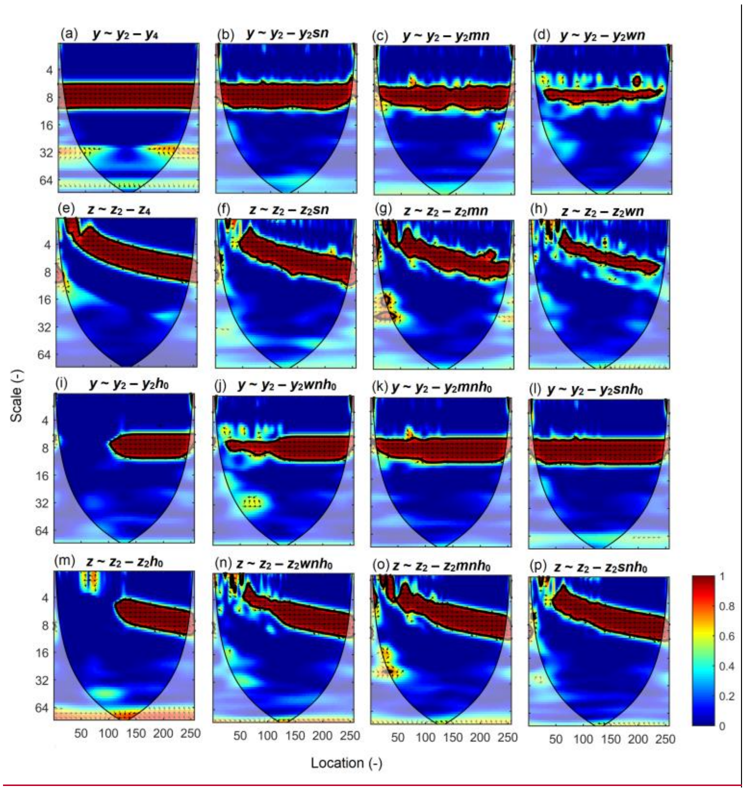

Figure 1.

Partial wavelet coherency (PWC) between response variable y (or z) and predictor variable y_2 (or z_2) after excluding the effect of variables y_4 (or z_4), y_2sn (or z_2sn), y_2mn (or z_2mn), y_2wn (or z_2wn), ~~and y_2 (or z_2)~~y_2h_0 (or z_2h_0), y_2wnh_0 (or z_2wnh_0), y_2mnh_0 (or z_2mnh_0), and y_2snh_0 (or z_2snh_0) for the stationary (or non-stationary) case using the new method~~ (a) and Mihanović et al. (2009) method (b)~~. Arrows represent the phase angles of the cross-wavelet power spectra between two variables after eliminating the effect of excluding variables. Arrows pointing to the right (left) indicate positive (negative) correlations. Thin and thick solid lines show the cones of influence and the 95% confidence levels, respectively. All variables were generated by following Yan and Gao (2007) and Hu and Si (2016) and are explained in Section 3.1 and ~~are~~ shown in Fig. ~~S1~~ S2 of Sect. ~~S4~~ S3 in the Supplement.

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Similar results were obtained by excluding either y_4 or the strongly noised series of y_2 (y_2sn). Compared with the case of excluding variable of y_4 (Fig. 1a), excluding the effect of y_2sn (Fig. 1b) results in slightly narrower band of significant PWC and slightly reduced mean PWC$_{sig}$ (0.94 versus 0.96). When less noise is included in the excluding variables (i.e., y_2mn and y_2wn) (Fig. 1c-d), the significant PWC band becomes narrower. The PASC values are 86%, 77%, and 32% for excluding y_2sn, y_2mn and y_2wn, respectively, at scales of 6–10. Moreover, the mean PWC$_{sig}$ decreases from 0.94 (y_2sn) to 0.93 (y_2mn) and 0.89 (y_2wn) when progressively more noise is added (Fig. 1b-d~~a~~). ~~If we exclude the predictor variable~~ ~~y_2~~ ~~itself, there are, as we expect, no correlations between~~ ~~y~~ ~~and~~ ~~y_2~~ ~~(Fig. 1a).~~ For the non-stationary case, similar results are obtained (Fig. 1e-h~~a~~). The only difference is that the scales with significant PWC values change with location, as is found for MWC (Hu and Si, 2016).

