Response to Anonymous Referee #3

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Comment #1:

In this paper, the authors presented an improved variant of PWC for identifying the relationship between variables. This should be reflected in the title (like Improved PWC etc to be included in the title) to convey novel contribution. Also at present it is misleading like the authors proposes PWC concept.

Response #1:
Many thanks for your comments. We will change the title to "Technical Note: Improved partial wavelet coherency for understanding scale-specific and localized bivariate relationships in geosciences".

Overall the paper is well written. I recommend for minor revision.

Comment #2:

Line 18– and producing more accurate results.- pl give quantitative statements

Response #2:
As the two methods in case of one excluding variables have theoretical differences, the outperformance is obvious. However, the degree of outperformance depends, in the case of our artificial dataset, the new method produces PWC values more close to 1 than the existing method as we expect although the difference is not big (e.g., PWC value of 1.0 versus 0.97 between y and y_2 at the scale of 8 after excluding the effect of y_4). However, the comparison of these two methods using real data indicated that the difference between the two methods can be big. For example, the differences in PWC between evaporation (E) and relative humidity (RH) after excluding the effect of mean temperature (T) can be 0.4-0.6 at the scales of about 1 year. For this reason, rather than giving quantitative statements, we would like to point out why the proposed method produces more accurate results by changing the sentence to "Compared with the previous PWC calculation, the new method produces more accurate results in case of one excluding variable because bivariate real coherence rather than the bivariate complex coherence was used in the previous PWC calculation."

Comment #3:
Line 31- provide the developments in chronological order – should be checked at all places What is the real advantage in bringing the phase information in practical cases? this should be mentioned in the introduction section

Response #3:
All citations will be changed in a chronological order.
The importance of phase information will be explained by adding "the types of correlation (i.e., positive or negative) especially at different locations and scales remains unclear without phase information."

Comment #4:

Line 109 .. sufficient number of times using : : :pl make it clear
Response #4:

As we also replied to RC#1, to address the "sufficient number" issue, different combinations of r1 (first-order autocorrelation coefficient) values (i.e., 0.0, 0.5, and 0.9) were used to generate 10 to 10 000 AR(1) series with three, four and five variables. Our results indicate that the noise combination has little impact on the PWC values at the 95% confidence level as also found by Grinsted et al. (2004) for the BWC case (data not shown). The relative difference of PWC at the 95% confidence level compared to that calculated from 10 000 AR(1) series decreases with increase in number of AR(1) series. When the number of AR(1) is above 300, very low maximum relative difference (e.g., <2%) is observed (Fig. RC1 which will be put in the Supplement as Fig. S1 of Sect. S3). Therefore, repeating number of 300 seems to be efficient for significance test. If calculation time is not a barrier, however, bigger repeating number such as ⩾1000 is recommended. This will be added into the revision.

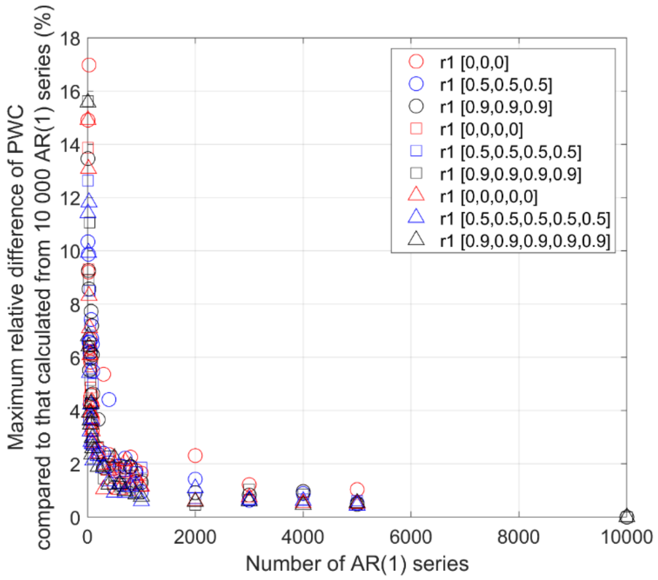

Figure RC1. Relationship between maximum relative difference (%) of PWC compared to that calculated from 10 000 AR(1) series (surrogate dataset) versus the number of AR(1) series during the significance test using the Monte Carlo test. Number of scales per octave is 12.

Comment #5:
Line 214- significance band
Response #5:

We will change it to significance band.

Comment #6:
Conclusion: Avoid the statements like – 'this new method produces slightly more accurate coherence'
Response #6:

As we explained in the Response #2, we will change it to "Compared with the previous PWC method, the new PWC method has the advantage of dealing with more than one excluding variable and providing the phase information associated with the PWC."