~~**Figure 2.**~~

~~Partial wavelet coherency (PWC) between response variable~~ ~~y~~ ~~(or~~ ~~z~~~~) and predictor variable~~ ~~y_2~~ ~~(or~~ ~~z_2~~~~) after excluding effect of variables~~ ~~y_2h_0~~ ~~(or~~ ~~z_2h_0~~~~),~~ ~~y_2wnh_0~~ ~~(or~~ ~~z_2wnh_0~~~~),~~ ~~y_2mnh_0~~ ~~(or~~ ~~z_2mnh_0~~~~), and~~ ~~y_2snh_0~~ ~~(or~~ ~~z_2snh_0~~~~), for the stationary (or non-stationary) case using the new method (a) and Mihanović et al. (2009) method (b). All variables are explained in Section 3.1 and are shown in Fig. S1 of Sect. S4 in the Supplement.~~

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When the second half of the excluding variable series is replaced by 0, the PWC values in that half are close to 1, while those in the first half of data series are 0 at scales corresponding to the predictor variable (Fig. ~~2~~1i and 1m~~a~~). For the stationary case, after excluding the effect of y_2h_0, the PWC values are close to 1 (0.98) and 0 in the second and first half of the data series, respectively, at the dimensionless scale of 8 (Fig. ~~2~~1i~~a~~). Similar results are observed for the non-stationary case (Fig. ~~2~~1m~~a~~). This is anticipated because the removing series of 0s from a portion of the predictor variable series does not affect their correlations at these locations. If different magnitudes of noises are added to the first half of the excluding variables (y_2 or z_2), the significant PWC band in the first half becomes wider as the magnitude of noises increases, while the significant PWC band in the second half remains almost unchanged (Fig. 1j-l and Fig. 1n-p). ~~Take~~ In the stationary case, for example, the PASC values at scales of 6–10 are 40% (y_2wnh_0), 74% (y_2mnh_0), and 86% (y_2snh_0) in the first half, ~~respectively,~~ while those values vary from 86% to 90% in the second half (Fig. 1j-l). Meanwhile, the mean PWC_{sig} in the first half at scales of 6–10 increases from 0.91 to 0.94 in both the stationary (Fig. 1j-l) and non-stationary (Fig. 1n-p) cases as more noises are added to the excluding variable y_2 or z_2. This indicates that the new PWC method can also capture the abrupt changes (Fig. 1i and 1m) in the data series, and has the ability to deal with localized relationships.

~~4.1.2~~3.2.2 PWC with two excluding variables using the new method

When both y_2 and y_4 (or z_2 and z_4) are considered in the predictor variables, there are two bands of wavelet coherence of 1 between y (or z) and y_2y_4 (or z_2z_4) (Hu and Si, 2016), which correspond to the scales of two predictor variables ~~(Hu and Si, 2016)~~. However, after the effect of y_4 (or z_4) is removed, only one band with PWC of around 1 occurs at the scale of the predictor variable y_2 (or z_2) (Fig. ~~3~~2a and 2f), which is identical to the PWC between y (or z) and y_2 (or z_2) after excluding the effect of variable y_4 (or z_4) (Fig. 1a and 1f). After both predictor variables y_2 and y_4 (or z_2 and z_4) are excluded (Fig. 2b and 2g), the PWC between y (or z) and y_2y_4 (or z_2z_4) is 0 at all location-scale ~~location~~ domains as we expect. When one of the excluding variables y_2 (or z_2) is added with noises, the relationship between response variable y (or z) and predictor variable y_2y_4 (or z_2z_4) becomes significant at scales of the excluding variable y_2 (or z_2) (Fig. 2c and 2h). Similar to the case of one excluding variable (Fig. 1), less noise in the excluding variable of y_2 (or z_2) results in a narrower significant PWC band, and reduced mean PWC_{sig} values (from 0.96 (y_2sn) to 0.90 (y_2wn) in the stationary case (Fig. 2c-e) and from 0.95 (z_2sn) to 0.92 (z_2wn) in the non-stationary case) (Fig. ~~3~~2h-j).

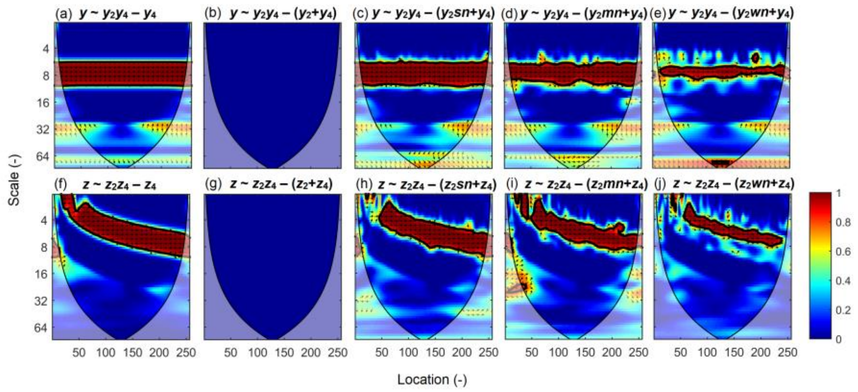

Figure ~~3~~2.

Partial wavelet coherency (PWC) between response variable y (or z) and predictor variable y_2y_4 (or z_2z_4) after excluding the effect of variables y_4 (or z_4), y_2+y_4 (or z_2+z_4), $y_2$sn$+y_4$ (or z_2sn+z_4), y_2mn+y_4 (or z_2mn+z_4), and y_2wn+y_4 (or z_2wn+z_4) for the stationary (or non-stationary) case using the new method. All variables were generated by following Yan and Gao (2007) and Hu and Si (2016) and are explained in Section 3.1 and ~~are~~ shown in Fig.

~~S1~~ S2 of Sect. ~~S4~~ S3 in the Supplement.

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~~4.1.3 Comparison of the new method with the Mihanović et al. (2009) method~~

~~In the case of one excluding variable, the corresponding PWC values calculated with the Mihanović et al. (2009) method are shown in Figs 1b and 2b. Except for the phase information, the two methods generally produce comparable coherence despite the differing numerators in their corresponding equations (Eq. 9 and 14). However, we notice that the new PWC method produces consistently slightly higher coherence than the Mihanović et al. (2009) method. For example, their mean PWCs between y and y_2 at the scale of 8 after excluding the effect of y_4 are 1.00 and 0.97, respectively. This may indicate that the new method slightly outperforms the Mihanović et al. (2009) method because we expect that the coherence between y and y_2 at the scale (8) of y_2 is exactly 1.~~

~~Note that some unexpected high PWC can be produced in some domains by the new method. For example, at a scale of 32, PWC values between y and y_2 after excluding y_4 are not significant, but relatively high, partly because of small octaves (default of 1/12) per scale. This spurious unexpected high PWC is caused by low values in both the numerator (partly associated with the low coherence between response y and predictor variables y_2 at scale of 32) and denominator (partly associated with the high coherence between response y and excluding variable y_4 at a scale of 32) in Eq. (9). The same problem also exists in the Mihanović et al. (2009) method (Fig. 1b and 2b). Particularly, the Mihanović et al. (2009) method produces some positive infinite coherence (small black zones) between y (or z) and y_2 (or z_2) after eliminating the effect of y_2h_0 (or z_2h_0) (Fig. 2b) because of extremely low~~

~~values in the both numerator and denominator term in Eq. (14). However, it seems that the domain with overestimation by the new method is very limited and it is located mainly outside of the cones of influence. Anyway, the unexpected results can be easily ruled out with knowledge of BWC between response and predictor variables.~~

~~Compared with the Mihanović et al. (2009) method, our new PWC method can be used to deal with situations with more than one excluding variable, which is a knowledge gap. Moreover, inclusion of phase information in the new PWC is another advantage of this method.(Hu et al., 2017)~~

4. Method application~~PWC~~ with real dataset

4.1 Description of free water evaporation dataset

~~Free water evaporation~~

The free water evaporation dataset ~~has been~~was used to test the MWC (Hu and Si, 2016). In brief, this dataset includes monthly free water evaporation (E), mean temperature (T), relative humidity (RH), sun hours (SH), and wind speed (WS) between January 1979 and December 2013 at Changwu site in Shaanxi province provided by the China Meteorological Administration. During this period, the average daily temperature was 9.4 °C, the average annual rainfall was 571 mm and annual ET_p was 883 mm. Being located in the transition between semi-arid and subhumid climates, agricultural production at the Changwu site is constrained by water availability. ~~The PWC between E and each meteorological variable is calculated by excluding the effect of each or all of the other meteorological variables.~~

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Results of wavelet power spectrum of E and BWC between every two variables are shown in Fig. S~~2~~3 and Fig. S~~3~~4 (Sect. S~~5~~3 in the Supplement), respectively.

4.2 PWC with free water evaporation dataset

~~Soil water content~~

~~Soil water datasets were obtained from the hummocky landscape of Canadian Prairies (Biswas and Si, 2011b; Hu et al., 2017). The sampling site is characterized by a subhumid continental climate with Dark Brown Chernozem soils. Data were collected from 128 locations with equal intervals (4.5 m) along a 576 m long transect. Soil water contents of top layer (0–0.2 m) were measured by a portable Tektronix TDR in spring (May 2, 2008) and summer (August 23, 2008). Other environmental variables measured were clay content, sand content, soil organic carbon content (SOC), bulk density (BD) of 0–0.2 m, depth to~~ ~~$CaCO_3$ layer (vertical distance between surface and the layer of first presence of $CaCO_3$), elevation, slope, aspect (calculated as cos(aspect)), and wetness index. Please refer to previous studies for detailed information on this dataset~~ (Biswas and Si, 2011a, b; Biswas et al., 2012)~~.~~

~~The PWC between SWC and each environmental variable is calculated by excluding the effect of another environmental factor. The BWC between SWC and each environmental factor (Fig. S4 and S5 of Sect. S5 in the Supplement), BWC between environmental factors (Fig. S6 of Sect. S5 in the Supplement), and MWC between SWC and environmental factors have been previously analyzed (Biswas and Si, 2011a; Hu et al., 2017).~~

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~~4.2.1 Free water evaporation~~

The PWC analysis indicates that the correlations between E and T after excluding the effect of each of other three variables (RH, SH, and WS) were almost the same as those indicated by the BWC (Fig. ~~4~~ 3a-c and Fig. ~~S3~~ S4 of Sect. ~~S5~~ S3 in the Supplement). For example, E and T, after excluding the effect of RH, ~~E and T~~ were positively correlated at the medium scales (8–32 months). The PASC was 61% and mean PWC_{sig} value was 0.94, which was identical to the case of BWC between E and T. ~~The~~ No significant correlations at scales around 64 months between E and T from 1979 to 1992 were ~~absent~~ found after eliminating the influence of RH (Fig. 3a-c). This implies that the influence of mean temperature on E at these scales and years may be associated with the negative influence of RH on both E and T (Fig. ~~S3~~ S4 of Sect. ~~S5~~ S3 in the Supplement).