Comment #7:

Line 450-455 should be explained better ; how can you overcome such problems ? I think better to provide a discussion section before conclusion where such references and unfamiliar terms can be explained in a better way. Then conclusion section should be presented as more specific

Response #7:

New discussion section will be added by moving this part to the discussion section. In terms of spurious correlations and multiple-testing problem, we will put it to a new section 5.2 weaknesses. Meanwhile, the advantages will be mentioned in section 5.1.

Here will be the changes:

"5. Discussion on the advantages and weaknesses of the new method

5.1 Advantages

We extend the partial coherence method from the frequency (scale) domain (Koopmans, 1995) to the time-frequency (location-scale) domain. The new method is an extension of previous work on PWC and MWC (Mihanović et al., 2009; Hu and Si, 2016). Method test and application has verified that it has the advantage of dealing with more than one excluding variable and providing the phase information associated with the PWC. In case of one excluding variable, Mihanović et al. (2009) has suggested to calculate PWC by an equation analogous to the traditional partial correlation squared (Eq. 14), which can be derived from our Eq. (9). However, their equation was widely used by replacing the complex coherence in Eq. (14) with real coherence as expressed in Eq. (15).

The differences between the new method (Eq.14) and the classical method (Eq. 15) are compared using both the artificial and real datasets. Except for the phase information, the two methods generally produce comparable coherence for the artificial dataset for the case of one excluding variable (Fig. S5 of Sect. S3 in the Supplement). However, the new PWC method produces consistently slightly higher coherence than the classical method. For example, their mean PWCs between y and y_2 at the scale of 8 after excluding the effect of y_4 are 1.00 and 0.97, respectively. This indicates that the new method produces coherence between y and y_2 at the scale (8) of y_2 closer to 1 as we expect. While the classical method produces similar PWC between E and other meteorological factors in most cases especially for the coherence between E and T after excluding the effects of others (Fig. S6 of Sect. S3 in the Supplement), large differences between these two methods can also be observed. For example, while the new method recognizes the strong coherence between E and RH after excluding the effect of T at scales of around 1 year (Fig. 3d), this coherence was negligible by the classical method (Fig. 5a). Mean PWC values by the new method were consistently higher than the classical method, and the differences ranged from 0.4 to 0.6 around the scale of 1 year (Fig. 5b). Considering the real coherence (Eq.15) rather than complex coherence (Eq.14) between every two variables in the numerators can potentially result in large underestimation of the partial wavelet coherence. Therefore, the new method produces more accurate results than the classical method is one of the advantages.

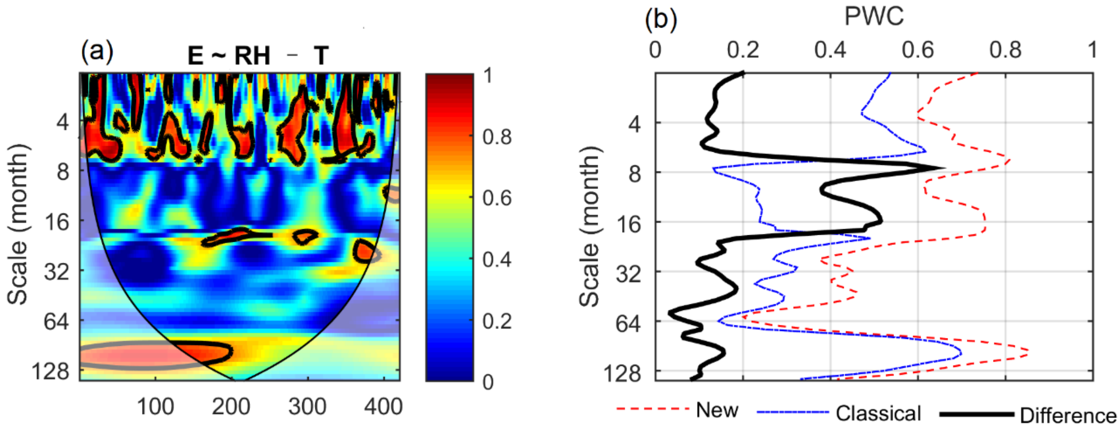

Figure 5.

Partial wavelet coherency (PWC) between evaporation (E) and relative humidity (RH) after excluding the effect of mean temperature (T) using the classical method (a) and differences in PWC between the new method and classical method as a function of scale (b).