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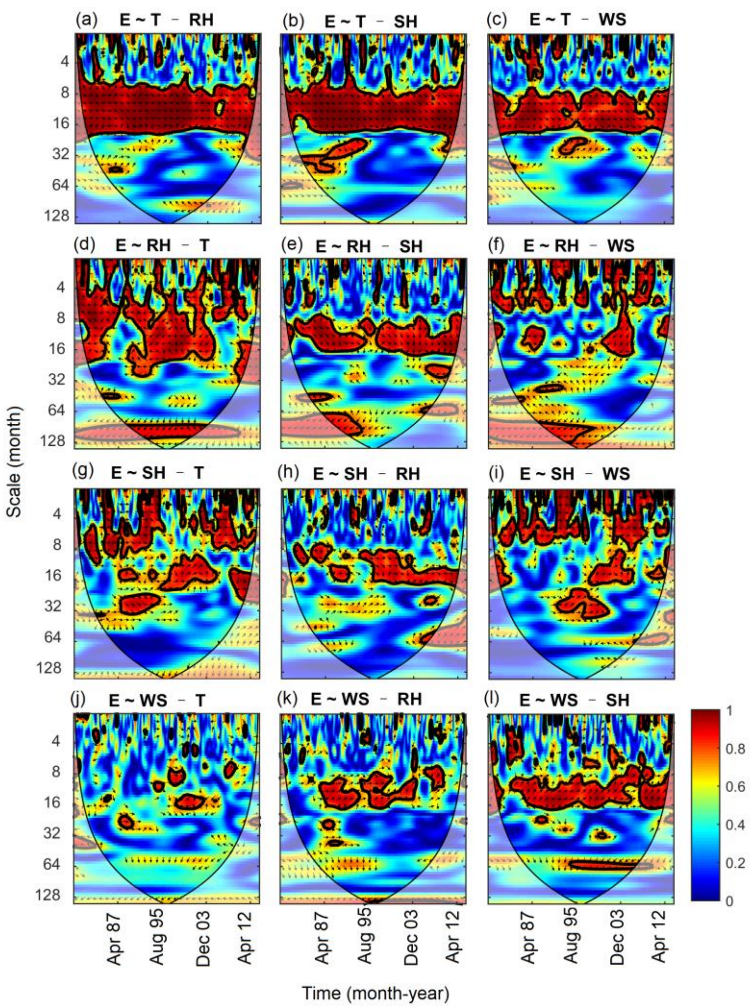

Figure 43.

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Partial wavelet coherency (PWC) between evaporation (E) and each meteorological factor (T, mean temperature; RH, relative humidity; SH, sun hours; WS, wind speed) after excluding the effect of each of other three meteorological factors.

The PWC between E and RH depended on the excluding variable and scale (Fig. ~~4~~3d-f). The mean PWC and PASC between E and RH after excluding T were 0.60 and 34%, respectively, which are comparable ~~to~~ with the mean BWC (0.62) and PASC (40%) between E and RH. The corresponding values after excluding SH and WS were 0.50 and 0.53 (PWC), 22% and 21% (PASC), respectively. In addition, compared with the BWC between E and RH (Fig. S4 of Sect. S3 in the Supplement), correlations between E and RH were ~~almost absent~~weak at small scales (<8 months) and medium scales (8–32 months) after eliminating the influence of SH and WS (Fig. 3e-f), respectively. Therefore, excluding the variable of T had less influence on the coherence between E and RH compared with excluding the variables of SH and WS. This is mainly because ~~relative humidity~~RH and ~~temperature~~ T are correlated with E at different scales (Fig. ~~S3~~ S4 of Sect. ~~S5~~ S3 in the Supplement), i.e., mean temperature affected E mainly at medium scales, while RH affected E across all scales. However, the domain where SH and WS were correlated with E was a subset of that where RH and E were correlated (Fig. S4 of Sect. S3 in the Supplement~~Fig. 4~~).

The relationships between E and ~~sun hours~~SH after excluding the other three factors were less consistent (Fig. 3g-h). The areas with significant corrections were scattered over the whole location-scale~~frequency-time~~ domain but differed with excluding factors. The PASC varied from 12% (excluding RH) to 20% (excluding T and WS), which is much lower than the PASC (28%) in the case of BWC. The significant relationships between E and WS were only limited to very small areas except for the case of SH being excluded, where E and ~~wind speed~~WS were positively correlated at scales of 8–16 months most of the time (Fig. 3j-l).

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In general, the PASC decreased after excluding the effects of more factors (data not shown). The correlations between E and each variable after eliminating the effects of all other variables are shown in Fig. ~~5~~4. The correlations between E and T were still significant at the medium scales (8–32 months) (Fig. 4a), where PASC value was 52% with mean PWC_{sig} of 0.92. The E was still correlated with RH at large scales (>32 months) (Fig. 4b), where PASC value was 35% with mean PWC_{sig} of 0.96. Interestingly, the domain with significant correlation between E and SH and WS was very limited (Fig. 4c-d). This indicates that the influences of SH and WS on E have already been covered by RH and T. This is in agreement with the MWC results that RH and T were the best to explain E variations at all scales (Hu and Si, 2016). Although the RH had the greatest mean wavelet coherence and PASC at the entire location~~scale location~~ scale domains, the PWC analysis seems to support that mean temperature was the most dominating factor for free water evaporation at the 1-year cycle (8–16 months), which is the dominant scale of E variation (Fig. ~~S2~~ S3 of Sect. ~~S5~~ S3 in the Supplement). ~~This further verifies the suitability of the Hargreaves model (only air temperature and incident solar radiation required) (Hargreaves, 1989) for estimating potential evapotranspiration on the Chinese Loess Plateau (Li, 2012).~~

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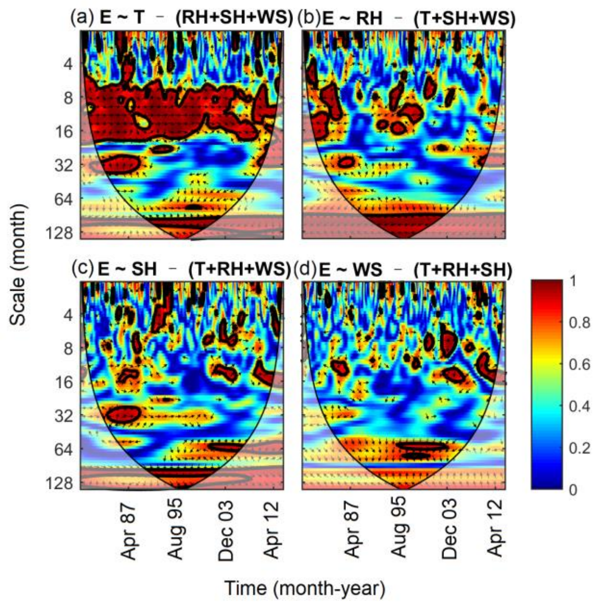

Figure ~~5~~4.

Partial wavelet coherency (PWC) between evaporation (E) and each meteorological factor (T, mean temperature; RH, relative humidity; SH, sun hours; WS, wind speed) after excluding the effects of all other three factors.

~~4.2.2 **SWC**~~

~~In spring, SWC at 0–0.2 m was significantly correlated with elevation, wetness index, depth to $CaCO_3$ layer, and SOC at large scales (72–144 m); it was significantly correlated with sand content, SOC, depth to $CaCO_3$ at medium scales (36–72 m) and bulk density at scales of 36–144 m in the first half of the transect (Fig. S4 of Sect. S5 in the Supplement). The PWC shows that SWC was not correlated with elevation after eliminating the effect of SOC or depth to $CaCO_3$ (Fig. 6). By contrast, after the removal of the elevation's effect, SWC was significantly correlated with SOC at scales of 36–144 m in the first half of the~~

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~~transect and significantly correlated with depth to $CaCO_3$ layer at large scales (>100 m) across the transect (Fig. 6). There were little correlations between SWC and wetness index after eliminating the effect of elevation (Fig. 6). Therefore, the influences of elevation and wetness index on SWC in spring might have been taken into account by SOC and depth to $CaCO_3$ layer. Although elevation and wetness index are important drivers of snowmelt run-off in spring (Hu et al., 2017), they did not contribute any more to explaining SWC variations than SOC or depth to $CaCO_3$ layer did. The same holds for bulk density and sand content whose influences on SWC were also limited after eliminating the effect of SOC (Fig. 6). This was because SOC was negatively correlated with sand content at medium scales (36–72 m) and bulk density at scales of 36–144 m in the first half of the transect (Fig. S5 of Sect. S5 in the Supplement). Interestingly, the significant correlations between SWC and SOC or depth to $CaCO_3$ layer still existed no matter what the excluding factors were. For example, SWC was significantly correlated with depth to $CaCO_3$ layer at scales >130 m after the effect of SOC was removed; SWC was significantly correlated with SOC at large scales (>130 m) across the transect and at scales of 36–90 m at locations from 45 to 200 m after eliminating the effect of depth to $CaCO_3$ layer (Fig. 6). This further validates that the combination of depth to $CaCO_3$ layer and SOC were the best to explain SWC variations in spring (Hu et al., 2017).~~

~~Figure 6.~~

~~Partial wavelet coherency (PWC) between soil water content (SWC) in spring and one~~

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~~environmental factor after excluding the effect of another environmental factor. SOC, soil organic carbon; $CaCO_3$, depth to the $CaCO_3$ layer; WI, wetness index; BD, bulk density.~~