Compared with the Mihanović et al. (2009) method, inclusion of phase information in the new PWC is another advantage of this method. This is because phase information is directly related to the type of correlation, i.e., in-phase and out-of-phase indicating positive and negative correlation, respectively. Different types of correlations were usually found at different locations and scales (Hu et al., 2017), the inclusion of phase information will be useful to understand the differences in associated mechanisms or processes at different locations and scales. In addition, the inclusion of phase information will allow us to detect the changes in not only degree of correlation (i.e., coherence) but also the type of correlation after excluding the effect of other variables. For example, E and RH were positively correlated at the 1-year cycle (8–16 months) from year 1979 to 1995 because higher evaporation usually occurs in summer when high T coincides with high RH as influenced by the monsoon climate in the area where data were collected (Fig. S4 of Sect. S3 in the Supplement). Interestingly, after excluding the effect of T, E was negatively correlated with RH at the scale of 1-year as we expect (Fig. 3d).

Moreover, our new PWC method can be used to deal with situations with more than one excluding variable, which is a knowledge gap. When multiple variables are correlated with both the predictor and responsible variables, the correlations between predictor and responsible variables may be misleading if the effects of all these multiple variable were not removed. For example, at the dominant scale (i.e., 1-year) of E variation, the effects of RH on E existed after excluding the effects of T or SH. However, their contrasting correlations (Fig. 3d-e) resulted in negligible effects of RH on E at this scale after the effects of all other variables were excluded (Fig. 4b). In this case, the dominant role of mean temperature in driving free water evaporation was proved at the 1-year cycle (Fig. 4a). This also further verifies the suitability of the Hargreaves model (only air temperature and incident solar radiation required) (Hargreaves, 1989) for estimating potential evapotranspiration on the Chinese Loess Plateau (Li, 2012).

5.2 Weaknesses

Similar to the Mihanović et al. (2009) method, the new method has the risk to produce spurious high correlations after excluding the effect from other variables. Take the artificial dataset for example, at a scale of 32, PWC values between y and y_2 after excluding y_4 are not significant, but relatively high, partly because of small octaves per scale (octave refers to the scaled distance between two scales with one scale being twice or half of the other, default of 1/12). This spurious unexpected high PWC is caused by low values in both the numerator (partly associated with the low coherence between response y and predictor variables y_2 at scale of 32) and denominator (partly associated with the high coherence between response y and excluding variable y_4 at a scale of 32) in Eq. (9). The same problem also exists in the Mihanović et al. (2009) method (Fig. S5 of Sect. S3 in the Supplement). So, caution should be taken to interpret those results. However, it seems that the domain with spurious correlation calculated by the new method is very limited and it is located mainly outside of the cones of influence. Anyway, the unexpected results can be easily ruled out with knowledge of BWC between response and predictor variables. We would expect that the correlation between two variables should not be increased after the effects of excluding variables are removed. Therefore, BWC analysis is suggested for better interpretation of the PWC results.

Similar to BWC and MWC, the confidence level of PWC calculated from the Monte Carlo is based on a single hypothesis which is tested one by one. But in reality, confidence level of PWC values at all locations and scales needs to be tested simultaneously. Therefore, the significance test suffers from the multiple-testing problem (Schaefli et al., 2007; Schulte et al., 2015). The new method may benefit from a better statistical significance testing method. Options for multiple-testing can be the Bonferroni adjusted p test (Westfall and Young, 1993) or false discovery rate (Abramovich and Benjamini, 1996; Shen et al., 2002) which is less stringent than the former. “

The conclusion section will be changed to " Partial wavelet coherency (PWC) is developed in this study to investigate scale-and location-specific bivariate relationships after excluding the effect of one or more variables in geosciences. Method tests using stationary and non-stationary artificial datasets verified the known scale- and localized bivariate relationships after eliminating the effects of other variables. Compared with the previous PWC method, the new PWC method has the advantage of dealing with more than one excluding variable and providing the phase information associated with the PWC. In case of one excluding variable, this new method produces more accurate coherence than the previous PWC method because the former considers complex coherence between every two variables while only real coherence is considered in the latter. Application of the new method to one temporal dataset (free water evaporation) has indicated the robustness of the new method in identifying the bivariate relationships and further convinced the MWC method in identifying the best combinations for explaining variations. The new method provides a much needed data-driven tool for unraveling underlining mechanisms in both temporal and spatial series. Thus, combining with wavelet transform, BWC, and MWC, the new PWC method can be used to detect various processes in geosciences, such as stream flow, droughts, greenhouse gas emissions (e.g., N2O, CO2, and CH4), atmospheric circulation, and oceanic processes (e.g., EI Niño-Southern Oscillation).".

Thanks again for your constructive comment.