~~In summer, SWC of 0–0.2 m tended to be significantly affected by aspect, slope, elevation, wetness index, clay, and sand at large scales (>90 m or 72–144 m) and by SOC, bulk density, and slope at medium scales (36–72 m) at locations from 45 to 450 m over the transect (Fig. S5 of Sect. S5 in the Supplement). The PWC analysis indicates that elevation, wetness index, sand (not shown), clay, and BD had little influences on SWC after eliminating the effect of slope in summer (Fig. 7). This is largely because slope was significantly correlated to BD at medium scales and to elevation, wetness index, sand, and clay at large scales (Fig. S6 of Sect. S5 in the Supplement). However, the influence of slope on SWC was also limited after eliminating the effect of SOC (Fig. 7). By contrast, the effect of SOC on SWC at the medium scales still existed at some locations after eliminating the effects of slope and aspect (Fig. 7). This highlights the dominant role of SOC as a surrogate of vegetation in driving evapo-transpiration loss at the slope (medium) scales (Hu et al., 2017). As we expect, the effect of SOC on SWC at the medium scales disappeared after eliminating the effect of BD because of the strong correlations between SOC and BD (Fig. S5 of Sect. S5 in the Supplement). However, the effect of SOC on SWC was amplified at large scales (>72 m) after excluding the effect of BD as also found in the artificial datasets (Fig. 7). Interestingly, the significant correlation between SWC and aspect at large scales (>90 m) persisted regardless the excluding variables (as an example, only PWC for excluding variable of SOC is shown in Fig. 7). This highlights the dominant role of aspect in driving soil water distribution at large scales in summer. Overall, the PWC analysis further confirms that a combination of aspect and SOC was the best to explain SWC variations in summer (Hu et al., 2017).~~

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~~Figure 7.~~

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5. ~~Partial wavelet coherency (PWC) between soil water content (SWC) in summer and one environmental factor by excluding another environmental factor. SOC, soil organic carbon; Aspect, Cos(Aspect); WI, wetness index; BD, bulk density.~~Discussion on the advantages and weaknesses of the new method

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5.1 Advantages

We extend the partial coherence method from the frequency (scale) domain (Koopmans, 1995) to the time-frequency (location-scale) domain. The new method is an extension of previous work on PWC and MWC (Mihanović et al., 2009; Hu and Si, 2016). The method test and application have verified that it has the advantage of dealing with more than one excluding variable and providing the phase information associated with the PWC. In the case of one excluding variable, Mihanović et al. (2009) has suggested to calculate PWC by using an equation analogous to the traditional partial correlation squared (Eq. 14), which can be derived from our Eq. (9). However, their equation was, unfortunately, widely used by replacing the complex coherence in Eq. (14) with real coherence as expressed in Eq. (15)Ng and Chan (2012a); (Ng and Chan, 2012b; Rathinasamy et al., 2017; Aloui et al., 2018; Altarturi et al., 2018b; Jia et al., 2018; Li et al., 2018; Mutascu and Sokic, 2020; Wu et al., 2020).

The differences between the new method (Eq.14) and the classical method (Eq. 15) are compared using both the artificial and real datasets. Except for the phase information, the two methods generally produce comparable coherence for the artificial dataset for the case of one excluding variable (Fig. S5 of Sect. S3 in the Supplement). However, the new PWC

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method produces consistently and slightly higher coherence than the classical method. For example, their mean PWCs between y and y_2 at the scale of 8 after excluding the effect of y_4 are 1.00 and 0.97, respectively. This indicates that the new method produces coherence between y and y_2 at the scale (8) of y_2 closer to 1 as we expect. While the classical method produces similar PWC between E and other meteorological factors in most cases especially for the coherence between E and T after excluding the effects of others (Fig. S6 of Sect. S3 in the Supplement), large differences between these two methods can also be observed. For example, while the new method recognizes the strong coherence between E and RH after excluding the effect of T at scales of around 1 year (Fig. 3d), this coherence was negligible by the classical method (Fig. 5a). Mean PWC values by the new method were consistently higher than the classical method, and the differences ranged from 0.4 to 0.6 around the scale of 1 year (Fig. 5b). Considering the real coherence (Eq.15) rather than complex coherence (Eq.14) between every two variables in the numerators can potentially result in large underestimation of the partial wavelet coherence. Therefore, the ability of the new method to produce more accurate results than the classical method is one of its advantages.

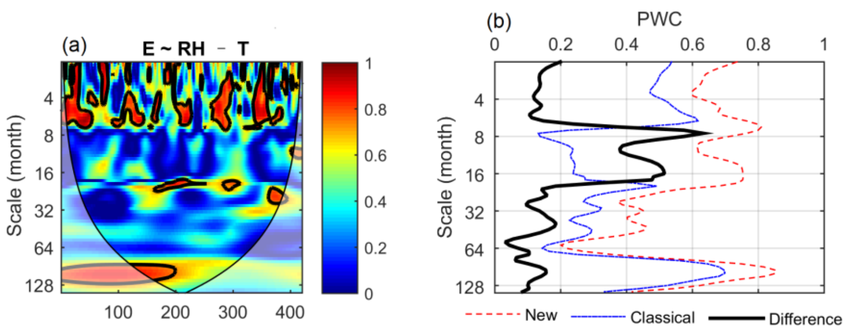

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Figure 5.

Partial wavelet coherency (PWC) between evaporation (E) and relative humidity (RH) after excluding the effect of mean temperature (T) using the classical method (Eq. 15) (a) and differences in PWC between the new method (Eq.14) and classical method as a function of scale (b).

Compared with the Mihanović et al. (2009) method, the additional phase information from the new PWC is another advantage of this new method. This is because phase information is directly related to the type of correlation, i.e., in-phase and out-of-phase indicating positive and negative correlation, respectively. Different types of correlations were usually found at different locations and scales (Hu et al., 2017). The phase information helps understand the differences in associated mechanisms or processes at different locations and scales. In addition, the phase information will allow us to detect the changes in not only the degree of correlation (i.e., coherence) but also the type of correlation after excluding the effect of other variables. For example, E and RH were positively correlated at the 1-year cycle (8–16 months) from year 1979 to 1995. This is because higher evaporation usually occurs in summer when high T coincides with high RH as influenced by the monsoon climate in the study area (Fig. S4 of Sect. S3 in the Supplement). Interestingly, after excluding the effect of T, E was negatively correlated with RH at the scale of 1-year as we expect (Fig. 3d).

Moreover, our new PWC method applies to cases with more than one excluding variable, which is a knowledge gap. When multiple variables are correlated with both the predictor

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and response variables, the correlations between predictor and response variables may be misleading if the effects of all these multiple variable were not removed. For example, at the dominant scale (i.e., 1-year) of E variation, the effects of RH on E existed after excluding the effects of T or SH. However, their contrasting correlations (Fig. 3d-e) resulted in negligible effects of RH on E at this scale after the effects of all other variables were excluded (Fig. 4b). In this case, the dominant role of mean temperature in driving free water evaporation was proved at the 1-year cycle (Fig. 4a). This also further verifies the suitability of the Hargreaves model (only air temperature and incident solar radiation required) (Hargreaves, 1989) for estimating potential evapotranspiration on the Chinese Loess Plateau (Li, 2012).

5.2 Weaknesses

Similar to the Mihanović et al. (2009) method, the new method has the risk to produce spurious high correlations after excluding the effect from other variables. Take the artificial dataset for example, at a scale of 32, PWC values between y and y_2 after excluding y_4 are not significant, but relatively high, partly because of small octaves per scale (octave refers to the scaled distance between two scales with one scale being twice or half of the other, default of 1/12). This spurious unexpected high PWC is caused by low values in both the numerator (partly associated with the low coherence between response y and predictor variables y_2 at scale of 32) and denominator (partly associated with the high coherence between response y and excluding variable y_4 at a scale of 32) in Eq. (9). The same problem also exists in the classical method (Fig. S5 of Sect. S3 in the Supplement). So, caution

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should be taken to interpret those results. However, it seems that the domain with spurious correlation calculated by the new method is very limited and it is located mainly outside of the cones of influence. Moreover, the unexpected results can be easily ruled out with knowledge of BWC between response and predictor variables. It is expected that the correlation between two variables should not increase after excluding one or more variables. Therefore, BWC analysis is suggested for better interpretation of the PWC results.

Similar to BWC and MWC, the confidence level of PWC calculated from the Monte Carlo simulation is based on a single hypothesis testing. But in reality, the confidence level of PWC values at all locations and scales needs to be tested simultaneously. Therefore, the significance test has the multiple-testing problem (Schaefli et al., 2007; Schulte et al., 2015). The new method may benefit from a better statistical significance testing method. Options for multiple-testing can be the Bonferroni adjusted *p* test (Westfall and Young, 1993) or false discovery rate (Abramovich and Benjamini, 1996; Shen et al., 2002) which is less stringent than the former.

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~~The new PWC method has been successfully tested with the artificial datasets. As we expect, regardless of the stationary and non-stationary case, there are no or reduced correlations between response and predictor variables in scale-location domains where the excluding variables are significantly correlated with the response variable. The new method also has the ability to deal with localized relationships. The new method was applied to two previously published datasets. The application has shown that the coherency between response and predictor variables was less affected by excluding other variables if the~~

~~predictor variable had dominating roles in explaining the variations in the response variable. This application further confirmed the best combinations for explaining temporal variations in free water evaporation at the Changwu site in China and spatial variations in soil water content in the hummocky landscape in Saskatchewan, Canada.~~

~~Like the Mihanović et al. (2009) method (a previous PWC method), the new method has the risk to produce spurious correlations after excluding the effect from other variables. But this spurious high coherence can be easily identified with knowledge of BWC. So, caution should be taken to interpret those results. Similar to BWC and MWC, the new PWC also suffers from the multiple-testing problem~~ (!!! INVALID CITATION !!! (Schaefli et al., 2007; Schulte et al., 2015))~~. Therefore, the new method can benefit from a better statistical significance testing method.~~

~~Our artificial datasets and two real-world datasets have verified that our PWC method provides an effective tool to untangle the bivariate relationships at multiple scale-location domains after eliminating the effects of other variables. The new method provides a much needed data-driven tool for unraveling underlining mechanisms in a spatial or temporal series. Thus, combining with wavelet transform, BWC, and MWC, the new PWC method can be used to detect various processes in geosciences, such as stream flow, droughts, greenhouse gas emissions (e.g., N_2O, CO_2, and CH_4), atmospheric circulation, and oceanic processes (e.g., El Niño-Southern Oscillation).~~

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~~5.~~6. Conclusions

Partial wavelet coherency (PWC) is developed in this study to investigate scale-specific and localized~~tion-specific~~ bivariate relationships after excluding the effect of one or more variables in geosciences. Method tests using stationary and non-stationary artificial datasets verified the known scale- and localized bivariate relationships after eliminating the effects of other variables. Compared with the previous PWC method, the new PWC method has the advantage of dealing with more than one excluding variable and providing the phase information (i.e., correlation type) associated with the PWC. In the case of one excluding variable, ~~This method was developed on the basis of partial coherence in the multivariate spectral case (Koopmans, 1995), and is an extension of previous work on PWC and WMC~~ (Mihanović et al., 2009; Hu and Si, 2016)~~. Compared with the previous PWC method (Mihanović et al., 2009),~~ this new method produces ~~slightly~~ more accurate coherence than the previous PWC method because the former considers complex coherence between every two variables, while the latter only considers the real coherence.~~.~~ Application of the new method to one temporal dataset (free water evaporation) has indicated the robustness of the new method in identifying the bivariate relationships and further convinced the MWC method in identifying the best combinations for explaining variations. The new method provides a much needed data-driven tool for unraveling underlying mechanisms in both temporal and spatial series. Thus, combining with wavelet transform, BWC, and MWC, the new PWC method can be used to detect various processes in geosciences, such as stream flow, droughts, greenhouse gas emissions (e.g., N_2O, CO_2, and CH_4), atmospheric circulation, and oceanic processes (e.g., EI Niño-Southern Oscillation).~~In addition, the new~~

~~PWC method has the advantage of dealing with more than one excluding variable and providing the phase information associated with the PWC.~~

~~The new PWC method has been successfully tested with the artificial datasets. As we expect, regardless of the stationary and non-stationary case, there are no or reduced correlations between response and predictor variables in scale-location domains where the excluding variables are significantly correlated with the response variable. The new method also has the ability to deal with localized relationships. The new method was applied to two previously published datasets. The application has shown that the coherency between response and predictor variables was less affected by excluding other variables if the predictor variable had dominating roles in explaining the variations in the response variable. This application further confirmed the best combinations for explaining temporal variations in free water evaporation at the Changwu site in China and spatial variations in soil water content in the hummocky landscape in Saskatchewan, Canada.~~

~~Like the Mihanović et al. (2009) method (a previous PWC method), the new method has the risk to produce spurious correlations after excluding the effect from other variables. But this spurious high coherence can be easily identified with knowledge of BWC. So, caution should be taken to interpret those results. Similar to BWC and MWC, the new PWC also suffers from the multiple-testing problem (Schaefli et al., 2007; Schulte et al., 2015). Therefore, the new method can benefit from a better statistical significance testing method.~~

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~~Our artificial datasets and two real-world datasets have verified that our PWC method provides an effective tool to untangle the bivariate relationships at multiple scale-location~~

~~domains after eliminating the effects of other variables. The new method provides a much needed data-driven tool for unraveling underlining mechanisms in a spatial or temporal series. Thus, combining with wavelet transform, BWC, and MWC, the new PWC method can be used to detect various processes in geosciences, such as stream flow, droughts, greenhouse gas emissions (e.g., N_2O, CO_2, and CH_4), atmospheric circulation, and oceanic processes (e.g., El Niño-Southern Oscillation).~~

Code/Data availability

The Matlab codes for calculating PWC, along with the updated MWC codes, are freely accessible (https://figshare.com/s/bc97956f43fe5734c784). The codes are developed based on those provided by Aslak Grinsted (http://www.glaciology.net/wavelet-coherence). The meteorological data sets can be obtained from the China Meteorological Administration.

Author contributions

WH wrote the paper, did the Matlab code development, and analyzed the data. Both authors conceived the study, interpreted the results, and revised the paper.

Competing interests

The authors declare that they have no conflict of interest.

Acknowledgements

~~The Matlab codes for calculating partial wavelet coherency are available in the Supplement (Sect. S1-S3). The codes are developed based on those provided by Aslak Grinsted~~

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~~(http://www.glaciology.net/wavelet-coherence) and Wei Hu and Bing Si (https://www.hydrol-earth-syst-sci.net/20/3183/2016/hess-20-3183-2016-supplement.pdf).~~ The preparation of this manuscript was ~~partly~~ supported by The New Zealand Institute for Plant and Food Research Limited under the Sustainable Agro-ecosystems programme.

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